Advanced Real-Time Systems

Lecture 5/6

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Outline

Space of feasible C_i

Theorem (Lemma 3 in [?])

$\mathcal{N}$ *is EDF-schedulable* **if and only if** $\sum_i U_i \leq 1$ *and:*

$$\forall t \in \mathcal{D} \quad \sum_{i=1}^{n} \max\left\{0, \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor\right\} C_i \leq t$$

with

$$\mathcal{D} = \{d_{i,k} : d_{i,k} = kT_i + D_i,\ i \in \mathcal{N},\ k \in \mathbb{N},\ d_{i,k} \leq D^*\}$$

and $D^* = \text{lcm}(T_1, \ldots, T_n) + \max_i\{D_i\}$*.*

$H = \text{lcm}(T_1, \ldots, T_n)$ is often called the *hyperperiod*.

- Since we are investigating the space of C_i, no smaller D^* can be considered.
- For given T_i and D_i, the space of EDF-schedulable C_i is convex!

An example

Let us assume $T_1 = 4, D_1 = 5$ and $T_2 = 6, D_2 = 5$.

$$\mathcal{D} = \{5, 9, 11, 13, 17\}$$

Hence equations are

$$\begin{bmatrix} 4 & 6 \\ 8 & 6 \\ 8 & 12 \\ 12 & 12 \\ 16 & 18 \\ 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \leq \begin{bmatrix} 5 \\ 9 \\ 11 \\ 13 \\ 17 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

An example: viewing the C_i

Reducing the set of deadlines

Hence only the equations

$$\begin{bmatrix} 4 & 6 \\ 16 & 18 \\ 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \leq \begin{bmatrix} 5 \\ 17 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

are needed to be checked, corresponding to $\mathcal{D} = \{5, 17\}$

- Finding a general method to reduce $\mathcal{D}$ without losing sufficiency would be an interesting contribution.

Optimal comp time

- Being the exact region convex the optimal assignment of computation times C_i does not present big difficulties.

Outline

Deadline space: qualitative analysis

- Example: $n = 2$, $T_1 = T_2$, $U_1 + U_2 \leq 1$
- We can reduce D_1 to C_1, but then $D_2 = C_1 + C_2$, or
- $D_2 = C_2$, but then $D_1 = C_1 + C_2$
- convex comb of these two points are not EDF-sched

- Let us assume $T_1 = T_2 = \ldots = T_n$, and $\sum_i U_1 \leq 1$.
- For any permutation $p : \mathcal{N} \to \mathcal{N}$, a "vertex" has coordinates

$$D_{p(i)} = \sum_{j=1}^{i} C_{p(j)}$$

Another exact test

Being the periods and deadlines into the set $\mathcal{D}$ and into the $\lfloor\cdot\rfloor$, dbf condition is unfit to show the space of feasible periods/deadline.

Theorem

The task set $\mathcal{N}$ is EDF-schedulable **if and only if***:*

$$\forall \mathbf{k} \in \mathbb{N}^n \setminus \{\mathbf{0}\} \quad \exists i \in I_{\mathbf{k}} \quad \sum_{j=1}^{n} C_j k_j \leq (k_i - 1)T_i + D_i$$

where

$$I_{\mathbf{k}} = \{j : k_j \neq 0\}$$

is the set of non-zero indexes in $\mathbf{k}$.

EDF-schedulablity as covering problem

$$\forall \mathbf{k} \in \mathbb{N}^n \setminus \{\mathbf{0}\} \quad \exists i : k_i \neq 0 \quad \sum_{j=1}^{n} C_j k_j \leq (k_i - 1)T_i + D_i$$

can be seen as the problem of finding a cover to $\mathbb{N}^n \setminus \{\mathbf{0}\}$ with halfspaces of equations

$$(C_i - T_i)k_i + \sum_{j \neq i} C_j k_j \leq -T_i + D_i$$

considering $(k_1, \ldots, k_n)$ as variables.

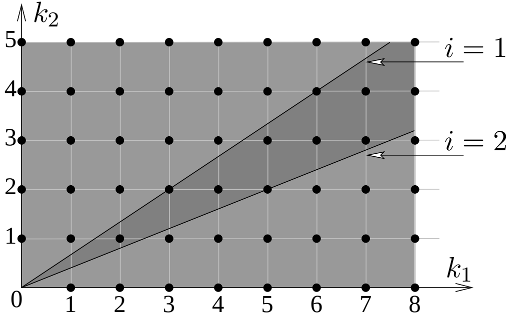
- Remember: $\exists i : k_i \neq 0$, not just $\exists i \in \mathcal{N}$
- If $U = 1$ linearly dependent.

Viewing the covering problem

If $T_1 = D_1 = 4$, $C_1 = 2$, and $T_2 = D_2 = 8$, $C_2 = 3$, then

$$-2k_1 + 3k_2 \leq 0$$
$$2k_1 - 5k_2 \leq 0$$

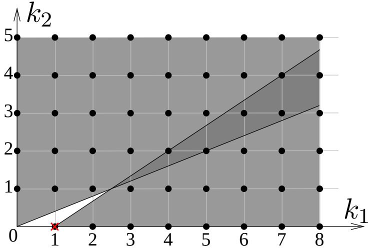
have to cover $\mathbb{N}^2 \setminus \{(0,0)\}$



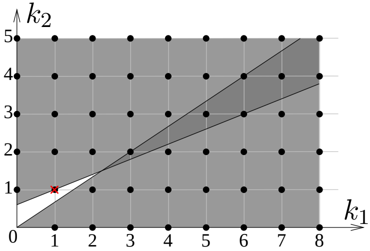
- If $U > 1$ cannot cover, ever.

Changing deadlines

- Deadline D_i appears at the RHS of the i-th inequality only
- Changing D_i means to translate boundary of the i-th halfspace

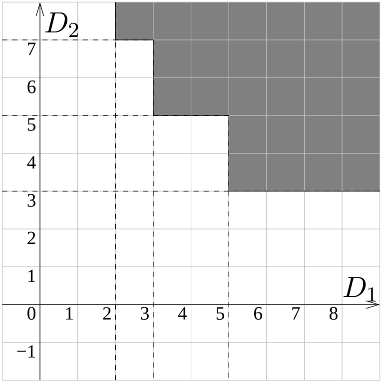


$D_1^{\mathsf{min}} = 2 \quad (= C_1)$



$D_2 = 5$

Space of feasible D_i



Changing the periods

- The i-th period has an effect only on the i-th equation
- however T_i appears in the coefficients and at the RHS
- changing T_i is a rotation of the i-th halfspace
- the points in common between two equations with T_i' and T_i'' are

$$(C_i - T_i')k_i + \sum_{j \neq i} C_j k_j = -T_i' + D_i$$

$$(C_i - T_i'')k_i + \sum_{j \neq i} C_j k_j = -T_i'' + D_i$$

$$k_i = 1$$

$$\sum_{j \neq i} C_j k_j = D_i - C_i$$

Changing periods ($n = 2$)

- As T_1 changes, the 1st boundary rotates around

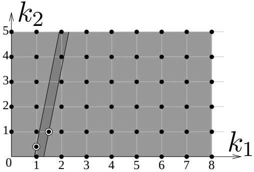
$$k_1 = 1, \quad k_2 = \frac{D_1 - C_1}{C_2}$$

- As T_2 changes, the 2nd boundary rotates around

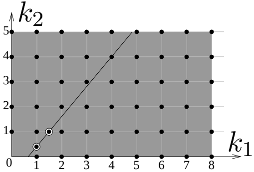
$$k_1 = \frac{D_2 - C_2}{C_1}, \quad k_2 = 1$$

Changing periods: example

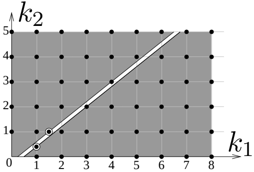
- $C_1 = 2,\ D_1 = 4$, and $C_2 = 5,\ D_2 = 8$
- changing T_1 rotates around $(1, 0.4)$
- changing T_2 rotates around $(1.5, 1)$



- $T_1 = 27$
- $T_2 = {}^{27}/_{5}$



- $T_1 = 8$
- $T_2 = {}^{20}/_{3}$



- $T_1 = 6$
- $T_2 = {}^{15}/_{2}$

- Draw the f-space

Space of deadlines: sufficient test

By making some more restrictive hypothesis, we can derive the following sufficient EDF-test

Theorem (Chantem et al. [?])

A task set is EDF-schedulable if:

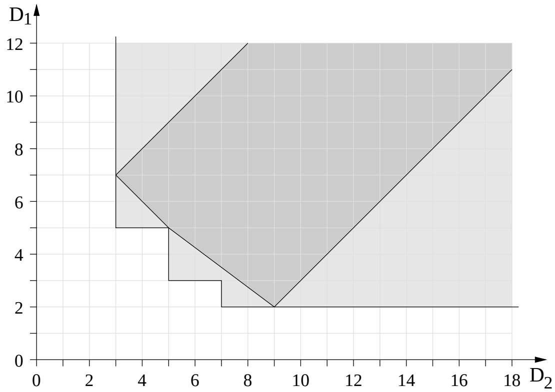
$$\begin{cases} \forall i, \quad D_i \leq T_i + D_{\min} \\ \sum_{i=1}^{n} U_i\, D_i \geq \sum_{i=1}^{n} C_i - D_{\min}\,(1 - \sum_{i=1}^{n} U_i). \end{cases}$$

with $D_{\min} = \min_i D_i$.

This condition has the advantage of being convex in the space of deadlines.

Viewing sufficient cond

If $C_1 = 2$, $T_1 = 4$ and $C_2 = 4$, $T_2 = 7$, we have



Outline

1. EDF: space of comp times
2. EDF: space of deadlines, periods
3. EDF: optimal design

Optimal T_i, D_i assignment

- $\mathcal{X} = \{T_1, D_1, \ldots, T_n, D_n\}$
- Typical problem in control systems:
  - given the execution times $\{C_1, \ldots, C_n\}$
  - choose the sampling periods and deadlines (=delays) of all controllers
  - such that a control cost (LQG?) is minimized

$$\min_{T_1, D_1, \ldots, T_n, D_n} J(T_1, D_1, \ldots, T_n, D_n)$$

s.t $\mathcal{N}$ is EDF-schedulable

Difficulty of the problem

We could try to find optimal T_i and then D_i

1. Suppose $C_1 = C_2 = 1$
2. Suppose that the optimal T_i are $T_1 = 2.1$, $T_2 = 1.91$
3. How much can we reduce the deadlines?
   - very little.
4. However by choosing non-optimal periods $T_1 = T_2 = 2$, we can actually reduce one of the two deadlines by 1, possibly making the cost J smaller

- To best of my knowledge it is still unsolved
- (due to bizarre shape of the feasible region)
- solving this problem optimally would be a decent contribution