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A Fusion Algorithm for Traffic Density Estimation using Sensors and Floating Car Data

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Abstract

This paper deals with data fusion of multiple sources of information for density estimation in traffic networks. A macroscopic approach is adopted by partitioning the network in cells and considering as state of the network the dynamically evolving vehicle densities in the cells. Estimation of the state of the network is of crucial importance in modern ITSs, but direct measurements can be obtained via expensive sensors only. To overcome this problem, this paper presents an algorithm for fusion of sensor measurements with Floating Car Data, which on the contrary have become recently available in large quantity and are relatively inexpensive. Furthermore, the paper provides methods for calibration of splitting ratios and of the Fundamental Diagram, and a discussion on optimal sensor placement. The prowess of the algorithm is shown in a real scenario using data from the sensor network along the Rocade Sud in Grenoble and INRIX Floating Car Data.

KEYWORDS: Transportation Systems, Data Fusion, Floating Car Data

1. Introduction

The last decades have witness an enormous increase of number of passenger and commercial vehicles, not matched by a comparable extension of roads infrastructures. Crucial highways and arterial roads have thus reached a state of near saturation, experiencing on a daily basis periods of congested traffic. Consequences are decreased safety, economical losses (fuel consumption, increased travel times), and health and psychological hazards (pollution, road rage). Development of Intelligent Transportation Systems (ITSs) is envisaged as a structured and long term solution to such a problem.

The ability of the traffic operator to monitor the state of the network by quantifying road usage, which namely the density of vehicles, is clearly of paramount importance. Such an information is used 1) to inform drivers in real time about the state of the network; 2) to feed, along with historical data, prediction algorithms to forecast travel time and traffic evolution, allowing drivers to take better navigation decisions; 3) to aid public authorities to monitor and maintain roads, e.g., allowing early detection of accidents; and 4) to compute control actions by standard means such as traffic lights, ramp metering, speed limits, or more recent means such as lane change and origin-destination navigation suggestions [1, 2, 3].

While monitoring is of crucial importance, density estimation is not an easy task. Classical measurement devices for traffic networks are static sensors such as induction loops or magnetometers, which provide measurements of flow, density (more precisely, occupancy, see Section 2), and, if deployed in pairs, speed, of the section of road in which they are deployed. These systems provide reliable measurements, but their require huge investments and planning. For this reason, measurements obtained using this method are “expensive” and thus usually sparse. Recently, technological advancements in the field of telecommunication devices have allowed a growing availability of Floating Car Data (FCD), namely, of measurement of position and speed of single vehicles by means of smartphones, navigation systems, and other devices able to track its GPS position. If enough vehicles are equipped with such devices and are willing to transmit their data, it is possible to obtain reasonably good measurements of the evolution of speed in the network. The most advanced systems, such as the one employed by INRIX, allow for very fine spatial granularity, up to one datum every 250 meters (source: [4]). This type of measurement is extremely appealing because it does not require any additional equipment other than already deployed communication systems, and is thus relatively “cheap” and covers all major traffic networks.

This paper is devoted to describing an algorithm that fuses these two sources of information to estimate traffic density. We model the network as a first order system whose dynamics depend on inflows and outflows at each cell in which the network is divided. We do not aim to model the latter, which depend in a complex way on the state of the network. Instead, we assume for estimation purposes a local relation between flow and density as captured by a bilinear Fundamental Diagram, and we assume that the inflow in a cell is a linear combination of the outflows from the cells immediately before it, with fixed weights known as splitting ratios. We propose a method to calibrate Fundamental Diagram and splitting ratios, and to estimate the dynamic evolution of the density making use of the aforementioned measurements of flow and speed. Furthermore, we discuss the problem of optimal sensor location and of trading-off between cost and estimation performance. Finally, we implement the proposed algorithm on the real case study of Grenoble Rocade Sud, using static sensor data from the Grenoble Traffic Lab and FCD provided by INRIX.

The problem of estimating the density of vehicles in a traffic network has been tackled in the literature using a number of techniques. A standard approach, partially employed in this paper, is to discretize the space so that roads are partitioned in cells, as in the celebrated Cell Transmission Model [5]. The dynamics of vehicles flowing from cell to cell are then modeled as a discretization of the celebrated Lighthill-Whitham and Richards (LWR) [6] model. Researchers have been then focusing on the problem of calibrating such models [7] and on system-theoretical problems such as observability and controllability [8]. As mentioned above, differently from these approaches the relation between flow and density, which is inherently nonlinear and complex and thus difficult to model precisely, is only used in this paper to design the observer of the state. Alternative approaches to the problem of fusion of data from heterogeneous sources employ signal processing techniques such as the generalized Treiber-Helbing filter [9]. Differently from this approach, we consider a dynamical model and system theoretical ideas. Very recent approaches avoid altogether discretizing the PDE model, and provide a way to cast the problem of estimation and control in traffic networks as convex problems [10]. This approach however requires additional assumptions on the initial conditions and the inputs in the system, and is more difficult to implement than ours.

The rest of the paper is organized as follows: Section 2 describes the setting and formulates the problem. Section 3 details the proposed solution. Section 4 is focused on optimal sensor placement. Section 5 illustrates the numerical study, and Section 6 draws the conclusions.

2. Problem formulation

A Traffic Network is a collection of interconnected *multi-lane roads*. Each lane is partitioned into several consecutive cells, and vehicles travel from cell to cell until they exit from the network (for example through an offramp). Origin cells act as gates through which vehicles enter into the network, and represent onramps or connectors. The network is then a graph $G = (V, E)$, where nodes $v \in V$ are junctions or *sections of roads* that separate *cells* $e \in E$. A stretch of road is a sequence of parallel cells separated by junctions (see Fig. 1). Two cells are parallel if they lie on different lanes of the same section of a road.

We consider a dynamical description of the system, where the densities of vehicles on the cells of the network change in time according to a discrete time scheme in which time is slotted in intervals of length $T > 0$. In our numerical study, $T = 15$ seconds. On each cell $e \in E$, let $\rho_e(t)$ be the density of vehicles, in number of vehicles per km, at time. The vector of densities $\rho(t) = [\rho_e(t)]_{e \in E}$ is the *state* of the network, and changes dynamically in time according to the first order equation

$$\rho_e(t+1) = \rho_e(t) + f^{in}_e(t)/L_e - f^{out}_e(t)/L_e \qquad (1)$$

where $f^{in}_e(t)$ and $f^{out}_e(t)$ are the inflows and outflows at time t in cell e, respectively, and L_e is the length of cell e (in km). Such an equation simply represents mass conservation of vehicles on cell e, as the change of traffic volume on cell e, $L_e(\rho_e(t+1) - \rho_e(t))$, is given by the difference of inflow and outflow on that cell during the last time slot.

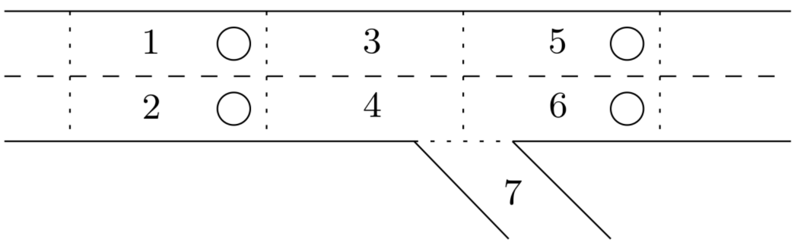

Figure 1: A stretch of road (traffic direction: left – right) with mainline and an offramp. Each lane is divided in cells (separated by dotted lines). Cells 1 and 2, 3 and 4, 5 and 6 are parallel. Vehicles exiting from 3 or 4 can proceed along the main line (cells 4 or 5) or turn into the offramp 7. Vehicles distribute according to splitting ratios R_{ej}, e.g., $f^{in}_3 = R_{13}f^{out}_1+R_{23}f^{out}_2$. Sensors (circles), are deployed at the end of some of the cells.

In the real system inflows and outflows $f^{in}_e(t)$ and $f^{out}_e(t)$ depend in a complex way on the density of vehicles of all the cells around e. When (1) is intended as a mathematical model of the real system, the most well known tool to describe the relation between density and flows is arguably the Cell Transmission Model (CTM) [5, 11], in which it is postulated that the flow from a cell to the following depends on the density of the former in freeflow, and on the density of the latter in congested state. In this paper, we do not aim to model in detail such a complex relation between flow and density. However, we maintain the standard assumption that vehicles on a cell split into the following ones according to given fractions, called *splitting ratios*. For every cell e and for every other cell j that follows e there exists a nonnegative value R_{ej} that specifies the fraction of vehicles that exiting from e enter into j.

Clearly $\sum_j R_{ej} = 1$, for every e. Splitting ratios specify the dependence of inflows on outflows, as the inflow on cell j will be given by the sum of the outflows from the preceding cells, say e, multiplied by the splitting ratio R_{ej}. The only exception is origin cells, whose inflows come from parts of the environment that are not modeled. In formulae, we can write

$$f^{in}_j(t) = \sum_e R_{ej} f^{out}_e(t) + \lambda_j(t), \quad (2)$$

where $\lambda_j(t)$ is the external inflow of cell j (zero for non-origin cells). In compact form, given the matrix $R = [R_{ej}]$ and the vector of external inflows $\lambda(t) = [\lambda_j(t)]_{j\in E}$, we have $f^{in}(t) = R^T f^{out}(t) + \lambda(t)$, for all t, where ^{T} denotes the transpose of a matrix.

As mentioned, we do not aim to model the way outflows $f^{out}_e(t)$, $e \in E$, depend on the state of the network. However, we postulate, for estimation purposes only, that there exists a deterministic relation between outflow and density on a cell, as in the LWR PDE model, the graph of such a relation being known under the name of Fundamental Diagram. In particular, we assume that the outflow $f^{out}_e = \varphi_e(\rho_e)$ on a cell e is given by the product of density ρ_e and speed $v_e(\rho_e)$, where the speed $v_e(\rho_e)$ is assumed to be a non increasing function of ρ_e. In particular, in the following, we assume the standard triangular Fundamental Diagram in which

$$v_e(\rho_e) = v^{ff}_e \qquad \rho_e \leq \rho_e^c \quad (3)$$
$$v_e(\rho_e) = C_e(\rho_e - \rho^{max})/(\rho_e^{c-} \rho^{max}) \qquad \rho_e > \rho_e^c \quad (3)$$

where v^{ff}_e is the freeflow speed on cell e, ρ_e^c is the critical density separating freeflow region (low densities) and congested region (high densities), ρ^{max} is the maximum density of vehicles, and C_e is the capacity, namely the maximum flow, on cell e. Notice that $C_e = v^{ff}_e/\rho_e^c$.

We immediately underline that, under the last assumption, (1) *does not* well describe the dynamics of a Traffic Network, except maybe for low densities. In fact, since (3) postulates that outflows only depend on the density on the same cell, (1) with (3) is not a good discretization of the continuous space LWR model, and cannot reproduce important phenomena such as shock waves. Indeed, in the following, (3) will only be a tool to reconstruct the densities in the network on the basis of flow and speed measurements.

Available measurements

In this paper we consider as sources of information both static sensor networks and FCDs.

Static sensor network

Induction loops and magnetometers are standard devices to measure traffic. They are buried in the ground and send information, through wired and wireless communication networks, to usually far away data centers, and they are thus considerably expensive.

In this paper we assume that roads are partitioned in cells and sensors are placed at their end in the sense of the traffic direction (see Fig. 1). Traffic sensors are able to:

- Count the number of vehicles crossing a section: such a value is the flow through the section and corresponds to the outflow from the cell where the sensor is deployed;
- Measure the time a vehicle is present above the sensor: such a value provides a noisy measurement of the occupancy of the road (and, for us, of teh cell) over the sensor. Occupancy is usually expressed as the percentage of time a vehicle is above a sensor

during a time slot, where 0 corresponds to no vehicles, and 100 full congestion. The density of vehicles is proportional to the occupancy by a constant given by $1/(100L_{ave})$, where L_{ave} is the average length of a vehicle expressed in km. For this reason, from now on we assume that sensors can measure density directly;
- If sensors are deployed in pairs at known distance, the two times at which a vehicle crosses the sensors: such information can be used to compute the speed of the vehicle and thus the average speed of the vehicles.

We assume from now that sensors obtain a new traffic measurement at every time slot. As such, we denote by $\varphi^m{}_e(t)$, $\rho^m{}_e(t)$, and $\nu^m{}_e(t)$ the flow, density and average speed noisy measurements, respectively, at time t and on the sensor on cell e. Sources of noise range from temporary inability to detect changes of the magnetic field, too fast or too slow vehicles, etc. While in the present paper we do not model such sources of noise, this will be subject for future research. Finally, we denote by M the set of cells in which sensor are deployed.

Remark: while in the Grenoble Traffic lab network sensors are deployed in pairs and thus able to obtain speed measurements, this is not usually the case due to the corresponding cost increase. As such, in the following we do not employ speed measurements $\nu^m{}_e(t)$.

Floating Car Data

Vehicles embedded with communication and tracking devices can measure their position and speed and communicate such information to ITS systems. For privacy reasons, single vehicle data cannot be directly used and are aggregated, usually in terms of average speed on *segments*. We refer to the latter quantities as Floating Car Data (FCD). While vehicles such as buses and taxis can be tracked, we do not consider them in this paper.

Depending on the technology, the spatial granularity of FCD can be very fine, with segments as short as 250 meters. Moreover, we assume that FCD cover the entire area of interest. However, differently from sensor networks, FCD do not usually distinguish among parallel cells. In addition, information provided via FCD is averaged over a relatively long period of time, of the order of minutes. For this reason, we assume that a new speed aggregate datum is available every N time slots, namely, at times N, 2N, 3N, ..., and corresponds to the average speed in periods [0, N-1], [N, 2N-1], [2N, 3N-1], ..., respectively. Thus, at each time t and at each cell e, the measurement of speed $v^m{}_e(t)$ is a noisy measurement of the average speed of vehicles in e and all cells parallel to e during the last interval [kN, (k+1)N-1] before time t.

As an example, in our numerical study we consider a scenario in which time is slotted with T = 15 seconds, and measurements of speed are obtained every 5 minutes starting from 00:00:00. Therefore, between 00:05:00 and 00:09:45 the speed measurement on cell e is the (noisy) average speed of vehicles on cell e and on cells parallel to e in the (N = 20 time slots long) period 00:00:00 – 00:04:45.

Remark: both $\nu^m{}_e(t)$ and $v^m{}_e(t)$ are measurements of speed, but their nature is different. In fact, the former represents the average speed of all vehicles crossing a certain section during a time slot, while the latter is the average speed of a subset of vehicles that were in a certain cell during several time slots. As such, the sources of noise are different, being related to sensors' parameters for the former, and to penetration rate for the latter.

The system observer

From now on, interpret $\rho_e(t)$ as an *estimate* of the density of vehicles on cell e at time t, not as its real (unknown) counterpart, and consider the following observer for a Traffic Network

$$\rho_e(t+1) = \rho_e(t) + f^{in,e}{}_e(t)/L_e - f^{out,e}{}_e(t)/L_e + \gamma(\rho^{fe}{}_e(t) - \rho_e(t)) \quad (4)$$

The observer has form similar to (1), however $f^{in,e}{}_e(t)$ and $f^{out,e}{}_e(t)$ denote here *estimates* of the real inflow and outflow, obtained on the basis of the measurements. Furthermore, a compensation term $\gamma(\rho^{fe}{}_e(t) - \rho_e(t))$ is added to the right hand side: its effect is to steer the density estimate towards a first estimate of the density $\rho^{fe}{}_e(t)$, function of the measurements. The tuning parameter γ trades off between model and first estimates. If $\gamma = 0$, we only trust the flow measurements, while if γ is large the trajectory $\rho_e(t)$ tends to track the first estimates. In our study we set $\gamma = 0.5$ to avoid high corrections and, in turn, possibly unrealistic negative estimates. The question of how to tune it will be matter for future research.

The problem that we tackle in this paper is how to design the maps $f^{in,e}{}_e$ and $f^{out,e}{}_e$, and $\rho^{fe}{}_e$ as functions of the measurements, in order to obtain good estimates of the real traffic densities.

3. Proposed solution

The proposed solution consists of an *offline* calibration step and an *online* update procedure.

Offline calibration

The offline calibration part consists in

- calibrating the Fundamental Diagram, that is, estimating the function $\varphi_e(\rho) = \rho v_e(\rho)$ where $v_e(\rho)$ has the form (3), for all cells $e \in M$
- calibrating the splitting ratios matrix R

Calibration of the Fundamental Diagram

For each $e \in M$, let $[\rho^m{}_e(t_i), \varphi^m{}_e(t_i)]_i$ be the pairs density-flow measured at given times t_i, i = 1, 2, ..., n_c used for calibration. Calibration of the Fundamental Diagram is cast into a minimization problem in which the variables are the parameters $\rho^c{}_e$ and C_e of the Fundamental Diagram, and in which the cost function penalizes square deviations of $\varphi_e(\rho(t_i))$ from the measured outflow $\varphi^m{}_e(t_i)$ on all available measured couples. The problem is solved making use of a gradient descent algorithm and the nonlinearity of the fundamental diagram is solved by identifying, at each step of the algorithm, which pairs $[\rho^m{}_e(t_i), \varphi^m{}_e(t_i)]_i$ are in freeflow and which in congested state using the previous critical density $\rho^c{}_e$. For all cells for which measurements of flow and density are not available, we compute the parameters $\rho^c{}_e$ and C_e of the Fundamental Diagram by linear interpolation using those of the closest cells in M.

Calibration of the splitting ratios matrix R

Let $[\varphi^m{}_e(t_i)]_{i,e\in E}$ be the set of all available flow measurements at given consecutive times t_i, i = 1, 2, ..., n_c used for calibration. By combining (1) and (2) yields that the rate of change of the state is given by a linear combination of the outflows, with weights equal to the splitting ratios. Under the assumption that the latter are constant, the difference between the state of the network at the end and the beginning of a time interval is given by the same linear combination of the outflows summed over the whole interval. If such a period is long (for example, a day), the latter sum is large, while the previous difference is small. As such, we can assume that the linear combination of summed outflows is zero, and one can then cast the problem of estimating the splitting ratios as an optimization program in which splitting ratios are the variables and the measured outflows define the constraints.

Online update

The Density Estimation Algorithm proceeds according to the following steps:

- Initialization: initialize the state of the network to an arbitrary value. In case the algorithm starts during night time, zero state is close to reality;
- At time t:
  - Receive the new flow measurements $[\varphi^m_e(t)]_{e \in E}$, and compute estimates of $f^{in}_e(t)$ and $f^{out}_e(t)$ using the matrix of splitting ratios;
  - Receive, if available, new speed measurements $[v^m_e(t)]_{e \in E}$, or hold the previous ones, on the whole network;
  - For each cell e, compute from the Fundamental Diagram the two possible densities ρ^1_e (freeflow) and ρ^2_e (congested) compatible with the estimated outflow $f^{out}_e(t)$;
  - For each cell e, compute the two corresponding speeds $v_e(\rho^1_e)$ and $v_e(\rho^2_e)$;
  - Set as *first estimate* $\rho^{fe}_e(t)$ that among ρ^1_e and ρ^2_e whose associate speed is the closest to $v^m_e(t)$;
- Update $\rho_e(t+1)$ according to (4).

The most critical steps of the algorithm are flow estimate and speed computations. In the next two paragraphs we provide more details on such steps.

Flow estimate

While part of the flows are directly measured by sensors, in most of the network they are not directly available. To overcome this problem, we cast the outflow estimate as a further optimization problem in which the optimization variables are the outflows. Aside from non negativity, we constrain the outflows on the cells $e \in M$ to be equal to the measured flows. The cost function is designed in such a way that it penalizes deviations from zero of the linear combination of the flows having as weights the splitting ratios. In other terms, the optimization problem looks for a set of flows equal to those measured where possible, and such that the network is almost at steady-state, that is, inflow and outflow are equal in each cell. Clearly this is in general not the case, but the difference between the two in a time slot (for example, in a period of 15 seconds in our numerical study) is nonetheless small, hence our choice. Different flow estimation methods will be subject for further research. Finally, inflows at each cell are computed via splitting ratios and outflows through (2).

Speed computation

Given an outflow estimate of f^{out}_e, we can easily obtain two candidate densities ρ^1_e and ρ^2_e from the Fundamental Diagram: $\rho^1_e = f^{out}_e \rho^c_e / C_e$, the freeflow candidate density, and $\rho^2_e = \rho^{max} + f^{out}_e (\rho^c_e - \rho^{max})/C_e$, the congested candidate density. The corresponding speed is computed through the relation $\varphi_e(\rho) = \rho v_e(\rho)$, which yields a freeflow speed $v_e(\rho^1_e) = \alpha C_e / \rho^c_e$, and a congested speed $v_e(\rho^2_e) = \alpha C_e \ / (C_e \rho^{max} / (\alpha f^{out}_e) + (\rho^c_e - \rho^{max}))$. Notice that the flows computed in the previous section are given in vehicles per time slot, and the parameter α is used to convert the flow in vehicles per hour. For example, T = 15 corresponds to $\alpha = 240$.

4. Optimal sensor placement

The proposed algorithm, and in general traffic monitoring and control, highly profits from good flow estimates. Since the latter are influenced by the locations at which sensors are deployed, one of the most crucial problems in designing a sensor network is to decide how many sensors to use, and where they are to be deployed, in order to meet the requirements. Let n be the number of sensors that we are allowed to deploy. A solution to the first problem comes from considering a static setting, in which we assume that the network is in steady state. Under the assumption that the splitting ratios are fixed and roughly known, let $[\varphi^{m}_{e}]_{e\in M}$ be a vector of flow measurements, where M is any subset of E of cardinality n. Then, as in the above algorithm, the problem of estimating the whole vector of flows from $[\varphi^{m}_{e}]_{e\in M}$ can be cast as a minimization problem aimed to find a vector $[\varphi^{M}_{e}]_{e\in E}$ that satisfies the linear constraints implied by the splitting ratios. If each φ^{m}_{e} is a noisy measurement of the true φ_{e}, then for each M we can measure the performance of the flow estimation in terms of the variance of the difference φ^{M}-φ, possibly weighted to improve performance on main roads. It can be shown then that the optimal placement corresponds to the minimization of the trace of such a variance, which is a combinatorial problem in which the variables are the cells in M.

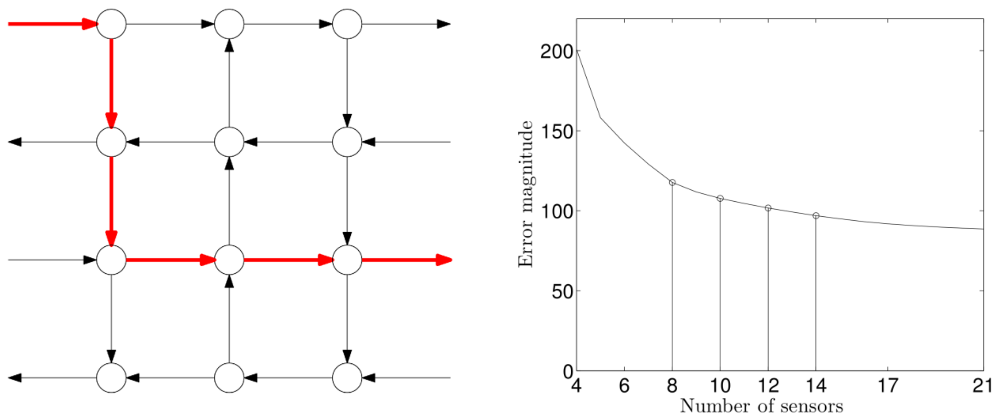

Figure 2: Trade-off between performance and cost. Left panel: the regular grid used in the study. Right panel: error magnitude versus number of sensors.

Assume now that an algorithm for optimal placement has been designed. The second problem is to study the trade-off between performance and cost of the network. Indeed, while increasing the number of sensors and placing them in the optimal position cannot but improve the estimates' performance, additional sensors also increase the cost of the network.

We have analyzed such phenomenon in the fictitious network in Fig. 2, composed of 24 cells (arrows indicate traffic direction), with uniform splitting ratios. Thick red cells form a main road and are weighted 100 times more than ordinary black cells. On the right panel of Fig. 2 we show the error magnitude for the best placement of sensors when the number of sensors ranges from 4 to 21. As expected, the performance improvement due to addition of one more sensor decreases with the number of sensors. In particular, after 12 sensors such an addition does not seem to be profitable from a cost-benefit trade-off point of view.

The combinatorial nature of the problem makes brute force search unfeasible in real cases of hundreds of roads. For this reason, in future research we aim to relaxed and/or tackle the problem via randomized algorithms with performance guarantee [12].

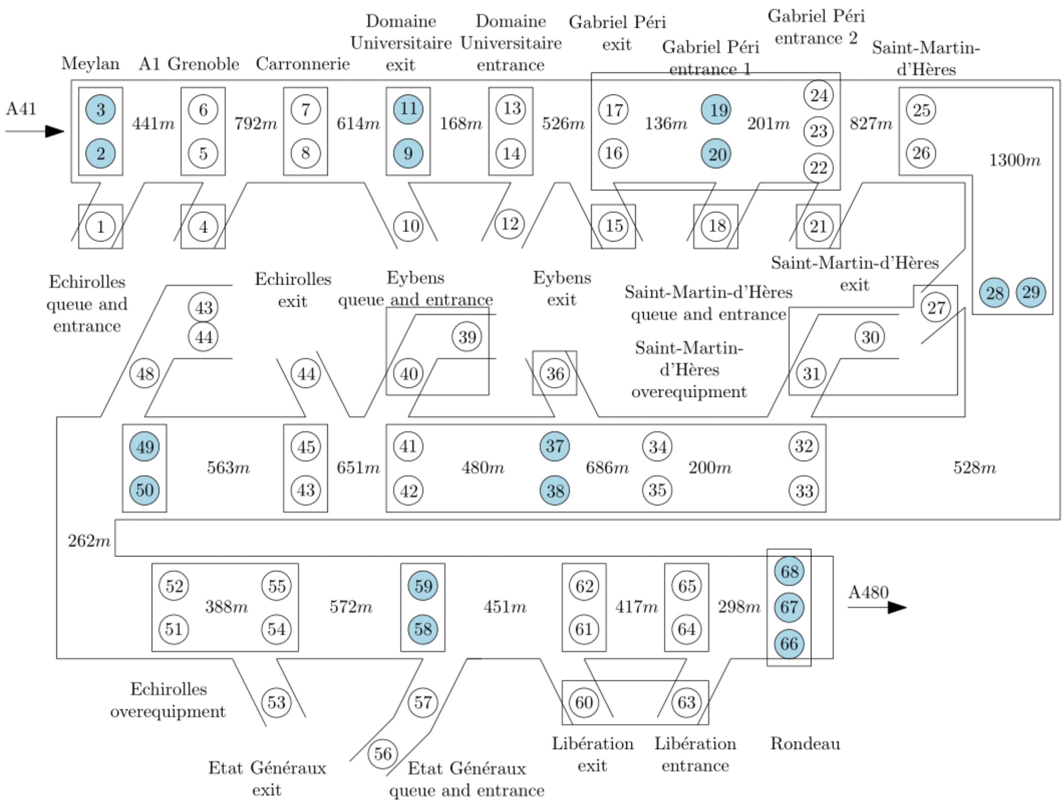

Figure 3: stylized depiction of the Rocade Sud. Each circle represents a cell/pair of sensors. Onramps and offramps are represented as stretches or roads entering and exiting the main line, respectively. The main line is composed of two lanes (except for Gabriel Péri entrance 2 and Rondeau). Light blue circles represent sensors used in the study ("expensive" measurements). Rectangles represent segments of roads in which average speed measurements are available ("cheap" FCD measurements).

5. Numerical experiments

The numerical study has been carried on making use of the Grenoble Traffic Lab, an extensive sensor network deployed along the Rocade Sud, a 10 km long km peri-urban freeway enclosing the city of Grenoble, France. Composed of two carriageways with two/three lanes, the Rocade is a connector of paramount importance, showing an average of 45000 vehicles (5% trucks) driving across each direction every day. The highway is operated by the Direction Interdepartementale des Routes Centre-Est (DIR-CE).

The Grenoble Traffic Lab (GTL) sensor network is currently deployed on the east – south direction. It is composed of more than 130 magnetometers buried in the pavement. In most locations, magnetometers are deployed in pairs distant 2.5 meters able to measure flow, density and speed. Pairs of magnetometers are deployed on all onramps and offramps, on the two lanes of the main line (three in two points of the Rocade) at sections distant on average 500 meters (minimum 200 meters, maximum 1300 meters), and on three connector queues that carry vehicles from the city to the onramps. For further information, we refer to [13].

In the study, we consider as cells: a) each onramp, b) each offramp, c) each queue, and d) each the segment of lane between two pairs of sensors on the same lane. The length of the cells is set to 200 meters for cells not on the main line, and to their real value for cells on the main line. In total, the network is composed of are 10 onramps, 7 offramps, 3 queues, and 22 sections of main lines, for a total of 67 cells, and is shown in Fig. 3.

The GTL is operating since October 2013 on the whole Rocade. Each pair of sensors provides every T = 15 seconds measurements of

- flow, as number of vehicles that have crossed the two sensors;
- density, via measurements of occupancy of the road above the two sensors;
- speed, as average speed of all vehicles that crossed the two sensors.

The two sources of information used by the algorithm are the following:

- Flow and density measurements every T = 15 seconds on cells $e \in M$, represented in light blue in Fig. 3. Since the corresponding measurement is "expensive", and to show the prowess of the algorithm, we only use one third of the available sensors, and only on the main line. Additional information provided by the sensor network is discarded and only used for validation purposes.
- Speed measurements every minute provided by INRIX. The Rocade has been divided in segments and a measurement of speed is available on each segment. Segments are represented as rectangles encircling cells in Fig. 3.

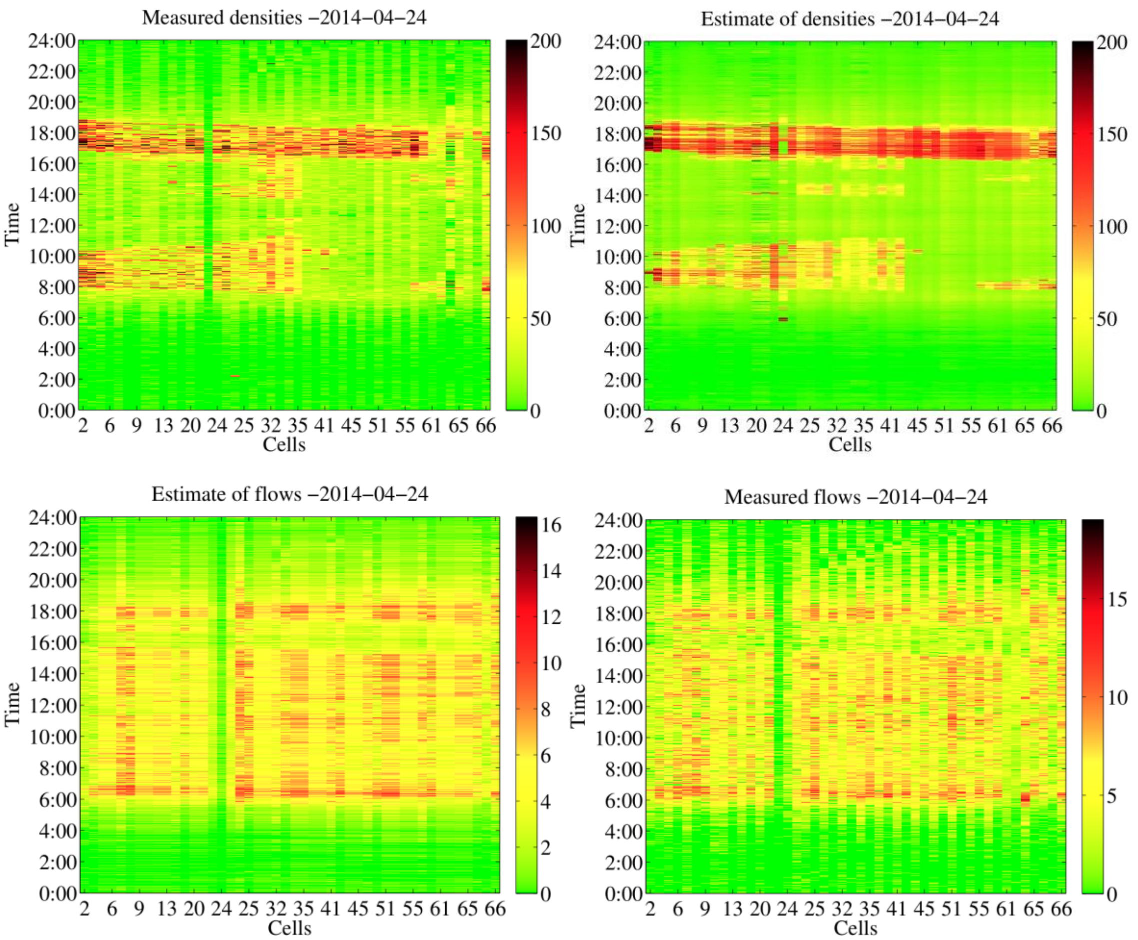

Figure 5: color plot of the measured densities (upper-left panel), estimated densities (upper-right panel), measured flows (lower-left panel) and estimated (out)flows (lower-right panel) along the main line of the Rocade Sud in Grenoble on April 24th, 2014. Cell numbers correspond to cells in Fig. 3.

For the validation of the algorithm, we considered data obtained two weeks later, on Thursday

April 24th, 2014, also a normal working day. We run the estimation algorithm starting at 00:00:00 and for the whole 24 hours, for a total of 5760 time slots. The state of the observer was initialized to zero density, a value close to reality during night time.

The algorithm was implemented in MatLab 2014b and run on a non dedicated commercial laptop HP EliteBook Folio 1040 G1 equipped with Intel Core i7-4600U processor. The optimization problems in the calibration step and in the update step are solved using CVX, a Matlab package for specifying and solving convex problems [14, 15]. The mean executing time was 0.0676 seconds per time slot, for a total of about 6 minutes for a whole day.

The results of the numerical study are presented in Fig. 4. In the upper-left and lower-left panels we provide density and flow measurements for all cells along the main line of the Rocade Sud during April 24th, 2014. The two traffic peaks, between 7:30 and 10:30 and between 16:00 and 19, are clearly visible. Congestion travels backwards in both cases, but in the morning we can notice two points in which congestion originates, namely at Rondeau (end of Rocade) and at Eybens (in the middle), while in the afternoon the traffic is so intense that congestion is continuous throughout the Rocade. Nonetheless, the area immediately before Eybens is more congested than the rest of the Rocade in the afternoon even before the peak – observe for example a smaller congestion that lasts from 14:00 to 15:00.

In the upper-right and lower-right panels we show density and flow estimates, respectively. The algorithm provides good reconstructions, well matching the reality and in particular detecting all major congestion waves. One can observe that the estimated density in congestion is often higher than the real density. The reason has to be found in characteristic of the Fundamental Diagram. Indeed, it should be noted that the relation described by Fundamental Diagram is meant to represent an *average* relation between density and flow in a cell, but in general it captures it in a rather rough manner. In particular, large part of density-flow pairs in congestion lie *below* the Fundamental Diagram in our case study, implying that for a given value of flow the real (congested) density is *lower* than the one predicted by the Fundamental Diagram. Early results show that the situation is greatly improved employing in the congestion region a convex Fundamental Diagram, better suited to follow the rapid decrease of the flow for high densities. This will be subject of further studies.

6. Conclusions

In this paper, we have designed an observer for traffic networks able to fuse heterogeneous information of flow and speed. Calibration algorithms for the Fundamental Diagram and the splitting ratios are discussed, as well as the problem of optimal sensor placement and the trade-off between cost and performance. Future research directions include and are not limited to development of a statistical model for the relationship between flow and densities, design of the algorithm’s parameters for mean-square minimization of the estimation error, and improvement of the algorithms for flow reconstruction. Furthermore, we plan to extend the numerical study to the city of Grenoble.

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