Persistence of disagreement in social networks

Daron Acemoglu, Giacomo Como, Fabio Fagnani, and Asuman Ozdaglar

Disagreement among individuals in a society, even on central questions that have been debated for centuries, is the rule; agreement is the rare exception. How can disagreement of this sort persist for so long? Existing models of communication and learning, based on Bayesian or non-Bayesian updating mechanisms, typically lead to consensus provided that communication takes place over a strongly connected network (see, e.g., [?, ?, ?, ?, ?, ?] and references therein) These models are thus unable to explain persistent disagreements.

In this paper, we use a stochastic model of communication combined with the assumption that there are some "stubborn" agents in the economy who never change their opinions. We show that the presence of these stubborn agents leads to persistent disagreements among the rest of the society—because different individuals are within the "sphere of influence" of distinct stubborn agents and are influenced to varying degrees. Under general conditions, there is no convergence to a consensus. Instead, the expected cross-sectional distribution of beliefs in the society converges (in distribution), and generally, the opinion of a single individual, and in fact that of the whole society, potentially fluctuates forever. This model provides a new approach to understanding persistent disagreements, and in the process, introduces new tools for the analysis of opinion formation and consensus models.

Briefly, we consider a society envisaged as a social network of n agents, communicating and exchanging information. Each agent starts with an opinion $x_i(0) \in \mathbb{R}$ and is then "recognized" according to a rate-1 independent Poisson process in continuous time. Following this event, she meets one of the individuals in her *social neighborhood* according to a stochastic matrix P. We shall identify agents with the vertices of a (possibly directed) graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, representing an underlying *social network*, where $(i, j) \in \mathcal{E}$ iff $P_{ij} > 0$. We distinguish between two types of individuals, *stubborn* and *regular*. Stubborn agents, which are typically few in number and whose set is denoted by $\mathcal{S} \subseteq \mathcal{V}$, do not change their opinion after a meeting, while regular agents, whose set is denoted by $\mathcal{A} := \mathcal{V} \setminus \mathcal{S}$ make up the great majority of the social networks, update their beliefs to some weighted average of their pre-meeting belief and the belief of the agent they met. Specifically, we shall assume that, if agent $a \in \mathcal{A}$ is recognized at time $t \geq 0$, then her belief jumps from its current value $x_a(t^-)$ to $x_a(t) = (1 - \eta)x_a(t^-) + \eta x_b(t^-)$, while all other individuals' opinions remain constant. The parameter $\eta \in (0, 1)$ is a measure of the trust that each regular individual puts on other individuals' beliefs, which, for the sake of simplicity, will be assumed constant over $\mathcal{A}$ and in time. This information exchange generates a Markov process $x(t)$ over $\mathbb{R}^{\mathcal{V}}$, and we study its long-run behavior, under the mild connectivity assumption that there exists a path in $\mathcal{G}$ from every $a \in \mathcal{A}$ to every $v \in \mathcal{V}$.

It can be shown that, unless $x_s(0) = x_{s'}(0)$ for all $s, s' \in \mathcal{S}$, $x(t)$ does not converge with probability one. Nevertheless, the following weak convergence result holds.

Theorem 1. *There exists an random variable $x(\infty)$, such that $\lim_{t\to+\infty} x(t) = x(\infty)$ in distribution. Moreover, $h := \mathbb{E}[x(\infty)] \in \mathbb{R}^{\mathcal{V}}$ is the unique solution of the following Laplace equation $((P-I)h)_a = 0$ for all $a \in \mathcal{A}$, with boundary conditions $h_s = x_s(0)$ for all $s \in \mathcal{S}$.*

Theorem 1 suggests to study the expected asymptotic opinion vector h. In fact, h admits the following standard representation [**?**][Ch. 2, Lemma 27]:

$$h_v = \sum_{s\in\mathcal{S}} \mathbb{P}_v\left(\tau_{\mathcal{S}} = \tau_s\right) x_s(0), \qquad \forall v \in \mathcal{V}, \tag{1}$$

where $\mathbb{P}_v(\,\cdot\,)$ denotes the probability measure associated to a continuous-time random walk $V(t)$, with initial state $V(0) = v$, transition rates P, while $\tau_{\mathcal{W}} := \inf\{t \geq 0 : V(t) \in \mathcal{W}\}$ denotes the hitting time of an arbitrary subset $\mathcal{W} \subseteq \mathcal{V}$. Formula (1) allows one to compute h exactly for certain social network topologies, including trees and Cayley graphs.

Example 2. *Let $\mathcal{G}$ be an undirected tree, $P_{ij} = 1/\deg(i)$ for all $(i,j) \in \mathcal{E}$, and let $\mathcal{S} = \{s_0, s_1\}$, with $x_{s_0}(0) = 0$, and $x_{s_1}(0) = 1$. Then, the expected asymptotic opinion vector h can be computed as follows: For all vertices v lying on the path between s_0 and s_1, one has $h_v = d(v, s_0)/\left(d(v, s_0) + d(v, s_1)\right)$, where $d(\,\cdot\,,\,\cdot\,)$ denotes distance on $\mathcal{G}$. On the other hand, for all vertices v such that the path from v to s_1 (respectively, to s_0) passes through s_0 (through s_1), one has $h_v = 0$ ($h_v = 1$).*

In some cases when the expected asymptotic opinion vector h cannot be explicitly computed in a simple way, it is possible to provide bounds on its dispersion.

Theorem 3. *Assume that the stochastic matrix P is reversible with invariant measure π. Then, for all $\varepsilon > 0$, it holds*

$$\pi\left(a : |\mathbb{E}[X_a(\infty)] - \sum_v \pi_v \mathbb{E}[X_v(\infty)]| \geq \alpha\varepsilon\right) \leq \frac{2}{\varepsilon} \log(2e^2/\varepsilon) \frac{\tau_1}{\mathbb{E}_\pi[\tau_{\mathcal{S}}]} \,.$$

*where $\alpha := \sum_{s\in\mathcal{S}} |X_s(0)|$, τ_1 is the variation threshold time [**?**, Ch. 4, p. 1] of $V(t)$, and $\mathbb{E}_\pi[\,\cdot\,]$ denotes the expectation for the Markov chain $V(t)$ with initial distribution π.*

Theorem 3 has the following intuitive meaning. If the Markov chain $V(t)$ mixes in a time faster than the expected hitting time of the stubborn agents set $\mathcal{S}$, then it will eventually hit any of them with approximately equal probability, and thus the expected asymptotic opinions do not vary much over the network. As a corollary to Theorem 3, using known estimations of τ_1 and $\mathbb{E}[\tau_{\mathcal{S}}]$ for Abelian Cayley graphs [**?**, Ch.s 5 and 7], one finds for instance that, if $\mathcal{G}$ is a d-dimensional torus with $d \geq 2$, $|\mathcal{V}| = n$, and $|\mathcal{S}|$ constant in n, then, for all $\varepsilon > 0$, the fraction of $v \in \mathcal{V}$ such that $|h_v - n^{-1} \sum_w h_w| > \varepsilon$ vanishes as n grows.

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