HOME ASSIGNMENT 6

Problem 1. Consider the DC-servo in Fig. 1. A mathematical model for the relation between the voltage, u, and the angle of the shaft, θ, is

$$\ddot{\theta}(t) + \dot{\theta}(t) = u(t),$$

with $\theta(0) = 0$ and $\dot{\theta}(0) = 1$. We want to design a voltage signal such that the cost functional

$$J(u) = \int_0^\infty \left(\theta^2(t) + 6\dot{\theta}^2(t) + \frac{1}{2}\theta(t)\dot{\theta}(t) + u^2(t) \right) dt$$

is minimized. Solve the problem by finding the optimal voltage input $u^*(t)$ and determine the closed loop poles of the system.

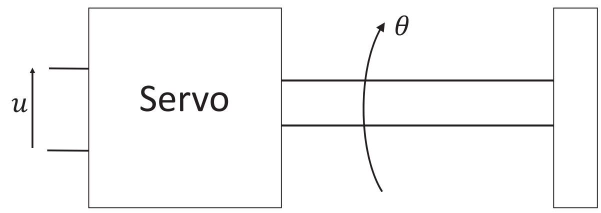

FIGURE 1. The DC-servo in Problem 1.

Problem 2. Consider the optimal control problem

$$\min_u \int_0^\infty (x^\top(t)Qx(t) + u^2(t))dt \quad \text{subj. to} \quad \dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0.$$

where

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & q^2 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and q is an arbitrary positive number. The observability and controllability criteria can be assumed to hold.

a) Determine the optimal feedback control and the optimal cost.

b) Determine the closed loop dynamics matrix. Consider the closed loop system, what does the elements of $x(t)$ converge to and how is this convergence affected when q tends to zero.