Feedback Fundamentals

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A Perspective on Control

- Servomechanism theory 1945
  - Drivers: gun control, radar, ...
  - A holistic view: theory, simulation and implementation
  - Block diagrams, Transfer functions, analog computing
- The second wave 1965
  - Drivers: space race, digital control, mathematics
  - Subspecialities: linear, nonlinear, optimal, stochastic, ...
  - Design methods: state feedback, Kalman filter, H_∞-control
  - Computational tools emerged
  - Impressive theory development but the holistic view was lost
- The third wave 2005
  - Embedded systems, control over/of communication networks, (systems biology)
  - Recover the holistic view

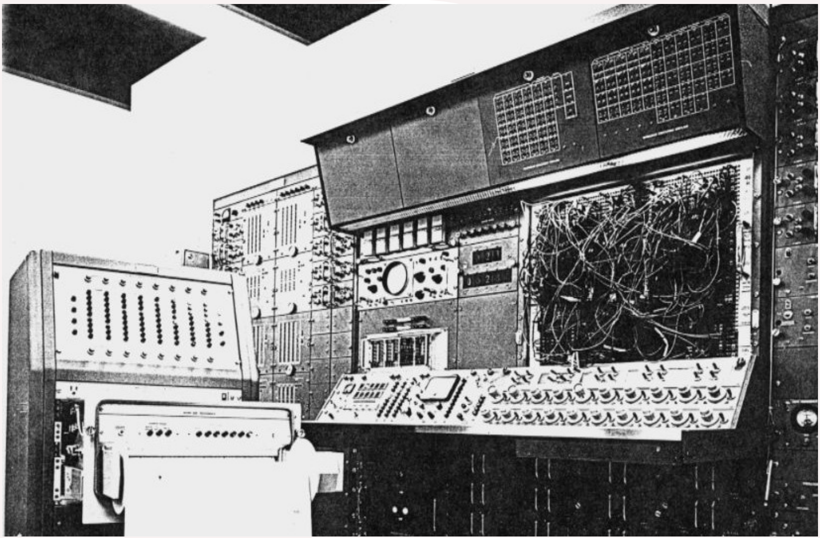
Control, Computing and Communication

- Bode, Nyquist and Shannon 1945
- Close connections during the analog era
- Essential to get systems engineers with a broad view and a deep specialization
- Generic knowledge: control, computing, communication
- Specific knowledge: process, sensing and actuation
- Practical skills: implementation, commisioning, operation
- Essential to compactify current knowledge for different users
- The Bologna process

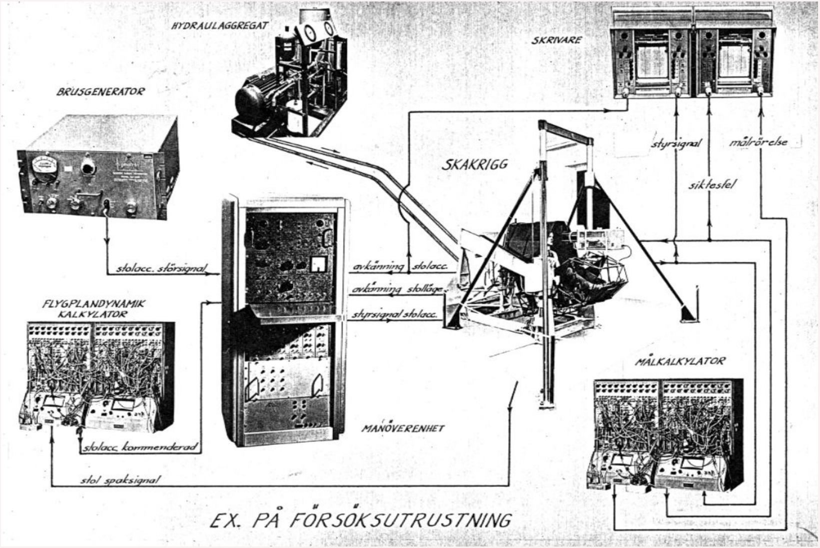
The Role of Computing

- Vannevar Bush 1927. *Engineering can proceed no faster than the mathematical analysis on which it is based. Formal mathematics is frequenly inadequate for numerous problems, a mechanical solution offers the most promise.*
- Herman Goldstine 1962: *When things change by two orders of magnitude it is revolution not evolution.*
- Gordon Moore 1965: *The number of transistors per square inch on integrated circuits has doubled approximately every 18 months.*
- Moore+Goldstine: *A revolution every 15 year!*

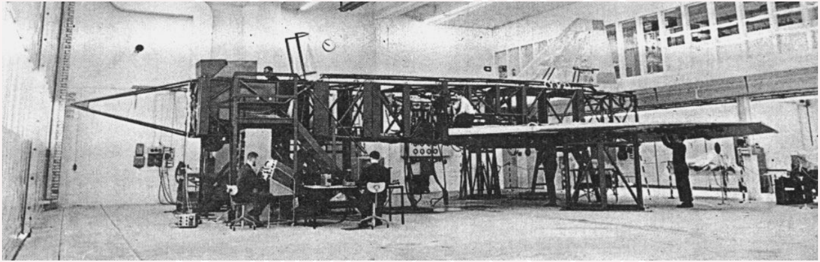
Analog Computing EAI 231



Hardware in the Loop Simulation



The Iron Bird



Feedback Fundamentals

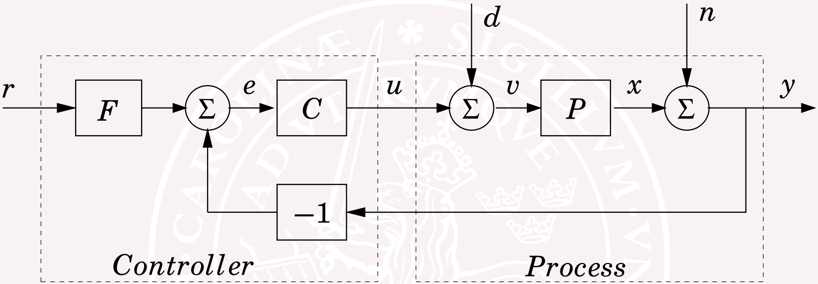
1. Introduction
2. Controllers with Two Degrees of Freedom
3. The Gangs of Four and Six
4. The Sensitivity Functions
5. Consequences for Design
6. Fundamental Limitations
7. PID Control
8. Summary

Themes: Understanding the basic feedback loop. Systems with two degrees of freedom. The gangs of four and six. Sensitivity functions. Fundamental limitations.

Introduction

- A basic feedback system
- Effects of
  - Load disturbances
  - Measurement noise
  - Process variations
  - Command signals
- How to capture a complex reality in tractable mathematics
- Assessment of the properties of a control system
- Concepts and insights
- A basis for analysis, specification and design
- Insight into fundamental limitations

A Basic Control System

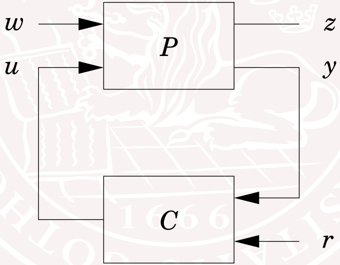


Ingredients:

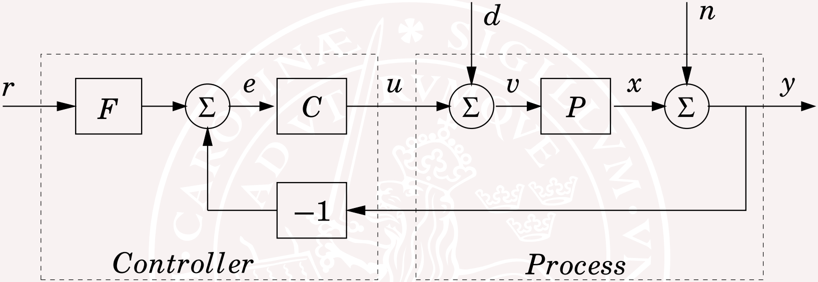
- Controller: feedback C, feedforward F
- Load disturbance d : Drives the system from desired state
- Measurement noise n : Corrupts information about x
- Process variable x should follow reference r

A Remark on Load Disturbances

Load disturbances are assumed to enter at the process input and measurement noise at the process output. The same idea can be applied to other configurations. A general structure is given below.



Criteria for Control Design

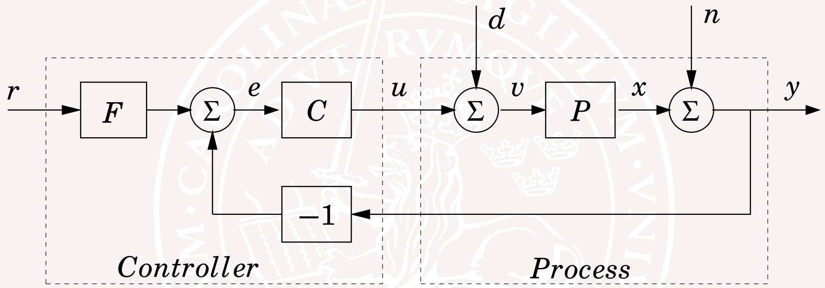


Ingredients

- Attenuate effects of load disturbance d
- Do not feed in too much measurement noise n
- Make the system insensitive to process variations
- Make state x follow command r

Feedback Fundamentals

System with Two Degrees of Freedom



The controller has two degrees of freedom 2DOF because the signal transmissions from reference r to control u and from measurement y to u are different. Horowitz 1963.

A Separation Principle for 2DOF Systems

Design the feedback C to achieve

- Low sensitivity to load disturbances d
- Low injection of measurement noise n
- High robustness to process variations

Then design the feedforward F to achieve the desired response to command signals r

Notice

- Many books and papers show only the set point response
- Interactive learning modules

Process Control

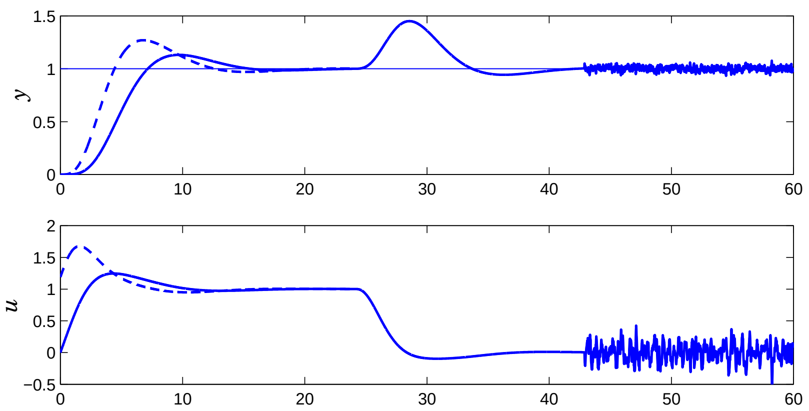
The tuning debate: Should controllers be tuned for set-point response or for load disturbance response?

- Different tuning rules for PID controllers
- Shinskey: Set-point disturbances are less common than load changes.
- Resolved by set-point weighting (poor mans 2DOF)

$$u(t) = k\big(\beta r(t) - y(t)\big) + k_i \int_0^t \big(r(\tau) - y(\tau)\big)d\tau + k_d\Big(\gamma \frac{dr}{dt} - \frac{dy_f}{dt}\Big)$$

- Tune k, k_i, and k_d for load disturbances, filtering for measurement noise and β, and γ for set-points

PID Control with Set-Point Weighting



Interactive Learning Modules

Learning is better than teaching because it is more intense: the more is being taught, the less can be learned.

Josef Albers 1888-1976

Demonstrate Interactive Learning Module

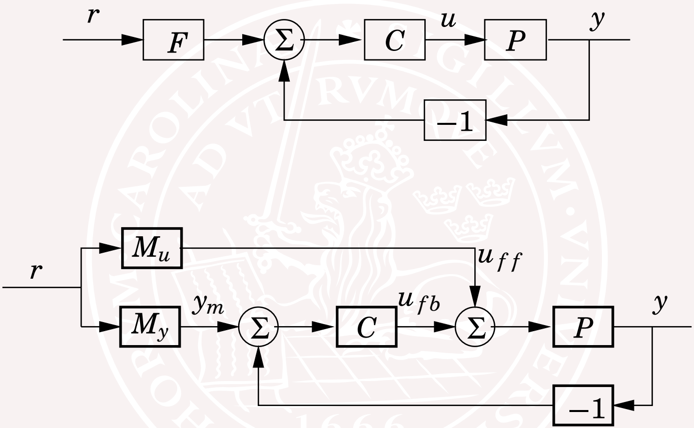
Designing Systems with 2DOF

Design procedure:

- Design the feedback C to achieve
  - Small sensitivity to load disturbances d
  - Low injection of measurement noise n
  - High robustness to process variations
- Then design the feedforward F to achieve desired response to command signals r

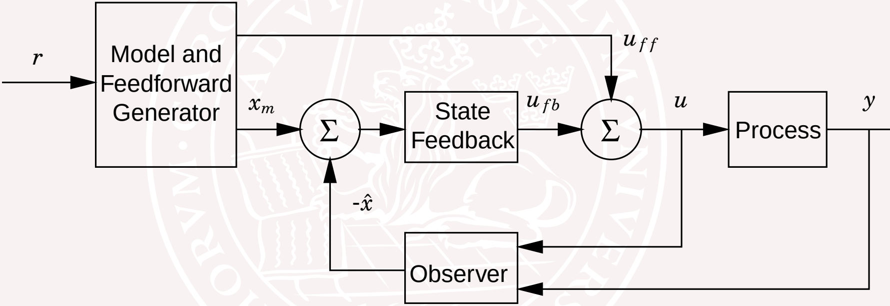
For many problems in process control the load disturbance response is much more important than the set point response. The set point response is more important in motion control. Few textbooks and papers show more than set point responses.

Many Versions of 2DOF



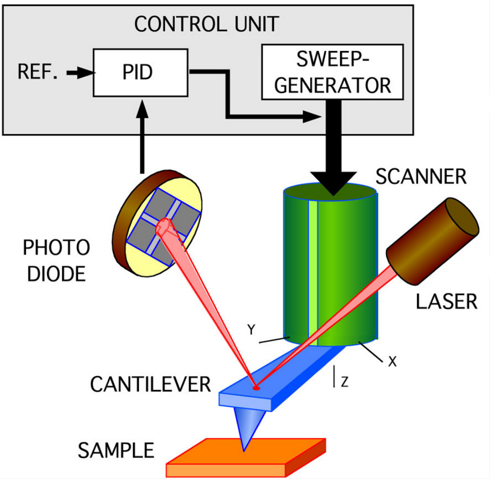
For linear systems all 2DOF configurations have the same properties. For the systems above we have $CF = M_u + CM_y$

A More General Structure



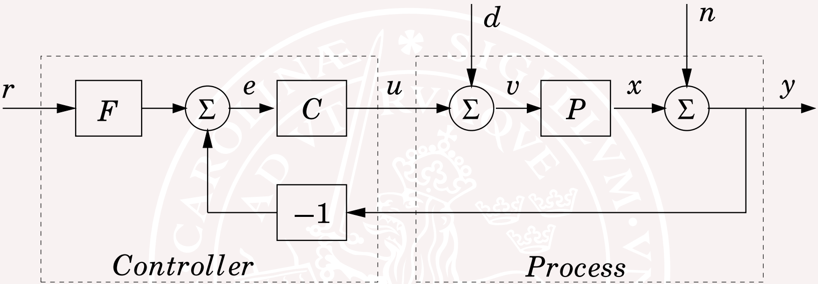
Some Systems only Allow Error Feedback

There are systems where only the error is measured, and the controller then has to be restricted to error feedback.



Feedback Fundamentals

The Gangs of Four and Six



$$X = \frac{P}{1+PC}D - \frac{PC}{1+PC}N + \frac{PCF}{1+PC}R$$

$$Y = \frac{P}{1+PC}D + \frac{1}{1+PC}N + \frac{PCF}{1+PC}R$$

$$U = -\frac{PC}{1+PC}D - \frac{C}{1+PC}N + \frac{CF}{1+PC}R$$

Some Observations

- To fully understand a system it is necessary to look at all transfer functions
- A system based on error feedback is characterized by *four* transfer functions *The Gang of Four*
- The system with a controller having two degrees of freedom is characterized by *six* transfer function *The Gang of Six*
- It may be strongly misleading to only show properties of a few systems for example the response of the output to command signals. This is a common omission in papers and books.
- The properties of the different transfer functions can be illustrated by their transient or frequency responses.

A Possible Choice

Six transfer functions are required to show the properties of a basic feedback loop. Four characterize the response to load disturbances and measurement noise, compare $\mathcal{H}_\infty$-theory.

$$\begin{array}{cc} \dfrac{PC}{1+PC} & \dfrac{P}{1+PC} \\ \dfrac{C}{1+PC} & \dfrac{1}{1+PC} \end{array}$$

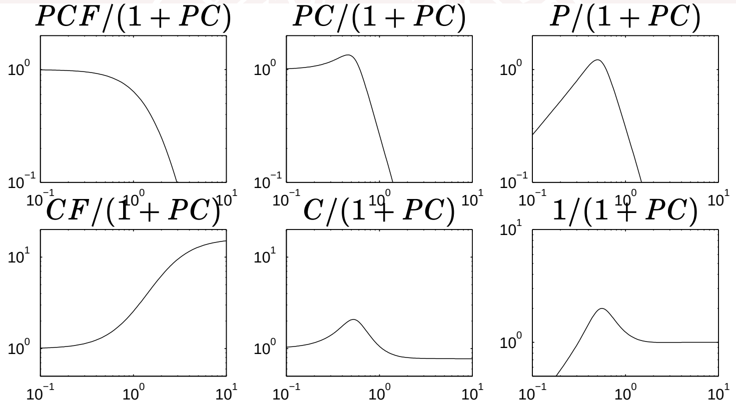
Two more are required to describe the response to set point changes.

$$\frac{PCF}{1+PC} \qquad \frac{CF}{1+PC}$$

Physical interpretations!

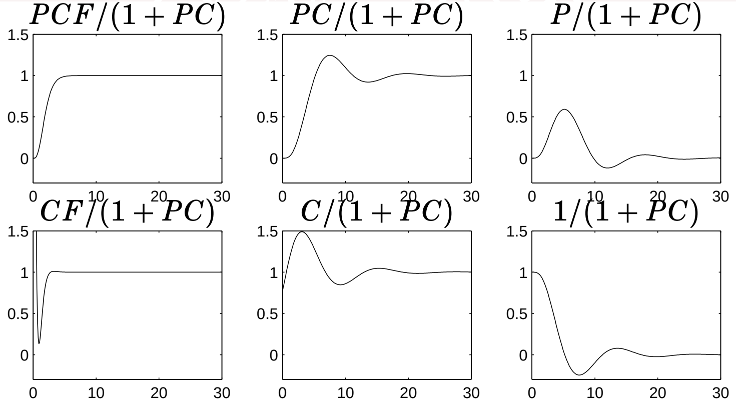
Amplitude Curves of Frequency Responses

PI control $k = 0.775$, $T_i = 2.05$ of $P(s) = (s+1)^{-4}$ with $M(s) = (0.5s+1)^{-4}$



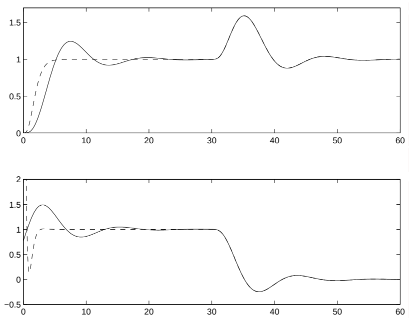
Step Responses

PI control $k = 0.775$, $T_i = 2.05$ of $P(s) = (s+1)^{-4}$ with $M(s) = (0.5s+1)^{-4}$



An Alternative

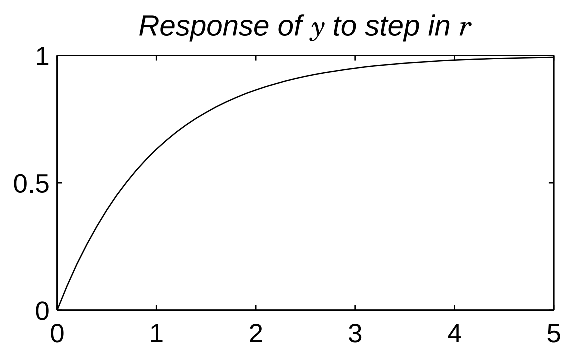
Show the responses in the output and the control signal to a step change in the reference signal for system with pure error feedback and with feedforward. Keep the reference signal constant and make a unit step in the process input. Show the response of the output and the control signal.



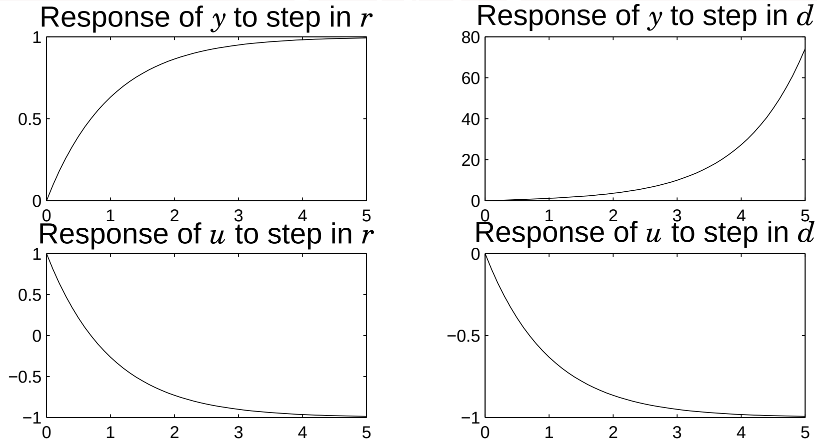
Interactive Learning Modules!

A Warning!

Remember to always look at all responses when you are dealing with control systems. The step response below looks fine but ...



Four Responses



What is going on?

The System

Process $P(s) = \dfrac{1}{s-1}$

Controller $C(s) = \dfrac{s-1}{s}$

The system has error feedback sufficient to consider The Gang of Four

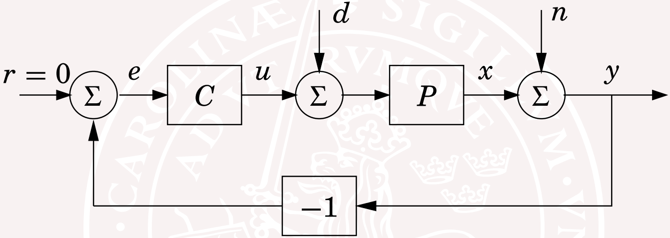
$$\frac{PC}{1+PC} = \frac{1}{s+1} \qquad \frac{P}{1+PC} = \frac{s}{(s+1)(s-1)}$$

$$\frac{C}{1+PC} = \frac{s-1}{s+1} \qquad \frac{1}{1+PC} = \frac{1}{s+1}$$

Response of y to step in disturbance d

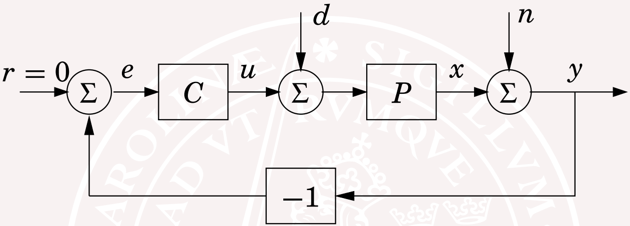
$$\frac{Y(s)}{D(s)} = \frac{P}{1+PC} = \frac{s}{(s+1)(s-1)}$$

Focus on Feedback



- Neglect following of reference signals (the feedforward problem) and focus on on the feedback problem, i.e.
  - Load disturbances
  - Measurement noise
  - Model uncertainty

The Feedback Problem

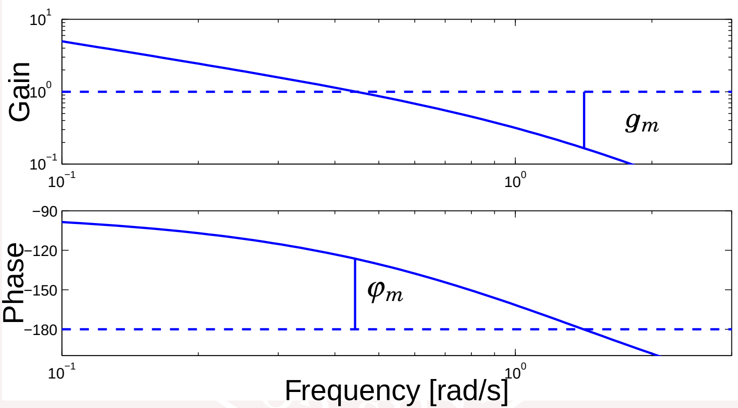


The signals have the following relations. Notice that there are only four transfer functions - The Gang of Four.

$$
\begin{aligned}
X &= \frac{P}{1+PC}D - \frac{PC}{1+PC}N \\
Y &= \frac{P}{1+PC}D + \frac{1}{1+PC}N \\
U &= -\frac{PC}{1+PC}D - \frac{C}{1+PC}N
\end{aligned}
$$

The Loop Transfer Function $L(s) = P(s)C(s)$

Tells a lot about the system, quantitative measures phase margin and gain margin



But it only tells about $1/(1+PC)$, and $PC/(1+PC)$ but not $P/(1+PC)$ and $C/(1+PC)$

The Gangs of Four and Six

Response of y to load disturbance d is characterized by

$$G_{yd} = \frac{P}{1+PC}$$

Response of u to measurement noise n is characterized by

$$-G_{un} = \frac{C}{1+PC}$$

Robustness to process variations is characterized by

$$S = \frac{1}{1+PC}, \quad T = \frac{PC}{1+PC}$$

Responses of y and u to reference signal r is characterized by

$$G_{yr} = \frac{PCF}{1+PC}, \quad G_{ur} = \frac{CF}{1+PC}$$

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The Sensitivity Functions

The transfer functions

- Sensitivity function $S = \dfrac{1}{1+PC} = \dfrac{1}{1+L}$
- Complementary sensitivity function $T = \dfrac{PC}{1+PC} = \dfrac{L}{1+L}$

are called sensitivity functions. They have interesting properties and useful physical interpretations. We have

- The functions S and T only depend on the loop transfer function L
- $S + T = 1$
- Typically $S(0)$ small and $S(\infty) = 1$ and consequently $T(0) = 1$ and $T(\infty)$ small

Poles, Zeros and Sensitivity Functions

The sensitivity functions depend only on the loop transfer function

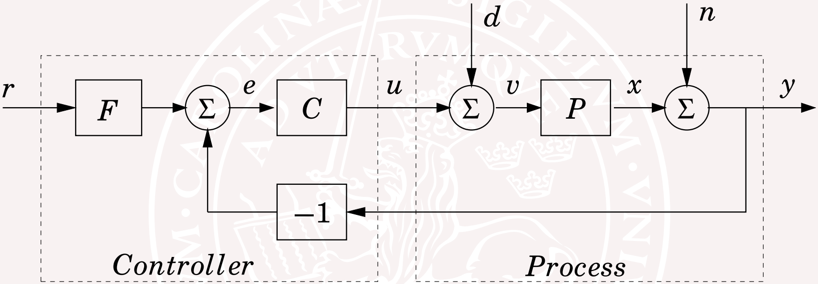
$$S = \frac{1}{1+L}, \quad T = \frac{L}{1+L}$$

Notice that

- The sensitivity function S is zero and the complementary sensitivity function is one at the poles of L
- The sensitivity function S is one and complementary sensitivity function T is zero at the zeros of L

Quiz

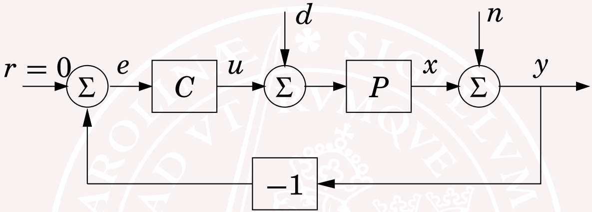
Look at the block diagram



Find all relations where the signal transmissions are equal to either the sensitivity function or the complementary sensitivity function

The Audience is Thinking ...

Disturbance Reduction



Output without control $Y = Y_{ol}(s) = N(s) + P(s)D(s)$

Output with feedback control

$$Y_{cl} = \frac{1}{1+PC}(N + PL) = \frac{1}{1+PC} Y_{ol} = SY_{ol}$$

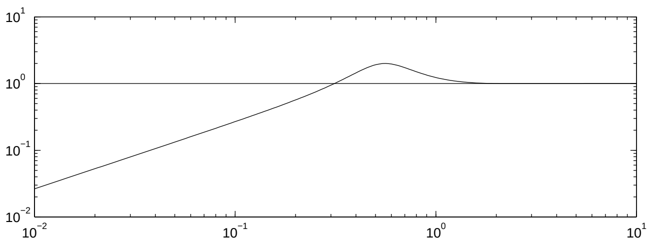
Disturbances with frequencies such that $|S(i\omega)| < 1$ are reduced by feedback, disturbances with frequencies such that $|S(i\omega)| > 1$ are amplified by feedback.

Assessment of Disturbance Reduction

We have

$$\frac{Y_{cl}(s)}{Y_{cl}(s)} = S(s) = \frac{1}{1+P(s)C(s)}$$

Feedback attenuates disturbances of frequencies ω such that $|S(i\omega)| < 1$. It amplifies disturbances of frequencies such that $|S(i\omega)| > 1$

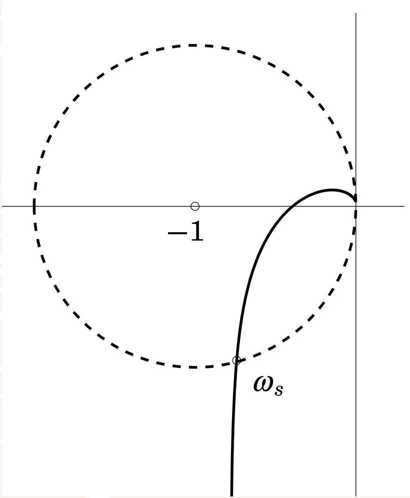


Assessment of Disturbance Reduction

$$\frac{Y_{cl}}{Y_{ol}} = \frac{1}{1+PC} = S$$

Geometric interpretation: Disturbances with frequencies inside the circle are amplified by feedback. Disturbances with frequencies outside are reduced.

Disturbances with frequencies less than ω_s are reduced by feedback.



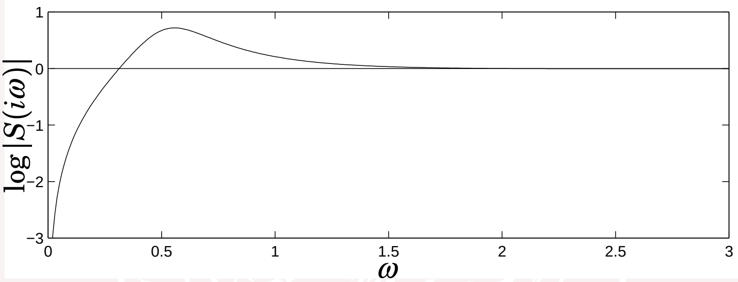
Properties of the Sensitivity function

- Can the sensitivity be small for all frequencies?
  - No we have $S(\infty) = 1$!
- Can we get $|S(i\omega)| \le 1$?
  - If the Nyquist curve of $L = PC$ is in the first and third quadrant! Passive systems!
- Bode's integral, p_k RHP poles of $L(s)$

$$\int_0^\infty \log|S(i\omega)|d\omega = \pi \sum \mathrm{Re}\, p_k - \frac{\pi}{2} \lim_{s\to\infty} sL(s)$$

- The "water-bed effect". Push the curve down at one frequency and it pops up at another!

The Water Bed Effect



$$\int_0^\infty \log|S(i\omega)|d\omega = \pi \sum \mathrm{Re}\; p_k - \frac{\pi}{2} \lim_{s\to\infty} sL(s)$$

The sensitivity can be decreased at one frequency at the cost of increase at another frequency.

Robustness

Effect of small process changes on $T = PC/(1+PC)$

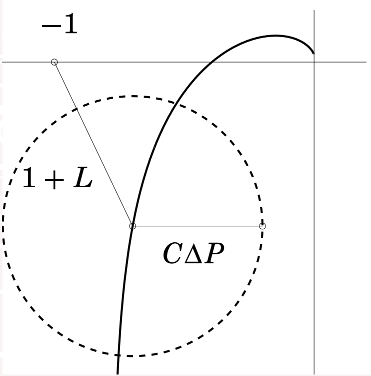
$$\frac{dT}{dP} = \frac{dP}{P} - \frac{CdP}{1+PC} = \frac{1}{1+PC}\frac{dP}{P} = S\frac{dP}{P}$$

How much can the process be changes without making the system unstable?

$$|C\Delta P| < |1+PC|$$

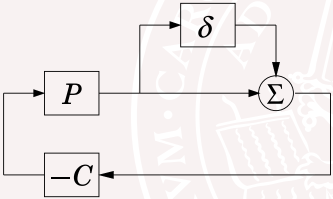
or

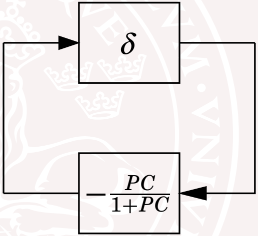
$$\frac{|\Delta P|}{|P|} < \frac{1}{|T|}$$



Another View of Robustness

A feedback system where the process has multiplicative uncertainty, i.e. $P(1+\delta)$, where δ is the relative error, can be represented with the following block diagrams





The small gain theorem gives the stability condition

$$|\delta P| < \left|\frac{1+PC}{PC}\right| = \frac{1}{|T|}$$

When are Two Systems Close

- For stable systems

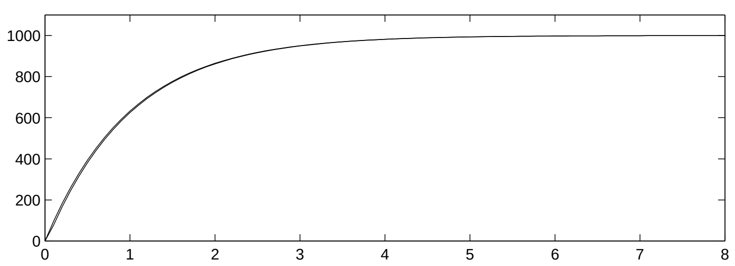
$$\delta(P_1, P_2) = \max_{\omega} |P_1(i\omega) - P_2(i\omega)|$$

  as a measure of of closeness of two processes.
- Is this a good measure?
- Are there other alternatives?
- A long story

  Gap metric (Zames)
  Graph metric coprime factorization (Vidyasagar) $G = N/D$
  Vinnicombe's metric

Similar Open Loop Different Closed Loop

$$P_1(s) = \frac{1000}{s+1}, \qquad P_2(s) = \frac{1000a^2}{(s+1)(s+a)^2}$$



Complementary sensitivity functions with unit feedback $C = 1$

$$T_1 = \frac{1000}{s+1001}, \qquad T_2 = \frac{10^7}{(s-287)(s^2+86s+34879)}$$

Different Open Loop Similar Closed Loop

The systems

$$P_1(s) = \frac{1000}{s+1}, \qquad P_2(s) = \frac{1000}{s-1}$$

are very different because P_1 is stable and P_2 unstable. The complementary sensitivity functions obtained with unit feedback are

$$T_1(s) = \frac{1000}{s+1001} \qquad T_2(s) = \frac{1000}{s+999}$$

These closed loop systems are very similar.

The Graph Metric

We know how to compare stable systems. What to do with unstable systems? Let

$$P(s) = \frac{B(s)}{A(s)}$$

where A and B are polynomials. Choose a stable polynomial C whose degree is not lower than the degrees of A and B, then

$$P(s) = \frac{\dfrac{B(s)}{C(s)}}{\dfrac{B(s)}{C(s)}} = \frac{N(s)}{D(s)}$$

Compare the numerator and denominator transfer functions jointly.

Many Ways to Choose D

Two rational functions D and N are called coprime if there exist rational functions X and Y which satisfy the equation

$$XD + YN = 1$$

The condition for coprimeness is essentially that $D(s)$ and $N(s)$ do not have any common factors.

Let $D^*(s) = D(-s)$. A factorization $P = N/D$ such that

$$DD^* + NN^* = 1$$

is called a coprime factorization of P.

Vinnicombe's Metric

Consider two systems with the normalized coprime factorizations

$$P_1 = \frac{D_1}{N_1}, \qquad P_2 = \frac{D_2}{N_2}$$

To compare the systems it must be required that

$$\frac{1}{2\pi}\Delta \arg_\Gamma (N_1 N_2^* + D_1 D_2^*) = 0$$

where Γ is the Nyquist contour. In the polynomial representation this condition implies

$$\frac{1}{2\pi}\Delta \arg_\Gamma (B_1 B_2^* + A_1 A_2^*) = \deg A_2$$

The winding number constraint!

Vinnicombe's Metric

If the winding number constraint is satisfied Vinnicombe's Metric can be defined as

$$\delta_\nu(P_1, P_2) = \sup_\omega \frac{|P_1(i\omega) - P_2(i\omega)|}{\sqrt{(1 + |P_1(i\omega)|^2)(1 + |P_2(i\omega)|^2)}}$$

Feedback Interpretation

Consider systems with the transfer functions P_1 and P_2. Compare the complementary sensitivity functions for the closed loop systems obtained with a controller C that stabilizes both systems.

$$\delta(P_1, P_2) = \left|\frac{P_1 C}{1 + P_1 C} - \frac{P_2 C}{1 + P_2 C}\right| = \left|\frac{(P_1 - P_2)C}{(1 + P_1 C)(1 + P_2 C)}\right|$$

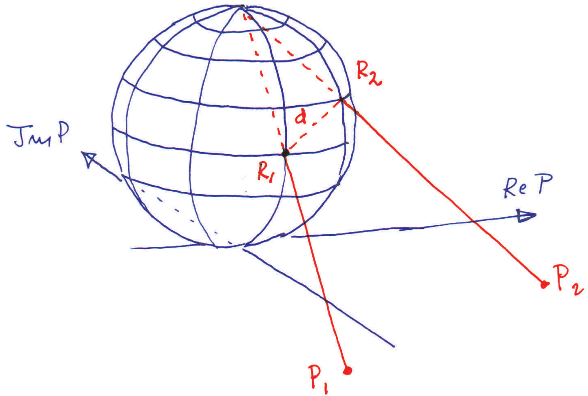
For frequencies where the maximum sensitivity is large we have

$$\delta(P_1, P_2) \approx M_{s1} M_{s2} |C(P_1 - P_2)|$$

It can be shown that δ is a good measure of closeness of processes.

Vinnicombes metric corresponds to $C = 1$, i.e. unit feedback.

Geometric Interpretation



Robustness

Additive perturbations $P \to P + \Delta P$, ΔP stable

$$\frac{|\Delta P(i\omega)|}{|P(i\omega)|} < \frac{|P(i\omega)C(i\omega)|}{|1 + P(i\omega)C(i\omega)|} = \frac{1}{|T(i\omega)|}$$

For normalized Co-prime factor perturbations
$P = N/D \;\to\; (N + \Delta N)(D + \Delta D)$ this generalizes to

$$||(\Delta N(i\omega), \Delta D(i\omega))|| < \frac{1}{\gamma(\omega)}$$

where

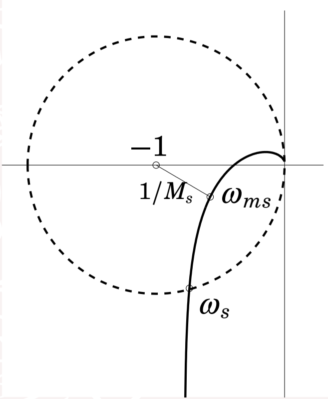
$$\gamma = \bar{\sigma} \begin{pmatrix} \frac{1}{1+P(i\omega)C(i\omega)} & \frac{P(i\omega)}{1+P(i\omega)C(i\omega)} \\ \frac{P(i\omega)}{1+P(i\omega)C(i\omega)} & \frac{P(i\omega)C(i\omega)}{1+P(i\omega)C(i\omega)} \end{pmatrix} = \frac{\sqrt{(1+|P(i\omega)|^2)(1+|C(i\omega)|^2)}}{|1+P(i\omega)C(i\omega)|}$$

Maximum Sensitivity

The number

$$M_s = \max |S(i\omega)|$$

is a measure of robustness, because $1/M_s$ is the smallest distance from the Nyquist curve to the critical point -1.

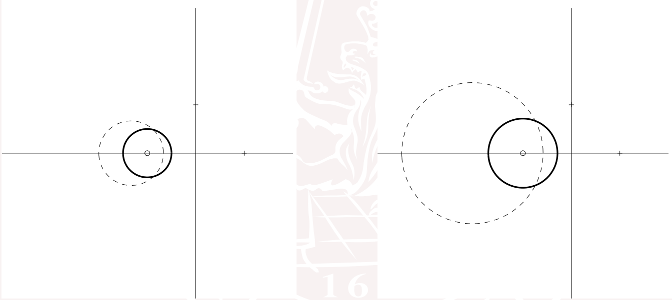


Reasonable values are between 1.2 and 2.

Maximum Sensitivities

Specifications on maximum sensitivities give require the Nyquist curve to be outside circles around the critical point

$M_s = M_t = 2$ $\qquad$ $M_s = M_t = 1.4$



The circles show the loci of constant sensitivities, full lines for M_s and dashed lines for M_t.

Maximum Sensitivities

A maximal sensitivity M_s guarantees a gain margin

$$g_m \geq \frac{M_s}{M_s - 1}$$

and a phase margin

$$\varphi_m \geq \arcsin \frac{1}{M_s}$$

Constraints on both gain and phase margins can be replaced by constraints on M_s.

- $M_s = 2$ guarantees $g_m \geq 2$ and $\varphi_m \geq 30°$
- $M_s = \sqrt{2} \approx 1.41$ guarantees $g_m \geq 3.4$ and $\varphi_m \geq 45°$
- $M_s = 2/\sqrt{3} \approx 1.15$ guarantees $g_m \geq 7.5$ and $\varphi_m \geq 60°$

Summary of the Sensitivity Functions

$$S = \frac{1}{1+L}, \quad T = \frac{L}{1+L}, \quad M_s = \max|S(i\omega)|, \quad M_t = \max|T(i\omega)|$$

The value $1/M_s$ is the shortest distance from the Nyquist curve of the loop transfer function $L(i\omega)$ to the critical point -1.

$$S = \frac{\partial \log T}{\partial \log P} = \frac{Y_{cl}(s)}{Y_{ol}(s)}$$

How much can the process be changed without making the system unstable?

$$\frac{|\Delta P|}{|P|} < \frac{1}{|T|}$$

Bode's integral the water bed effect.

$$\int_0^\infty \log|S(i\omega)| d\omega = \pi \sum \mathrm{Re}\, p_k - \frac{\pi}{2} \lim_{s\to\infty} sL(s)$$

Summary of Sensitivity Functions

$$S = \frac{1}{1+L},\ T = \frac{L}{1+L},\ M_s = \max|S(i\omega)|,\ M_t = \max|T(i\omega)|$$

The value $1/M_s$ is the shortest distance from the Nyquist curve of the loop transfer function $L(i\omega)$ to the critical point -1.

$$S = \frac{\partial \log T}{\partial \log P} = \frac{Y_{cl}(s)}{Y_{ol}(s)}, \qquad \frac{|\Delta P|}{|P|} < \frac{1}{|T|}$$

Bode's integral and the water bed effect.

$$\int_0^\infty \log|S(i\omega)|d\omega = \int_0^\infty \log|\frac{1}{1+L(i\omega)}|d\omega = \pi\sum p_i$$
$$\int_0^\infty \log|T(\frac{1}{i\omega})|d\omega = \int_0^\infty \log|\frac{L(1/i\omega)}{1+L(1/i\omega)}|d\omega = \pi\sum \frac{1}{z_i}$$

Feedback Fundamentals

Performance

Disturbance reduction by feedback

$$\frac{Y_{cl}(s)}{Y_{ol}(s)} = \frac{1}{1+PC}$$

Load disturbance attenuation (typically low frequencies)

$$G_{xd} = G_{yd} = \frac{P}{1+PC}, \qquad G_{ud} = -\frac{PC}{1+PC}$$

Measurement noise injection (typically high frequencies)

$$G_{xn} = \frac{PC}{1+PC}, \qquad G_{un} = -\frac{C}{1+PC}$$

Command signal following

$$G_{xr} = \frac{Y}{R} = \frac{PCF}{1+PC}, \qquad G_{ur} = \frac{CF}{1+PC}$$

Robustness

Robustness to process variations (large, additive, stable ΔP)

$$\left|\frac{\Delta P}{P}\right| < \frac{|1+PC|}{|PC|} = \frac{1}{|T|}$$

Sensitivity of command signal response (small variations)

$$\frac{dG_{xr}}{G_{xr}} = \frac{1}{1+PC}\frac{dP}{P}$$

Consequences for Design

Consider a first order system with PI control

$$P(s) = \frac{b}{s+a}, \quad C(s) = k + \frac{k_i}{s}$$

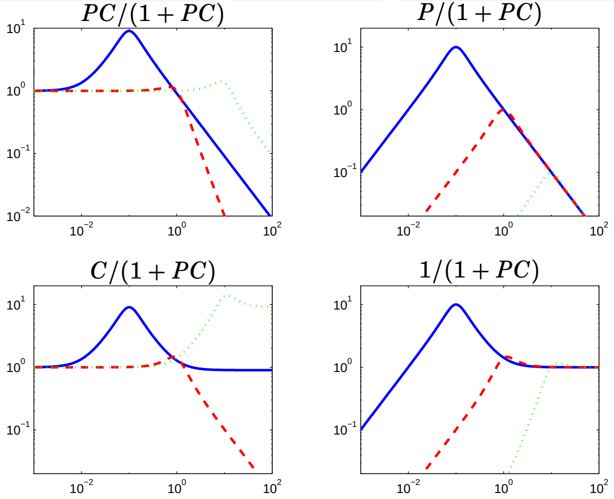
where the controller parameters are chosen to give a closed loop system with the characteristic polynomial $s^2 + \omega_0 s + \omega_0^2$. The Gang of Four is given by

$$\frac{PC}{1+PC} = \frac{(\omega_0 - a)s + \omega_0^2}{s^2 + \omega_0 s + \omega_0^2} \qquad \frac{P}{1+PC} = \frac{bs}{s^2 + \omega_0 s + \omega_0^2}$$

$$\frac{C}{1+PC} = \frac{((\omega_0 - a)s + \omega_0^2)(s+a)}{b(s^2 + \omega_0 s + \omega_0^2)} \qquad \frac{1}{1+PC} = \frac{s(s+a)}{s^2 + \omega_0 s + \omega_0^2}$$

We will investigate the properties of the Gang of Four for $\omega_0/a = 0.1$, 1 and 10.

Amplitude Curves for the Gang of Four



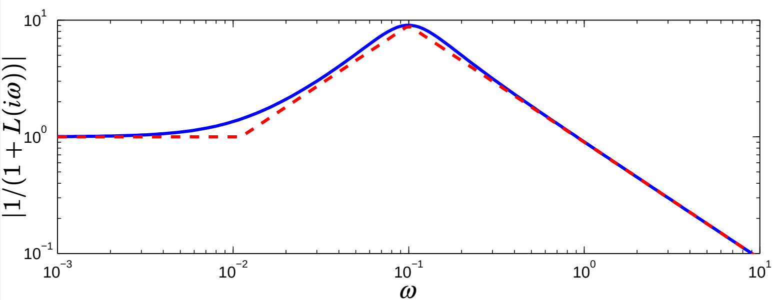
Comments

- Attenuation of load disturbances increases with increasing ω_0.
- Amplification of high frequency disturbances increases with ω_0
- The sensitivity and the complementary sensitivities are very large for $\omega_0 = 0.1$. Designs with small values of ω_0 are useless because of their extreme sensitivity to modeling errors.
- The ability to follow command signals increases with increasing ω_0.
- The closed loop poles cannot be chosen arbitrarily even in a simple case like this.

Estimating Maximum Sensitivity

We have for $a = 1$ and $\omega_0 = 0.1$

$$S = \frac{s(s+a)}{s^2 + \omega_0 s + \omega_0^2} = \frac{s(s+1)}{s^2 + 0.1s + 0.01}$$

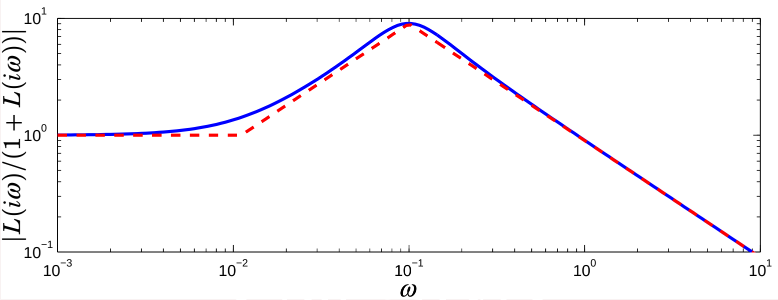


We have approximately $M_s \approx \dfrac{0.1}{0.011} = 9$ (9.4)

Estimating Maximum Complementary Sensitivity

We have for $a = 1$ and $\omega_0 = 0.1$

$$T = \frac{(\omega_0 - 1)s + \omega_0^2}{s^2 + \omega_0 s + \omega_0^2} = \frac{-0.9s + 0.01}{s^2 + 0.1s + 0.01}$$



We have approximately $M_t \approx \frac{0.1}{0.01} = 10$ (10.04)

A Simple Pole Placement Design

Consider a stable first order system

$$Y(s) = \frac{b}{s+a} U(s),$$

PI controller with set point weighting

$$U(s) = -k\beta Y(s) + k_i (R(s) - Y(s))$$

The transfer function from reference to output is

$$G_{yr}(s) = \frac{k\beta s + bk_i}{s^2 + (a + bk)s + bk_i}$$

Desired closed loop characteristic polynomial

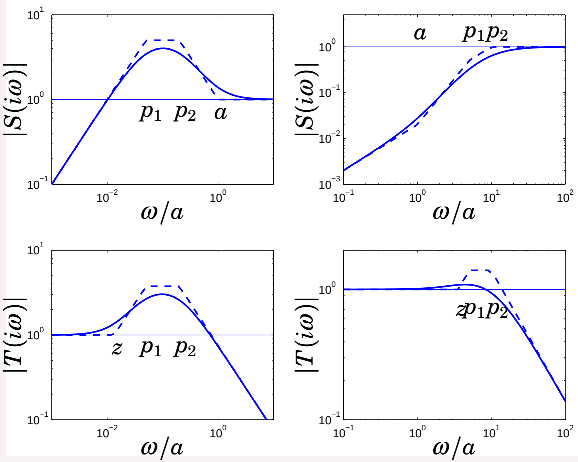
$$(s + p_1)(s + p_2),$$

Controller parameters

$$k = \frac{p_1 + p_2 - a}{b} \qquad k_i = \frac{p_1 p_2}{b}$$

Sensitivity Functions

$$S(s) = \frac{s(s+a)}{(s+p_1)(s+p_2)} \qquad T(s) = \frac{(p_1+p_2-a)s + p_1p_2}{(s+p_1)(s+p_2)}$$



A Reasonable Choice

Closed loop system slower than process $p_1 < a$: choose $p_2 = a$, which implies that controller cancels fast pole.

Closed loop faster than process $p_1 \geq a$: choose $p_2 = p_1$

The controller parameters then becomes

$$k = \begin{cases} p_1/b & \text{if } p_1 < a \\ (2p_1 - a)/b & \text{if } p_1 \geq a. \end{cases}$$

$$k_i = \begin{cases} ap_1/b & \text{if } p_1 < a \\ p_1^2/b & \text{if } p_1 \geq a \end{cases}$$

$$\beta = \begin{cases} 1 & \text{if } p_1 < a \\ p_1/(2p_1 - a) & \text{if } p_1 \geq a \end{cases}$$

This controller parameters gives a robust closed loop system. Transfer function from reference to output is $G_{yr} = p_1/(s + p_1)$.

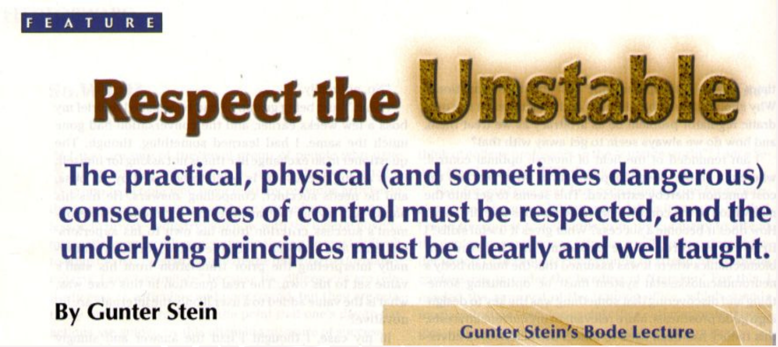
Design Rules

The following rules give designs with low sensitivities

- Determine desired closed loop bandwidth
- Cancel fast stable process poles by controller zeros
- Approximate cancellation obtained by eliminating poles in model before design
- Cancel slow stable process zeros by controller poles
- Unstable poles and zeros cannot be canceled and they give rise to fundamental limitations

Feedback Fundamentals

The First IEEE Bode Lecture 1989


FEATURE

Respect the Unstable

The practical, physical (and sometimes dangerous) consequences of control must be respected, and the underlying principles must be clearly and well taught.

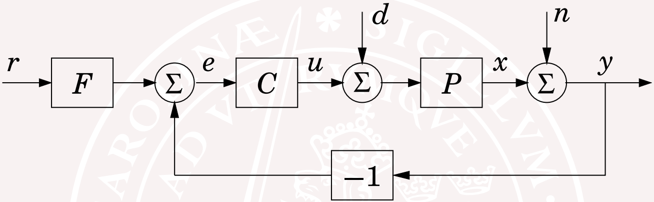
By Gunter Stein

Gunter Stein's Bode Lecture

A video was made by IEEE and the Lecture was finally printed in the IEEE Control Systems Magazine in August 2003!


IEEE
Control Systems
Magazine
August 2003
Volume 23 Number 4
Respect
the Unstable

Fundamental Limitations



Important factors

- Load disturbances and measurement noise
- Actuation power
- System dynamics with time delays, RHP poles and zeros imposes severe limitations of what can be achieved
- Recognize the difficult problems

Minimum Phase Systems

Any transfer function can be realized. No limitations because of system dynamics. High bandwidth attenuates disturbances effectively but measurement noise is also amplified. Gain crossover frequency ω_{gc} captures

- Disturbance attenuation

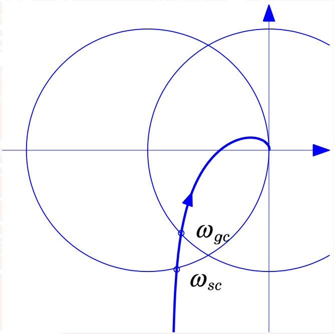
$$Y_{cl} = SY_{ol}$$

- Noise injection to state

$$X = -TN$$

- How about noise injection to u?

$$U = -CSN$$



Effect of Noise on Control Signal

Loop shaping design

- Determine desired crossover frequency ω_{gc}
- Required phase lead at crossover frequency

$$\varphi_l = \pi - \varphi_m - \arg P(i\omega_{gc})$$

- Add phase lead to give desired phase margin
- Adjust gain to make loop gain 1 at ω_{gc}

Phase lead is requires gain.

Gain of a Simple Lead Networks

$$G_n(s) = \Big(\frac{s+a}{s/\sqrt[n]{K}+a}\Big)^n.$$

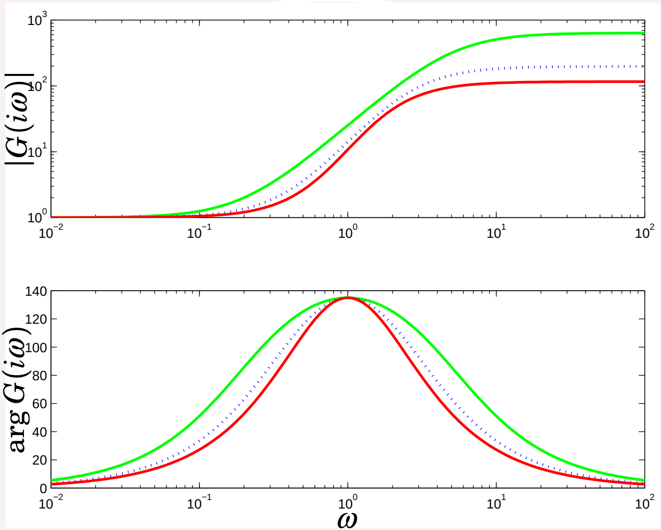
Phase lead $\varphi = n \arctan \dfrac{\sqrt[n]{K}-1}{2\sqrt[2n]{K}}$.

Gain $K_n = \Big(1 + 2\tan^2\frac{\varphi}{n} + 2\tan\frac{\varphi}{n}\sqrt{1+\tan^2\frac{\varphi}{n}}\ \Big)^n$

Phase lead	*n=2*	*n=4*	*n=6*	*n=8*	*n=*∞
$90°$	34	25	24	24	23
$180°$	-	1150	730	630	540
$225°$	-	14000	4800	3300	2600

As n goes to infinity $K_n \to K_\infty = e^{2\varphi}$, exponential increase

Lead Networks of 2nd 3rd and 10th Order



Bode's Phase Area Formula

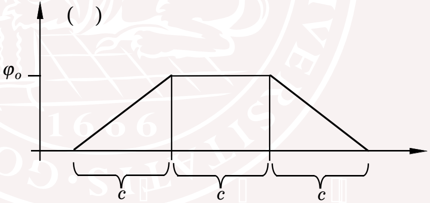
Let $G(s)$ be a transfer function with no poles and zeros in the right half plane. Assume that $\lim_{s\to\infty} G(s) = G_\infty$. Then

$$\log \frac{G(\infty)}{G(0)} = \frac{2}{\pi}\int_0^\infty \arg G(i\omega)\frac{d\omega}{\omega} = \frac{2}{\pi}\int_{-\infty}^{\infty} \arg \bar{G}(iu)du$$

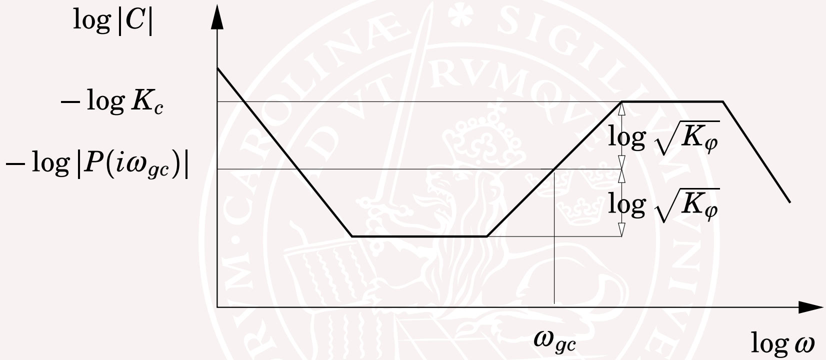
The gain K required to obtain a given phase lead φ is an exponential function of the area under the phase curve

$$K = e^{4c\varphi_0/\pi} = e^{2\gamma\varphi_0}$$

$$\gamma = \frac{2c}{\pi}$$



Estimate of Controller Gain



$$K_c = \max_{\omega \geq \omega_{gc}} |C(i\omega)| = \frac{\sqrt{K_\varphi}}{|P(i\omega_{gc})|} = \frac{e^{\gamma\varphi_l}}{|P(i\omega_{gc})|} = \frac{e^{\gamma(-\pi+\varphi_m-\arg P(i\omega_{gc}))}}{|P(i\omega_{gc})|}.$$

Right hand side only depends on the process!

Estimating Controller Gain

This largest high frequency gain of the controller is approximately given by ($\gamma \approx 1$)

$$K_c = \max_{\omega \geq \omega_{gc}} |C(i\omega)| = \frac{e^{\gamma\varphi_l}}{|P(i\omega_{gc})|} = \frac{e^{\gamma(-\pi+\varphi_m-\arg P(i\omega_{gc}))}}{|P(i\omega_{gc})|}$$

Notice that K_c only depends on the process

- Compensation for process gain $1/|P(i\omega_{gc})|$
- Gain required for phase lead: $e^{\gamma(-\pi+\varphi_m-\arg P(i\omega_{gc}))}$

The largest allowable gain is determined by sensor noise and resolution and saturation levels of the actuator. Results also hold for NMP systems but there are other limitations for such systems.

Example - Two Lags

For the process $P(s) = \frac{1}{(s+1)^n}$ we have

$$K_c = \frac{1}{|P(i\omega_{gc})|} e^{\gamma(-\pi+\varphi_m-\arg P(i\omega_{gc}))} = \left(1+\omega_{gc}^2\right)^{n/2} e^{\gamma(n\arctan\omega_{gc}-\pi+\varphi_m)}$$

Choose $n = 2$, $\gamma = 1$ and $\varphi_m = \pi/4$.

ω_{gc}	10	20	50	100	200
K_c	181.5	796	$5.3\ 10^3$	$2.2\ 10^4$	$8.7\ 10^4$
φ_l	33.6	39.3	42.7	43.8	44.4
$\arg P(i\omega_{gc})$	-168	-174	-178	-179	-179

Essentially compensation for the drop in process gain.

Example - Eight Lags

For the process $P(s) = \frac{1}{(s+1)^n}$ we have

$$K_c = \frac{1}{|P(i\omega_{gc})|} e^{\gamma(-\pi+\varphi_m-\arg P(i\omega_{gc}))} = \left(1+\omega_{gc}^2\right)^{n/2} e^{\gamma(n\arctan\omega_{gc}-\pi+\varphi_m)}$$

Choose $n = 8$, $\gamma = 1$ and $\varphi_m = \pi/4$.

ω_{gc}	0.5	1.0	1.2	1.4	1.5
K_c	9.4	812	3.7 10^3	1.5 10^4	2.7 10^4
φ_l	78	225	266	300	315
$\arg P(i\omega_{gc})$	-212	-360	-401	-435	-450

Much gain is needed to compensate for the phase lag!

A Classic Problem

- For linear systems it follows Bode's phase area formula that phase advance requires gain
- An observation: higher order compensator gives lower gain
- A key question: Can we get a given phase advance with less gain by using a nonlinear systems?
- The Clegg integrator
- A problem worth revisiting?

Limitations due to NMP Dynamics

Process dynamics can impose severe limitations on what can be achieved. Notice that dynamic phenomena do not show up in a traditional static analysis.

- An important part of recognizing the difficult problems
- Time delays and RHP zeros limit the achievable bandwidth
- Poles in the RHP requires high bandwidth
- Systems with poles and zeros in the right half plane can be very difficult or even impossible to control robustly. Think about the bicycle with rear wheel steering!

Remedies:

- Add sensors and actuators (changes and removes zeros) or redesign the process

Robustness and Gain Crossover Frequency

Factor process transfer function as $P(s) = P_{mp}(s)P_{nmp}(s)$ such that $|P_{nmp}(i\omega)| = 1$ and P_{nmp} has negative phase. Requiring a phase margin φ_m we get

$$\begin{aligned}\arg L(i\omega_{gc}) &= \arg P_{nmp}(i\omega_{gc}) + \arg P_{mp}(i\omega_{gc}) + \arg C(i\omega_{gc})\\ &\geq -\pi + \varphi_m\end{aligned}$$

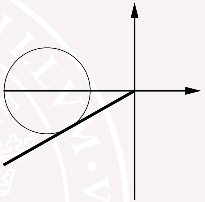
But $\arg P_{mp}C \approx n\pi/2$, where n is the slope at the crossover frequency. (Exact for Bodes ideal loop transfer function $P_{mp}(s)C(s) = (s/\omega_{gc})^n$). Hence

$$\arg P_{nmp}(i\omega_{gc}) \geq -\pi + \varphi_m - n\frac{\pi}{2}$$

The phase crossover inequality implies that robustness constraints for NMP systems can be expressed in terms of ω_{gc}.

Bode's Ideal Cut-off Characteristics

The repeater problem. Large gain variations in vacuum tube amplifiers. What should a loop transfer function look like to make the properties independent of open-loop gain?



$$L(s) = \left(\frac{s}{\omega_{gc}}\right)^n$$

Phase margin invariant with loop gain. For this transfer function we have $\arg L(i\omega) = n\pi/2$.

The slope $n = -1.5$ gives the phase margin $\varphi_m = 45°$.

Horowitz extended Bodes ideas to deal with arbitrary plant variations not just gain variations in the QFT method.

The Crossover Frequency Inequality

The inequality

$$\arg P_{nmp}(i\omega_{gc}) \geq -\pi + \varphi_m - n_{gc}\frac{\pi}{2}$$

implies that robustness requires that the phase lag of the non-minimum phase component P_{nmp} at the crossover frequency is not too large!

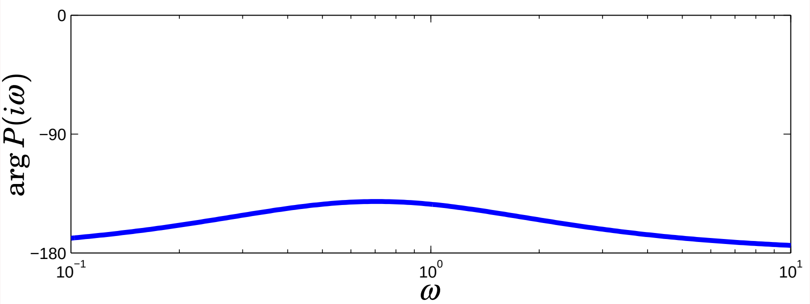
Simple rule of thumb:

- $\varphi_m = 45°,\ n_{gc} = -1/2 \Rightarrow -\arg P_{nmp}(i\omega_{gc}) \leq \frac{\pi}{2}\ (90°)$
- $\varphi_m = 60°,\ n_{gc} = -2/3 \Rightarrow -\arg P_{nmp}(i\omega_{gc}) \leq \frac{\pi}{3}\ (60°)$
- $\varphi_m = 45°,\ n_{gc} = -1 \Rightarrow -\arg P_{nmp}(i\omega_{gc}) \leq \frac{\pi}{4}\ (45°)$

Useful to Plot the Phase of P_{nmp}

Example from Doyle, Francis and Tannenbaum 1992 and the Bhattacharya fragility debate.

$$P(s) = \frac{s-1}{s^2+0.5s-0.5}, \qquad P_{nmp} = \frac{(1-s)(s+0.5)}{(1+s)(s-0.5)}$$



System with RHP Zero

$$P_{nmp}(s) = \frac{z-s}{z+s}$$

Cross over frequency inequality

$$\arg P_{nmp}(i\omega_{gc}) = -2\arctan\frac{\omega_{gc}}{z} \geq -\pi + \varphi_m - n_{gc}\frac{\pi}{2}$$

Hence

$$\frac{\omega_{gc}}{z} \leq \tan(\frac{\pi}{2} - \frac{\varphi_m}{2} + n_{gc}\frac{\pi}{4})$$

Requiring that phase lag of P_{nmp} is less than $90°$ gives

$$\omega_{gc} < z$$

System with Time Delay

$$P_{nmp}(s) = e^{-sT}$$

Cross over frequency inequality

$$\omega_{gc} T \leq \pi - \varphi_m + n_{gc}\frac{\pi}{2}$$

Requireing that phase lag of P_{nmp} is less than $90°$ gives

$$\omega_{gc} T \leq \frac{\pi}{2}$$

System with RHP Pole

$$P_{nmp}(s) = \frac{s+p}{s-p}$$

Cross over frequency inequality

$$-2\arctan\frac{p}{\omega_{gc}} \geq -\pi + \varphi_m - n_{gc}\frac{\pi}{2}$$

Hence

$$\omega_{gc} \geq \frac{p}{\tan(\frac{\pi}{2} - \frac{\varphi_m}{2} + n_{gc}\frac{\pi}{4})}$$

Requiring that phase lag of P_{nmp} is less than 90° gives $\omega_{gc} \geq p$

Time Delay and RHP Pole

$$P_{nmp}(s) = \frac{s+p}{s-p} e^{-sT}.$$

$$\arg P_{nmp}(i\omega_{gc}) = \pi - 2\arctan\frac{\omega_{gc}}{p} - \omega_{gc} T > -\pi + \varphi_m - n_{gc}\frac{\pi}{2}$$

Hence

$$2\arctan\sqrt{\frac{2}{pT} - 1} - pT\sqrt{\frac{2}{pT} - 1} > \varphi_m - n_{gc}\frac{\pi}{2}$$

Necessary for stability to have $pT < 2$.

Requiring that phase lag of P_{nmp} is less than $90°$ gives $pT < 0.33$.

Stabilizing an Inverted Pendulum with Delay

Right half plane pole at

$$p = \sqrt{\frac{g}{\ell}}$$

The inequality $pT < 0.33$ gives $T\sqrt{\frac{g}{\ell}} < 0.33$ or

$$\ell > \frac{gT^2}{0.33^3} \approx 90T^2$$

A neural lag of 0.07 gives $\ell > 0.44$ m.

A vision based system with sampling rate of 50 Hz gives a time delay of 0.02 s, this gives $\ell > 0.04$ m.

System with RHP Pole and Zero Pair

$$P_{nmp}(s) = \frac{(z-s)(s+p)}{(z+s)(s-p)}$$

For $z > p$ the cross over frequency inequality becomes

$$\frac{\omega_{gc}}{z} + \frac{p}{\omega_{gc}} \leq (1 - \frac{p}{z}) \tan\left(\frac{\pi}{2} - \frac{\varphi_m}{2} + n_{gc}\frac{\pi}{4}\right)$$

$$\varphi_m < \pi + n_{gc}\frac{\pi}{2} - 2\arctan\frac{\sqrt{p/z}}{1-p/z}$$

With $n_{gc} = -0.5$ we get

z/p	2	2.24	3.86	5	5.83	8.68	10	20
φ_m	-6.0	0	30	38.6	45	60	64.8	84.6

An Example

Doyle, Francis Tannenbaum 1992
Keel and Bhattacharyya 1997 (fragile control)

$$P(s) = \frac{s-1}{s^2 + 0.5s - 0.5}$$

- Pole at $s = 0.78$
- Zero at $s = 1.0$
- $\dfrac{z}{p} = 1.28$
- Hopeless to control robustly
- You don't need any more calculations

Example - The X-29

Advanced experimental aircraft. Much design effort was done with many methods and much cost. Specifications $\varphi_m = 45°$ could not be reached. Here is why!

Non-minimum phase part of the transfer function

$$P_{nmp}(s) = \frac{s - 26}{s - 6}$$

The zero pole ratio is $z/p = 4.33$ with $n_{gc} = -1/2$ we get

$$\varphi_m = 32.4$$

It is extremely difficult to obtain a phase margin of $45°$!

Bicycle with Rear Wheel Steering

Transfer function

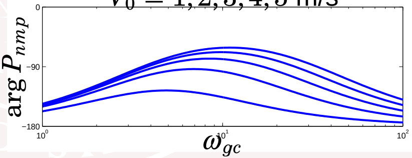
$$P(s) = \frac{am\ell V_0}{bJ} \frac{-s + \dfrac{V_0}{a}}{s^2 - \dfrac{mg\ell}{J}}$$

RHP pole at $\sqrt{mg\ell/J}$

RHP zero at V_0/a

Kleins bike

$V_0 = 1, 2, 3, 4, 5$ m/s



Other Criteria

There are several alternatives to the phase margin

$$M_s = \max_\omega |S(i\omega)|$$

$$M_t = \max_\omega |T(i\omega)|$$

Combined sensitivity

$$M_{sp} = \max_\omega (|T(i\omega)| + |S(i\omega)|)$$

$\mathcal{H}_\infty$ norm

$$M = \max_\omega \frac{\sqrt{(1+|C|^2)(1+|P|^2)}}{|1+PC|}$$

Essentially the same results but numerical values are different.

Summary of Limitations - Part 1

- A RHP zero z gives an upper bound to bandwidth

$$\frac{\omega_{gc}}{z} \leq \begin{cases} 0.5 & \text{for } M_s,\, M_t < 2 \\ 0.2 & \text{for } M_s,\, M_t < 1.4. \end{cases}$$

- A time delay T gives an upper bound to bandwidth

$$\omega_{gc} T \leq \begin{cases} 0.7 & \text{for } M_s,\, M_t < 2 \\ 0.4 & \text{for } M_s,\, M_t < 1.4. \end{cases}$$

- A RHP pole p gives a lower bound to bandwidth

$$\frac{\omega_{gc}}{p} \geq \begin{cases} 2 & \text{for } M_s,\, M_t < 2 \\ 5 & \text{for } M_s,\, M_t < 1.4. \end{cases}$$

Summary of Limitations - Part 2

- RHP poles and zeros must be sufficiently separated

$$\frac{z}{p} \geq \begin{cases} 7 & \text{for } M_s,\, M_t < 2 \\ 14 & \text{for } M_s,\, M_t < 1.4. \end{cases}$$

- RHP poles and zeros must be sufficiently separated

$$\frac{p}{z} \geq \begin{cases} 7 & \text{for } M_s,\, M_t < 2 \\ 14 & \text{for } M_s,\, M_t < 1.4 \end{cases}$$

- The product of a RHP pole and a time delay cannot be too large

$$pT \leq \begin{cases} 0.16 & \text{for } M_s,\, M_t < 2 \\ 0.05 & \text{for } M_s,\, M_t < 1.4. \end{cases}$$

Design Issues and Tradeoffs

- Load disturbances
- Measurement noise
- Command signals
- Process variations
- Process dynamics, time delays, RHP poles and zeros
- Actuator resolution and saturation
- Sensor resolution and range

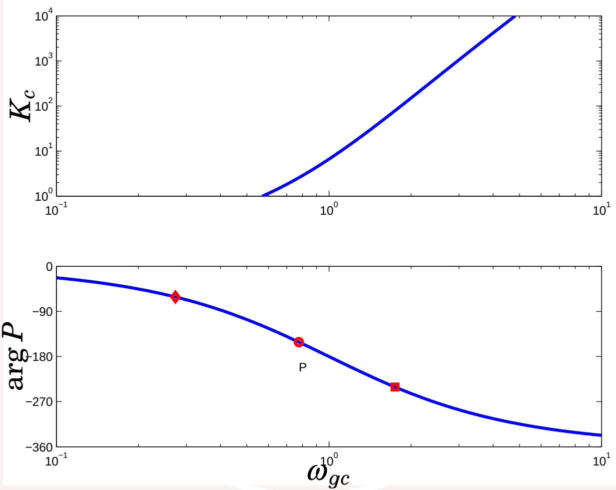
Results can be summarized in an assessment plot that can be generated from the process transfer function

The Assessment Plot

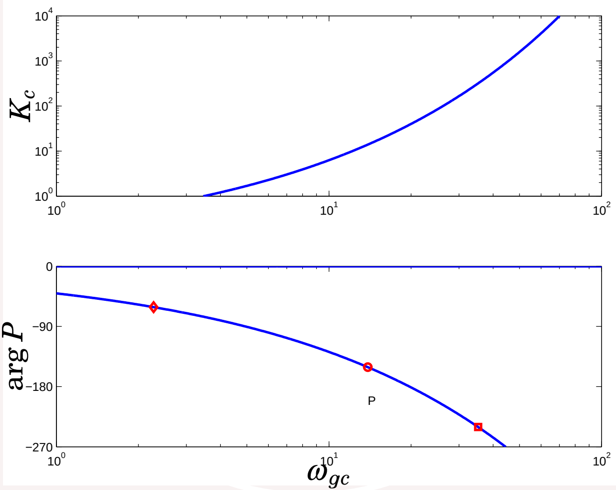
The *assessment plot* has a gain curve $K_c(\omega_{gc})$ and two phase curves $\arg P(i\omega)$ and $\arg P_{nmp}(i\omega)$

- Attenuation of disturbance captured by ω_{gc}
- Injection of measurement noise captured by the high frequency gain of the controller $K_c(\omega_{gc})$
- Robustness limitations due to time delays and RHP poles and zeros captured by $\arg P_{nmp}(\omega_{gc})$
- Controller complexity is captured by $\arg P(i\omega_{gc})$

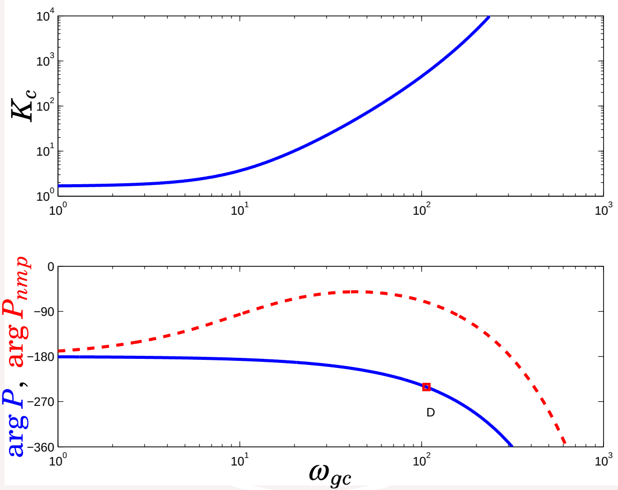
Assessment Plot for $P(s) = 1/(s+1)^4$



Assessment Plot for $P(s) = e^{-\sqrt{s}}$



Assessment Plot for $P(s) = e^{-0.01s}/(s^2 - 100)$



Summary

For non-minimum phase systems the limitations can be expressed by the crossover frequency inequality

$$\arg P_{nmp}(i\omega_{gc}) \geq -\pi + \varphi_m - n_{gc}\frac{\pi}{2}$$

Simple Rule of Thumb: $-\arg P_{nmp}(i\omega_{gc}) \leq 45^\circ - 90^\circ$

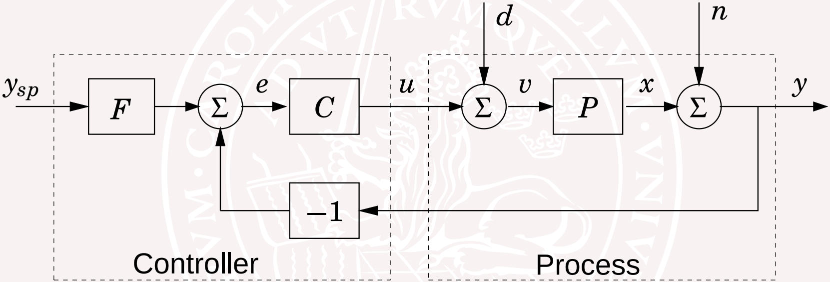
- RHP zeros and time delays give upper bound on ω_{gc}
  - Long time delays are bad
  - Slow unstable zeros are bad
- RHP poles gives a lower bound on ω_{gc}
  - Fast unstable poles are bad
- RHP poles and zeros cannot be too close

The tradeoff plot puts it all together!

Feedback Fundamentals

PID Control

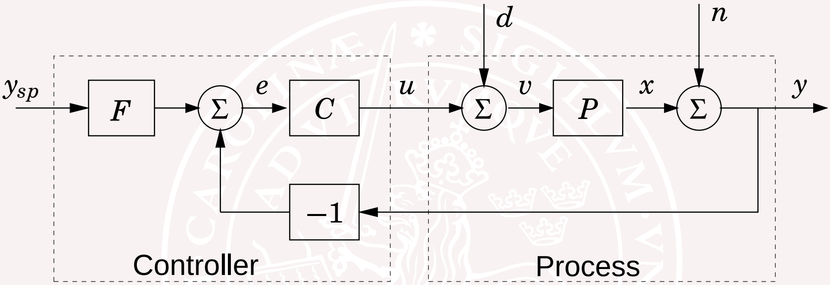
Look at traditional PID control from the perspective of feedback fundamentals.



$$u(t) = k\big(\beta y_{sp}(t) - y_f(t)\big) + k_i \int_0^t \big(y_{sp}(\tau) - y_f(\tau)\big)d\tau + k_d\Big(\gamma \frac{dy_{sp}}{dt} - \frac{dy_f}{dt}\Big)$$

Tune k, k_i, k_d, and filtering $Y_f = G_f Y$ for load disturbances and measurement noise and β, and γ for set-point response

Recall Criteria for Control Design



Ingredients

- Attenuate effects of load disturbance d
- Do not feed in too much measurement noise n
- Make the system insensitive to process variations
- Make state x follow set-point y_{sp}

Performance

Disturbance reduction by feedback

$$Y_{cl} = \frac{1}{1+PC} Y_{ol}$$

Load disturbance attenuation (typically low frequencies)

$$G_{yd} = \frac{P}{1+PC} \approx \frac{1}{sk_i}, \qquad -G_{ud} = \frac{PC}{1+PC}$$

Measurement noise injection (typically high frequencies)

$$G_{xd} = \frac{PC}{1+PC}, \qquad -G_{un} = \frac{C}{1+PC} \approx C = G_f\left(k + \frac{k_i}{s} + k_d s\right)$$

Command signal following

$$G_{xr} = \frac{PG_f(\gamma k_d s^2 + \beta k s + k_i)}{s + PG_f(k_d s^2 + \beta k s + k_i)}, G_{ur} = \frac{G_f(\gamma k_d s^2 + \beta k s + k_i)}{s + PG_f(k_d s^2 + \beta k s + k_i)}$$

Robustness

The sensitivity function

$$S = \frac{1}{1 + PC}$$

Complementary sensitivity

$$T = \frac{PC}{1 + PC}$$

Combined sensitivties

A Design Methodology

- Maximize integral gain subject to constraints on robustness and high frequency gain of the controller MIGO (M-constrained Integral Gain Optimization)
- Follow the footsteps of Ziegler and Nichols
  - Test batch of processes
  - Find optimized controllers
  - Correlate with dynamics features
- Results
  - Insight and tuning rules
  - Characterization of process dynamics

$$P(s) = \frac{K}{1 + sT} e^{-sL}$$

  - Lag dominance and delay dominance

How to Characterize Process Dynamics?

Standard model for PID control

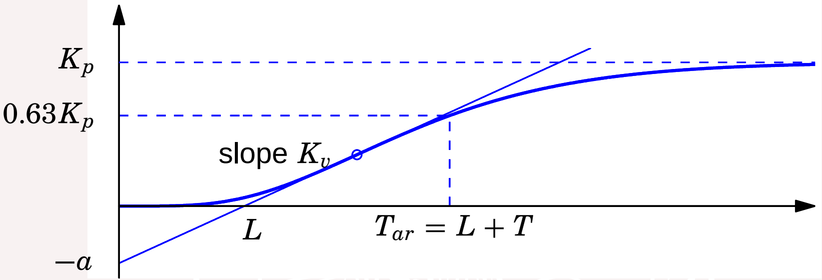
$$G(s) = \frac{K}{1+sT} e^{-sL}$$

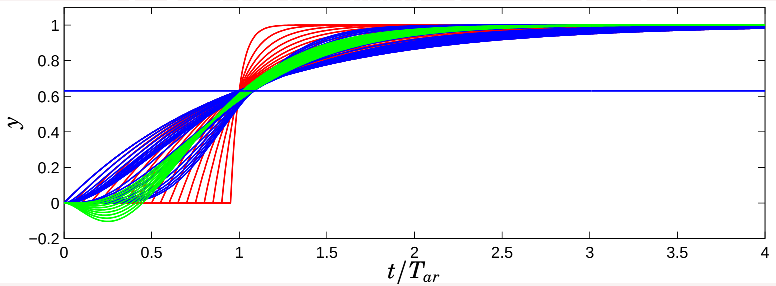
- K static gain
- T apparent time constant
- L apparent time delay

Ziegler and Nichols used two parameters K/T and L

Is this enough?

Process Dynamics - Step Responses





Essentially Monotone Step Responses

$$P_1(s) = \frac{e^{-s}}{1+sT}, \qquad P_2(s) = \frac{e^{-s}}{(1+sT)^2}$$

$$P_3(s) = \frac{1}{(s+1)(1+sT)^2}, \qquad P_4(s) = \frac{1}{(s+1)^n}$$

$$P_5(s) = \frac{1}{(1+s)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)}$$

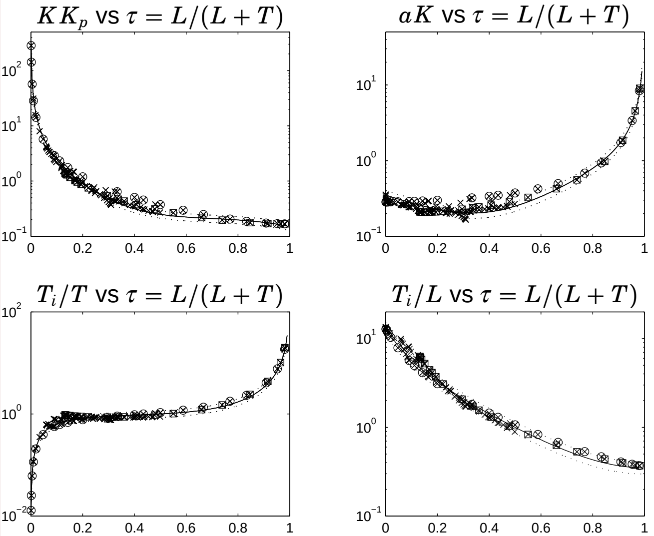
$$P_6(s) = \frac{1}{s(1+sT_1)} e^{-sL_1}, \quad T_1 + L_1 = 1$$

$$P_7(s) = \frac{T}{(1+sT)(1+sT_1)} e^{-sL_1}, \qquad T_1 + L_1 = 1$$

$$P_8(s) = \frac{1-\alpha s}{(s+1)^3}$$

$$P_9(s) = \frac{1}{(s+1)((sT)^2 + 1.4sT + 1)}$$

PI Control



The AMIGO Tuning Rule

Robustness criterion: $M_s = M_t = 1.4$

$$K = \frac{0.15}{K_p} + \left(0.35 - \frac{LT}{(L+T)^2}\right)\frac{T}{K_p L}$$

$$T_i = 0.35L + \frac{13LT^2}{T^2 + 12LT + 7L^2},$$

For integrating processes, K_p and T go to infinity and $K_p/T = K_v$, and he tuning rule is be simplified to

$$K = \frac{0.35}{K_v L}$$

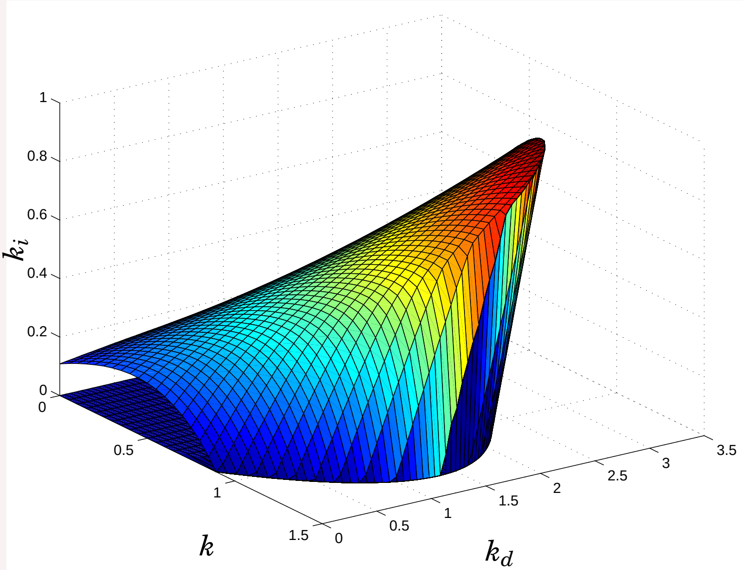
$$T_i = 13.4L.$$

Works for delay dominant as well as lag dominant processes

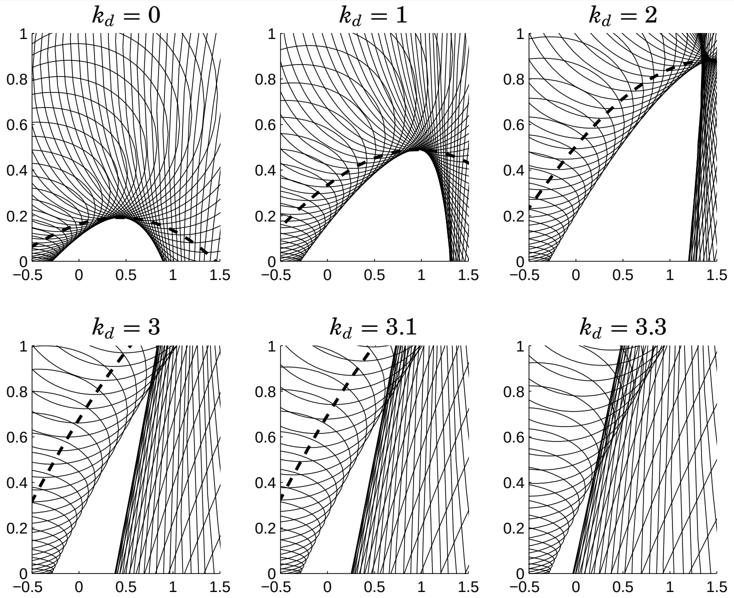
PID Control

- Looks straight forward, but ...
- A difficulty
- Derivative action is a real cliffhanger
- Understanding what goes on
- Fixing the problem
- Tuning rules

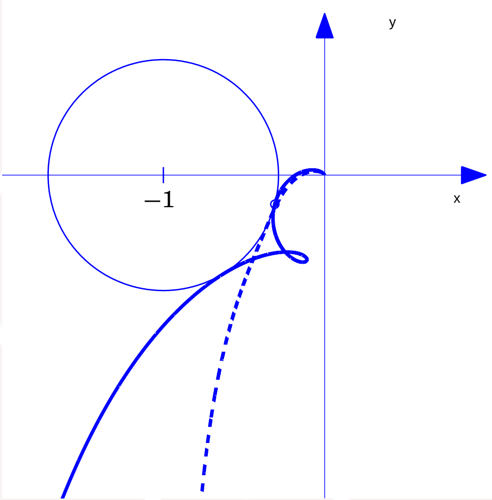
Derivative Action - A Cliffhanger $P(s) = (1+s)^{-4}$



Derivative Action - A Cliffhanger $P(s) = (1+s)^{-4}$

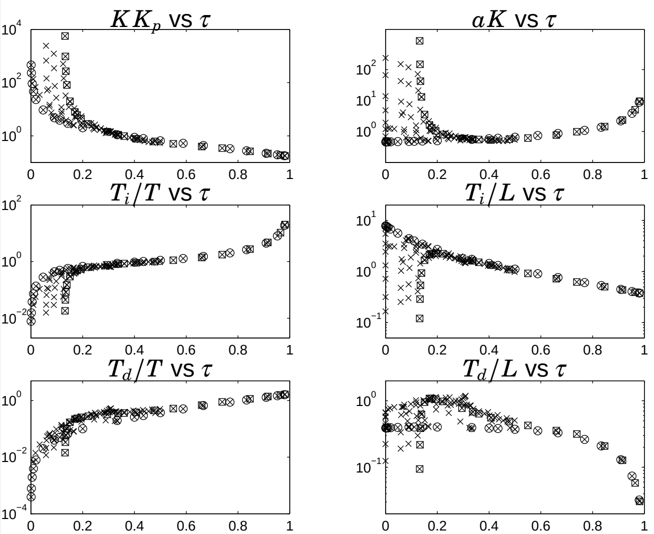


$P(s) = (1+s)^{-4}$



$k = 0.925$, $k_i = 0.9$, and $k_d = 2.86$

PID Control



KK_p vs τ
aK vs τ
T_i/T vs τ
T_i/L vs τ
T_d/T vs τ
T_d/L vs τ
0
0.2
0.4
0.6
0.8
1
10^4
10^2
10^0
10^1
10^{-1}
10^{-2}
10^{-4}

A Conservative Tuning Rule

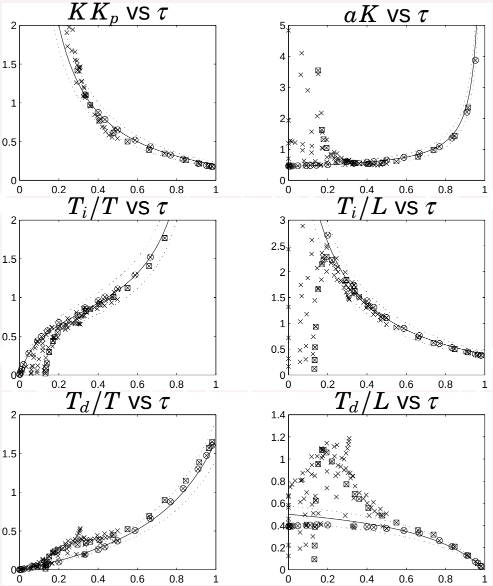
AMIGO (Approximate MIGO) for PID control

$$
\begin{aligned}
K &= \frac{1}{K_p}\left(0.2 + 0.45\frac{T}{L}\right)\\
T_i &= \frac{0.4L + 0.8T}{L + 0.1T}L\\
T_d &= \frac{0.5LT}{0.3L + T}.
\end{aligned}
$$

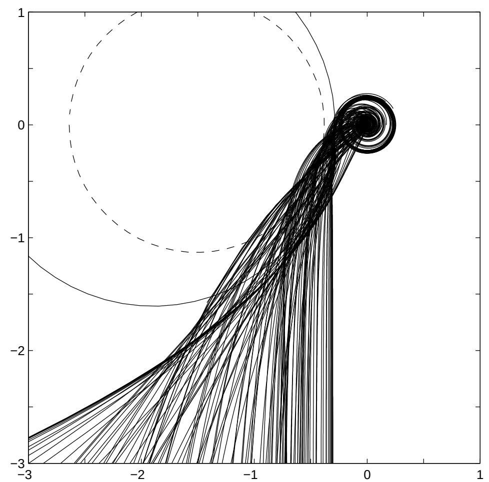
For integrating processes the equations becomes

$$
\begin{aligned}
K &= 0.45/K_v\\
T_i &= 8L\\
T_d &= 0.5L.
\end{aligned}
$$

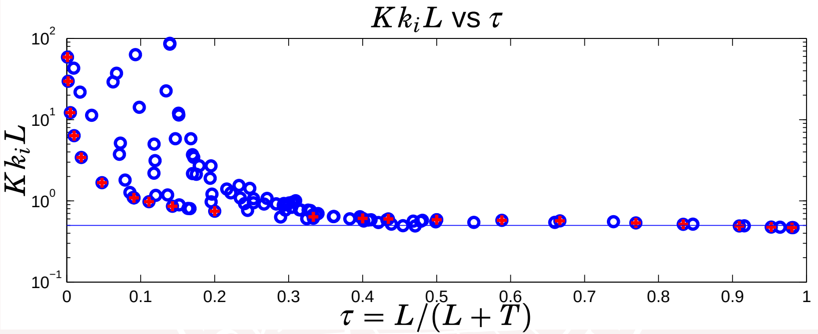
A Conservative Tuning Rule



PID Control

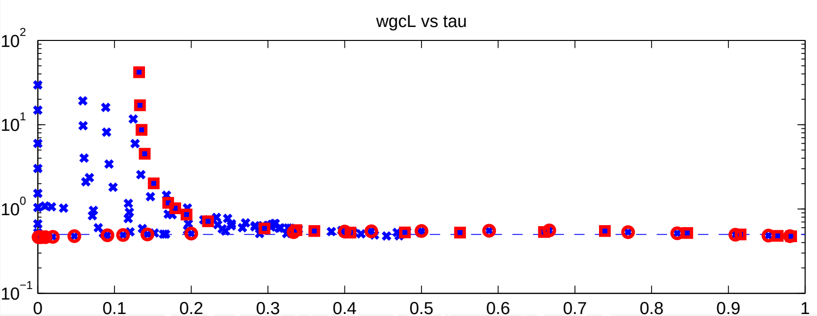


Delay Dominant Processes



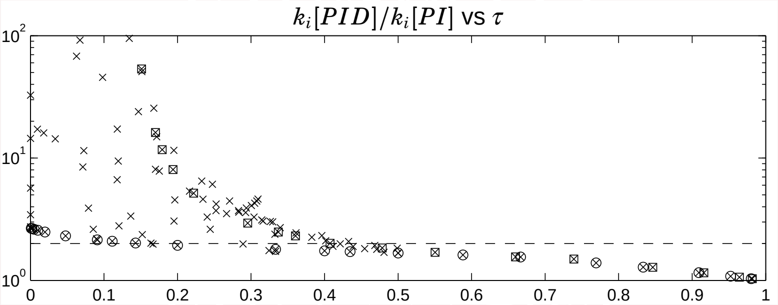
- The simple rule $Kk_iL = 0.5$ works well for $\tau > 0.4$

An Observation

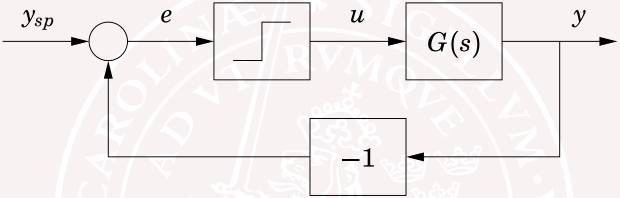


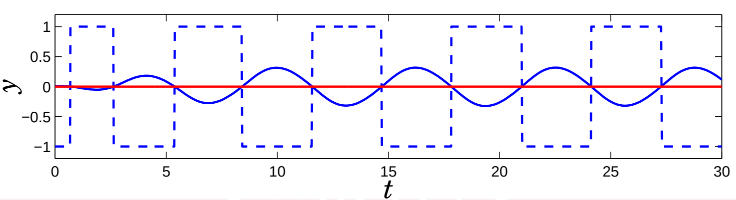
- Fundamental limitation $\omega_{gc}L \leq 0.4$ for $M_s = 1.4$
- Why different for small τ?

Benefits of Derivative Action

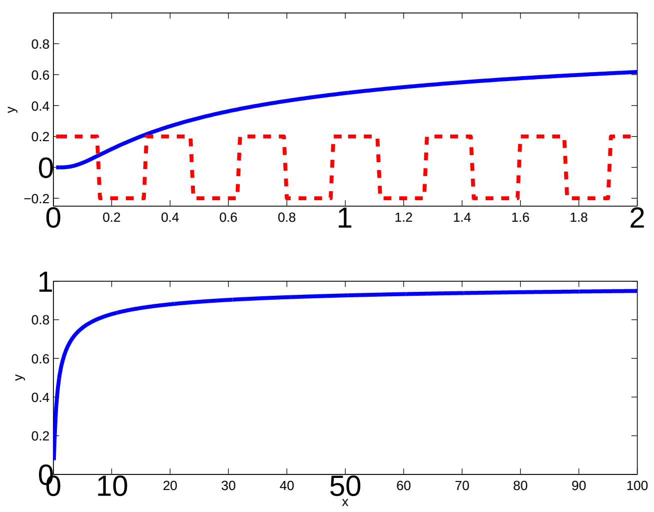


Better Modeling by Relay Feedback

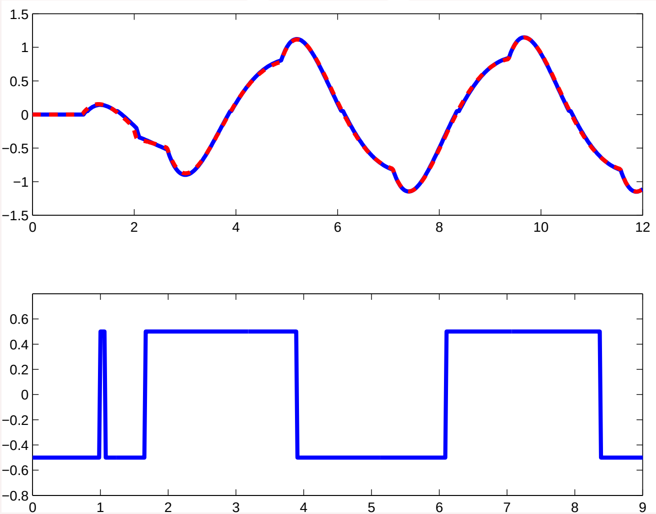




Short Experiment Time $G(s) = exp(-\sqrt{s})$



Good Excitation



Summary

- Derivative action - a cliffhanger
- The importance of auto-tuning
- Lag dominance (τ small) or delay dominance (τ large)
- Simple tuning rules work well for $\tau > 0.2$
- What happens for small τ?
  - Notice that L is the apparent time delay
  - Important to separate true time delay from time constants
  - Tuning can be improved with better modeling
  - Relay auto-tuning gives good excitation
- Sensor noise and detuning

Feedback Fundamentals

Summary

- Error feedback and systems with two degrees of freedom
- A system with error feedback is characterized by four transfer functions (Gang of Four GoF) S, T, PS, CS
- A system with two degrees of freedom is characterized by six transfer functions (Gang of Six = GoF + FT+ FCS)
- Systems with two degrees of freedom allow a complete separation of responses to reference signals and disturbances
- Design feedback for disturbances and robustness, then design feedforward F to give desired response to reference signals
- Analysis and specifications should cover all transfer functions!
- The assessment plot and PID control