HOME ASSIGNMENT 5

Problem 1. A dog is chasing a hare. The dog starts at position $(0, y)$ and the hare starts at $(x, 0)$. The hare is running with speed v_h along the positive x-axis and the dog is running with speed $v_d (> v_h)$ and always in the direction pointing towards the hare. Derive the boundary value problem that is solved by the time $f(x, y)$ that the dog needs to catch the hare.

Problem 2. Prove the principle of optimality.

Problem 3. Show that the value function is a viscosity *super*solution to the HJB equation.

Problem 4. Consider the nonlinear optimal control problem

$$\min_u \; x(1)^2 + \int_0^1 (x(t)u(t))^2 dt \quad \text{subj. to} \quad \dot{x}(t) = u(t)x(t), \quad x(0) = 1.$$

Solve the problem using dynamic programming by making the ansatz $V(t, x) = q(t)x^2$.