Advanced Real-Time Systems

Lecture 4/6

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November 2, 2012

Outline

Earliest Deadline First

- Task model is still the same $\tau_i = (C_i, T_i, D_i)$;
- In FP, priorities are per task: all jobs of same task that have the same priority;
- In EDF, priorities are per job: jobs are prioritized according to their absolute deadline.

Draw an example of schedule.

Most interesting feature

Theorem (Liu and layland, 1973 [?])

If a task set is feasible, then it is EDF-schedulable.

Theorem (Liu and Layland, 1973 [?])

If $D_i = T_i$ (implicit deadline) then a task set is EDF-schedulable **if and only if***:*

$$\sum_{i=1}^{n} U_i \leq 1$$

- Any FP-schedulable task set is also EDF-schedulable task set.

Outline

Demand bound function

If $D_i \neq T_i$ the schedulability condition becomes more complicated.

Theorem (Lemma 3 in [?])

The task set $\mathcal{N}$ is EDF-schedulable **if and only if***:*

$$\forall t \geq 0 \quad \sum_{i=1}^{n} \max\left\{0, \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor\right\} C_i \leq t$$

The LHS is called *demand bound function* dbf(t) of the task set at t.

- dbf(t) is the maximum amount of work of jobs with both activation and deadline in $[0, t]$.
- no per-task condition: any task task may influence others
- $\max\{0, \cdot\}$ only needed for i with $D_i > T_i$

Making it more practical

- Obviously, checking $\forall t > 0$ is not very practical
- By observing the step shape of the dbf we can check only at the points where the steps occur

Theorem (Lemma 3 in [**?**])

The task set $\mathcal{N}$ is EDF-schedulable **if and only if** $\sum_i U_i \leq 1$ *and:*

$$\forall t \in \mathcal{D} \quad \sum_{i=1}^{n} \max\left\{0, \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor\right\} C_i \leq t$$

with

$$\mathcal{D} = \{d_{i,k} : d_{i,k} = kT_i + D_i,\ i \in \mathcal{N},\ k \in \mathbb{N},\ d_{i,k} \leq D^*\}$$

and $D^* = \mathrm{lcm}(T_1, \ldots, T_n) + \max_i\{D_i\}$.

$H = \mathrm{lcm}(T_1, \ldots, T_n)$ is often called *hyperperiod* of the task set.

Reducing the number of points

- If $U < 1$, then the for large t the condition is always true
- then D^* can be computed [?] by upper bounding dbt(t) with a line and we find

$$D^* = \frac{U}{1-U} \max_i \{T_i - D_i\}$$

- what happen to D^* if $\max_i \{T_i - D_i\} \leq 0$?

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- what happen to D^* if $\max_i\{T_i - D_i\} \leq 0$?
- the task set is obviously EDF-schedulable,
  - $\max_i\{T_i - D_i\} \leq 0 \quad \Leftrightarrow \quad \forall i,\ D_i \geq T_i$
  - EDF is sustainable, hence if $D_i = T_i$ is sched then $D_i \geq T_i$ also sched
  - Since $U < 1$, the task set is sched

Faster exact test

- All deadlines in $[0, D^*]$ may be too many
- Zhang and Burns [?] proposed the Quick convergence Processor-demans Analysis (QPA)

```
d_min ← min{D_i}
t ← max{d_{i,k} : d_{i,k} ≤ D*}                ▷ initial assignment
while dbf(t) ≤ t ∧ dbf(t) > d_min do
    if dbf(t) < t then
        t ← dbf(t)
    else
        t ← max{d_{i,k} : d_{i,k} < t}   ▷ escape from fixed points
    end if
end while
if dbf(t) ≤ d_min then task set EDF-schedulable
else task set not EDF-schedulable
end if
```

Outline

Sufficient tests

- By replacing T_i with the more conservative value $\min\{T_i, D_i\}$, we find

$$\sum_{i=1}^{n} \frac{C_i}{\min\{T_i, D_i\}} \leq 1$$

the ratio $\frac{C_i}{\min\{T_i,D_i\}}$ is often called *density* of the task

More sophisticated suff test

Devi proposed the following sufficient test

- Assuming that tasks are sorted by non-decreasing relative deadline ($D_i \leq D_2 \leq \ldots \leq D_n$)

Theorem (Theorem 1 in [?])

The task set $\mathcal{N}$ is EDF-schedulable if:

$$\forall k = 1, \ldots, n \quad D_k \sum_{i=1}^{k} U_i + \sum_{i=1}^{k} \frac{T_i - \min\{T_i, D_i\}}{T_i} C_i \leq D_k$$

- Proved to strictly dominate the density test

FTPAS for EDF

- Albers et al. [?] proposed a Fully Polynomial Time Approximation Scheme.
- The i-th term in dbf(t) can be upper bounded by

$$\max\{0, \left\lfloor \frac{t - D_i + T_i}{T_i} \right\rfloor\} C_i \leq \mathsf{dub}_i(k,t)$$

$$= \begin{cases} \max\{0, \left\lfloor \frac{t-D_i+T_i}{T_i} \right\rfloor\} C_i & t \leq d_{i,k} = (k-1)T_i + D_i \\ U_i(t + T_i - D_i) & t > d_{i,k} \end{cases}$$

so that

$$\frac{k}{k+1} \sum_i \mathsf{dub}_i(k,t) \leq \mathsf{dbf}(t) \leq \sum_i \mathsf{dub}_i(k,t)$$

FTPAS for EDF

- This enables a quite interesting result

Theorem

If $U \leq 1$ *and*

$$\forall t \in \mathcal{D}(\overline{k}) = \{d_{i,k} \in \mathbb{R}^+ : d_{i,k} = (k-1)T_i + D_i, 1 \leq k \leq \overline{k}\}$$

$$\sum_{i=1}^{n} \mathsf{dub}_i(\overline{k}, t) \leq t$$

then the task set is schedulable.
Otherwise it is not schedulable over a CPU with speed $\frac{\overline{k}}{\overline{k}+1}$.

- In this way only $n\overline{k}$ evaluation of the dbf are needed.
- We can trade accuracy vs. complexity. As $\overline{k} \to \infty$ it becomes necessary and sufficient.
- FPTAS

Outline

Min EDF-schedulable speed

Similarly as in the FP case we can find the minimum EDF-schedulable speed as follows

$$r^{\mathsf{min}} = \max_{t\in\mathcal{D}} \frac{\sum_{i=1}^{n} \max\left\{0, \left\lfloor \frac{t+T_i-D_i}{T_i} \right\rfloor\right\} C_i}{t}$$

However $D^* = H + \max_i\{D_i\}$, because we are changing the speed and then altering the linear upper bound that motivates the expression

$$D^* = \frac{U}{1-U} \max_i \{T_i - D_i\}$$

Max EDF-schedulable comp time

The maximum EDF-schedulable C_k^{max} can be computed in a similar way as in FP

$$C_k^{\mathsf{max}} = \min_{t \in \mathcal{D}, t \geq D_k} \frac{t - \sum_{\substack{i=1 \\ i \neq k}}^{n} \max\left\{0, \left\lfloor \frac{t+T_i-D_i}{T_i} \right\rfloor\right\} C_i}{\left\lfloor \frac{t+T_k-D_k}{T_k} \right\rfloor}$$

Here too, $D^* = H + \max_i\{D_i\}$.

Outline

Sufficient tests

Let us have the following task set

T_i	D_i	C_i
3	5	1
8	8	2
20	10	5

Not DM-schedulable since $R_3 = 14 > D_3 = 10$

- Density test

$$\frac{1}{3} + \frac{2}{8} + \frac{5}{10} = \frac{13}{12} > 1$$

- Devi's test

$$k = 1 \quad D_1U_1 \leq D_1 \quad \Rightarrow \quad \text{OK}$$

$$k = 2 \quad D_2(U_1 + U_2) \leq D_2 \quad \Rightarrow \quad \text{OK}$$

$$k = 3 \quad D_3(U_1 + U_2 + U_3) + (T_3 - D_3)U_3 \leq D_3$$

$$U_1 + U_2 + \frac{C_3}{D_3} \leq 1 \quad \Rightarrow \quad \text{NO}$$

Example

Exact test:

$$\forall t \in \mathcal{D} \quad \sum_{i=1}^{n} \max\left\{0, \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor\right\} C_i \leq t$$

with

$$\mathcal{D} = \{d_{i,k} : d_{i,k} = kT_i + D_i,\ i \in \mathcal{N},\ k \in \mathbb{N},\ d_{i,k} \leq D^*\}$$

Computing the upper bound D^* to the set of deadlines $\mathcal{D}$

$$D^* = \frac{U}{1-U} \max T_i - D_i = \frac{\frac{5}{6}}{\frac{1}{6}} 10 = 50$$

$$\mathcal{D} = \{5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50, 8, 16, 24, 32, 40, 48, 10, 30, 50\}$$

Example: QPA

$$
\begin{aligned}
d_{\text{min}} &= 5\\
t = 50, &\quad \text{dbf}(50) = 16 + 6 \times 2 + 3 \times 5 = 43\\
t = 43, &\quad \text{dbf}(43) = 13 + 5 \times 2 + 2 \times 5 = 33\\
t = 33, &\quad \text{dbf}(33) = 10 + 4 \times 2 + 2 \times 5 = 28\\
t = 28, &\quad \text{dbf}(28) = 8 + 3 \times 2 + 5 = 19\\
t = 19, &\quad \text{dbf}(19) = 5 + 2 \times 2 + 5 = 14\\
t = 14, &\quad \text{dbf}(14) = 4 + 2 + 5 = 11\\
t = 11, &\quad \text{dbf}(11) = 3 + 2 + 5 = 10\\
t = 10, &\quad \text{dbf}(10) = 2 + 2 + 5 = 9\\
t = 9, &\quad \text{dbf}(9) = 2 + 2 = 4\\
&\quad \text{exit because dbf}(9) = 4 < d_{\text{min}} = 5
\end{aligned}
$$

dbf computed 9 times, instead of $|\mathcal{D}| = 22$ times

Example: FPTAS

$$
\begin{aligned}
\overline{k} &= 1 & \mathcal{D}(1) &= \{5, 8, 10\} \\
t &= 5 & 1 + 0 + 0 &\leq 5 \\
t &= 8 & 2 + 2 + 0 &\leq 8 \\
t &= 10 & 2.666 + 2.5 + 5 &> 10 \\
\overline{k} &= 2 & \mathcal{D}(2) &= \{5, 8, 10, 16, 30\} \\
t &= 5 & 1 + 0 + 0 &\leq 5 \\
t &= 8 & 2 + 2 + 0 &\leq 8 \\
t &= 10 & 2.666 + 2 + 5 &\leq 10 \\
t &= 16 & 4.666 + 4 + 5 &\leq 16 \\
t &= 30 & 9.333 + 7.5 + 10 &\leq 30
\end{aligned}
$$

- EDF-schedulable
- non EDF-schedulable over a speed $\frac{1}{2}$ processor