FRT105F Optimal Control
Course Syllabus 2014

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The latest news related to the course and sets of home assignments will be posted on the course web page.

Purpose

Optimal control is the problem of determining the control function for a dynamical system to minimize a cost related to the system trajectory. The subject has its roots in the calculus of variations but it evolved to an independent branch of applied mathematics and engineering in the 1950s.

The overall goal of the course is to provide an understanding of the main results in calculus of variations and optimal control, and how these results can be used in various applications such as in robotics, finance, economics, and biology.

Main content

The Euler-Lagrange equation. Pontryagin's maximum principle. Dynamic programming. The Hamilton-Jacobi-Bellman equation. Finite and infinite horizon optimal control problems. Viscosity solutions to partial differential equations. The Linear quadratic regulator. Necessary and sufficient conditions for optimality. Numerical methods for optimal control problems.

Objectives

The course aims at providing a fundamental understanding of the main results in optimal control, and how these results can be proved.

At the end of the course the student should be able to:

- use calculus of variations to solve infinite dimensional optimization problems,
- adapt the steps leading up to the Euler-Lagrange equation to account for different boundary conditions,

- explain the main steps in the proof of Pontryagin's Maximum Principle (PMP),
- use PMP to solve optimal control problems,
- explain the bang-bang principle in optimal control,
- use Dynamic Programming (DynP) to solve optimal control problems,
- understand the notion of viscosity solutions to the Hamilton-Jacobi-Bellman equation,
- solve Linear Quadratic Regulator (LQR) problems,
- use computer software to obtain numerical solutions to optimal control problems.

Course Literature

The following book is highly recommended for this course: Daniel Liberzon, *Calculus of Variations and Optimal Control: A Concise Introduction*.

Home Assignments

Throughout the course the students will have to solve a number of home assignments in order to pass. The solutions are presented at home assignment seminars where, for each problem, one of the students is chosen at random to present his/her solution on the whiteboard.

The home assignments are a mix of the more theoretical assignments from the recommended course literature and more practical assignments. Each set of home assignments will be posted on the course web page at the very least one week before the corresponding seminar.

Preliminary schedule for the seminars:

- Seminar 1: *Friday 28 March, 13:15-15:00*
- Seminar 2: *Friday 4 April, 13:15-15:00*
- Seminar 3: *Wednesday 23 April, 14:15-16:00*
- Seminar 4: *Friday 25 April, 13:15-15:00*
- Seminar 5: *Monday 5 May, 13:15-15:00* (Lab F, entrance floor)
- Seminar 6: *Friday 9 May, 13:15-15:00*

In the last home assignment the students will be asked to solve an optimal control problem numerically. This assignment will not be presented in a seminar, instead a report describing the work should be sent to Magnus before the student can pass the course.

Project

At the end of the course a three week period is scheduled where the students will work on a project assignment. In the project assignment the students will be given the opportunity to focus on a topic in optimal control of their own choice. This may, for example, be one of the topics of Chapter 7 in the course literature, or a topic related to their own research project.

For this assignment the students are allowed to work in groups of two. To pass the course the students should participate in a seminar where each group is given around 30 minutes to present their project-work to the rest of the class.

To make sure that two groups does not chose the same topic, one of the group members should send an email to magnus.perninge@control.lth.se with the names of the students in the group and the topic that the group has chosen.

The project assignments are presented in the

- Project seminar: *Tuesday 10 June 9:15-12:00*

Lectures

Preliminary schedule:

- Lecture 1: *Monday 24 March, 10:15-12:00*, Introduction, Calculus of Variations. Chapters 1-2 in Liberzon.
- Lecture 2: *Monday 31 March, 10:15-12:00*, Calculus of Variations. Chapters 2-3 in Liberzon.
- Lecture 3: *Monday 14 April, 10:15-12:00*, Pontryagin's Maximum Principle (PMP). Chapter 4 in Liberzon.
- Lecture 4: *Tuesday 15 April, 10:15-12:00*, Pontryagin's Maximum Principle (PMP). Chapter 4 in Liberzon.
- Lecture 5: *Monday 28 April, 10:15-12:00*, Dynamic programming (DynP). Chapter 5 in Liberzon.
- Lecture 6: *Monday 5 May, 10:15-12:00*, The Linear Quadratic Regulator (LQR). Chapter 6 in Liberzon.
- Lecture 7: *Monday 12 May, 9:15-12:00*, Numerical methods for optimal control problems.