Adaptive Dual Control

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1. Introduction

In all control problems there are certain degrees of uncertainty with respect to the process to be controlled. The structure of the process and/or the parameters of the process may vary in an unknown way. There are several ways to handle these types of uncertainties in the process. Feedback in itself makes the closed loop system, to some extent, insensitive against process variations. Fixed parameter controllers can also be designed to make the closed loop system robust against process variations. Such controllers must, by nature, be conservative in the sense that the bandwidth of the closed loop system has be decreased to reduce the influence of the variation in the process. Another way to handle uncertainties is to use an adaptive controller. In the adaptive controller there are attempts to identify or estimate the unknown parameters of the process. Most adaptive controllers have the structure shown in Figure 1, which is a self-tuning adaptive control system. The inputs and the outputs of the process are fed to the estimator block, which delivers information about the process to the controller design block. The design block uses the latest process information to determine the parameters of the controller. The adaptive controller thus consists of an ordinary feedback loop and a controller parameter updating loop. Different classes of adaptive controllers are obtained depending on the process information that is used in the controller and how this information is utilized.

To obtain good process information it is necessary to perturb the process. Normally, the information about the process will increase with the level of perturbation. On the other hand the specifications of the closed loop system are such that the output normally should vary as little as possible. There is thus a conflict between information gathering and control quality. This problem was introduced and discussed by A. A. Feldbaum in a sequence of four seminal papers from 1960 and 1961, see the references. Feldbaum's main idea is that in controlling the unknown process it is necessary that the controller has dual goals. First the controller must control the process as well as possible. Second, the controller must inject a probing signal or perturbation to get more information about the process. By gaining more process information better control can be achieved in future time. The compromise between probing and control or in Feldbaum's terminology investigating and directing leads to the concept of *dual control*.

Feldbaum showed that a functional equation gives the solution to the dual control problem. The derivation is based on dynamic programming and the resulting functional equation is often called the *Bellman equation*.

The solution to this equation is intractable from a numerical point of view and only a few very simple examples have been solved, analytically or numerically. There is thus a great need for looking at different approximations that can lead to simpler suboptimal solutions with dual features. In the suboptimal dual controllers it is necessary to introduce both cautious and probing features. Both parts of the control action can be obtained in numerous ways and different proposed schemes will be classified into

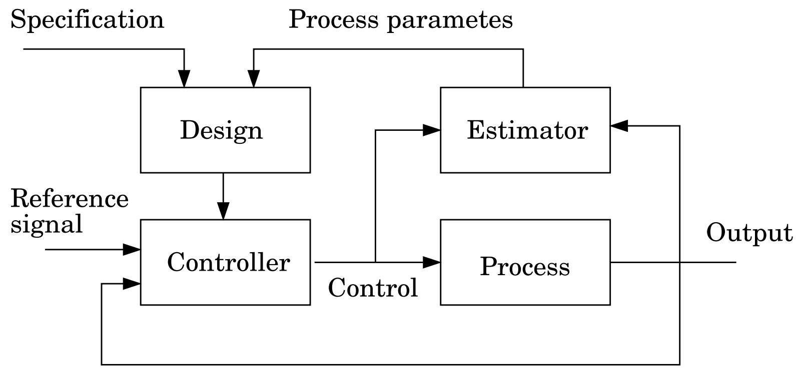

Figure 1 Self-tuning adaptive control system.

a handful of principles. This article gives an overview of adaptive dual control. To do so it is also necessary to introduce some concepts from the general field of adaptive control.

2. Stochastic adaptive control

To formulate the adaptive dual control problem we must specify the model for the process, the admissible control signals, and the specifications (loss function) for the closed loop system.

Introduce the following notations: $y(k)$ is the process output, $u(k)$ is the control signal, $\theta(k)$ is a vector of the unknown parameters of the process, $\hat{\theta}(k)$ is the current estimate of the process parameters, and $P(k)$ is the parameter uncertainty. Inputs up to time $k-1$ and outputs up to time k are collected into the vector

$$\mathcal{Y}_k = \begin{bmatrix} y(k) & y(k-1) & u(k-1) & \dots & y(0) & u(0) \end{bmatrix}$$

It is assumed that the process is described by the discrete time model

$$y(k+1) = f(u(k), \mathcal{Y}_k, \theta(k), \zeta(k))$$

where $\zeta(k)$ is a stochastic process driving the process and/or the parameters of the process. The probability distribution of ζ is assumed known. This implies that the output at the next sampling instance, $k+1$ is a, possibly nonlinear, function of the control signal to be determined at time k, some, not necessarily all, of the elements in $\mathcal{Y}_k$, and of the unknown process parameters. It is assumed that the function $f(\cdot)$ is known. This implies that the structure of the process is known but that there are unknown parameters, $\theta(k)$.

The admissible controllers are causal functions $g(\cdot)$ of all information gathered up to time k, i.e. $\mathcal{Y}_k$. If the parameters of the process are known the control signal at time k is also allowed to be a function of $\theta(k)$.

The performance of the closed loop system is measured by a loss function, that should be as small as possible. Assume that the loss function to be minimized is

$$J_N = E\left\{ \frac{1}{N} \sum_{k=1}^{N} h(y(k), u(k-1), y_r(k), k) \right\} \tag{1}$$

where y is the process output, y_r is the reference signal, $h(\cdot)$ is a positive convex function, and E denotes mathematical expectation taken over the distribution of ζ. This is called an *N-stage criterion*. The loss function should be minimized with respect to the admissible control signals $u(0)$, $u(1)$, ..., $u(N-1)$. A simple example of the loss function is

$$J_N = E\left\{ \frac{1}{N} \sum_{k=1}^{N} (y(k) - y_r(k))^2 \right\} \tag{2}$$

The parameters of the process can be described in several different ways, for instance, as

- Random walk
- Random walk with local and global trends
- Jump changes
- Markov chain

Random walk implies that the parameters are drifting due to an underlying stochastic process. In the Markov chain model the parameters are changing between a finite number of possible outcomes. Depending on the type of variation of the process parameters it is necessary to use different estimation methods. It is thus assumed that the parameter variation is described in stochastic terms where the probability distribution of the process is known.

Different types of prediction error methods can be used to obtain the probability distribution of the parameters. If the process is linear in the parameters and if the parameter variations can be described by a Gaussian process then the distribution is fully characterized by the mean value $\hat{\theta}(k)$ and the covariance matrix $P(k)$. The covariance matrix is used as a measure of the uncertainty of the parameter estimates. The future behavior of $P(k)$ depends on the choice of the control signal.

The model, with the description of its parameter variations, the admissible control laws, and the loss function are now specified. The adaptive control problem has been transformed into an optimization problem, where the control signals over the control horizon have to be determined. One of the difficulties in the optimization problem is to anticipate how the future behavior (or formally the behavior of the distribution function) of the parameter estimates will be influenced by the choice of the control signals. The controllers minimizing (1) are very different if $N = 1$ or if N is large.

The stochastic adaptive control problem can be attacked in many different ways. Many adaptive controllers are based on the *separation principle*. This implies that the unknown parameters are estimated separately from the design part. The separation is sometimes optimal and is in other cases used as an assumption. The separation principle holds, for instance, for the Gaussian case and when the process is linear in the unknown parameters, and the loss function is a quadratic function.

Assume that for the known parameter case the optimal controller is

$$u(k) = g_{known}(\mathcal{Y}_k, \theta(k))$$

The simplest adaptive controller is thus obtained by estimating the unknown process parameters $\hat{\theta}(k)$ and then use them as if they were the true ones, i.e. use the controller

$$u(k) = g_{known}(\mathcal{Y}_k, \hat{\theta}(k))$$

An adaptive controller of this kind is said to be based on the *certainty equivalence principle*. Self-tuning controllers are, in general, of this kind.

The control actions determined in the design block, when using the certainty equivalence principle, do not take any active actions that will influence the uncertainty.

An optimal adaptive controller should also take the quality of the parameter estimates into account when designing the controller. Poor estimates, or information, should lead to other control actions than good estimates. A simple modification of the certainty equivalence controller is obtained by minimizing the loss function (1) only one step ahead. This leads to a controller that also uses the uncertainties of the parameter estimate. This type of controller is called a *cautious controller*. The cautious controller has the form

$$u_{cautious}(k) = g_{cautious}(\mathcal{Y}_k, \hat{\theta}(k), P(k))$$

The cautious controller obtained when the control horizon in (1) is $N = 1$ is sometimes also called a myopic controller, since it is short-sighted and looks only one step ahead. The cautious controller hedges against poor process knowledge. A consequence of this caution is that the gain in the controller is decreased. With small control signals less information will be gained about the process and the parameter uncertainties may increase and even smaller control signals will be generated. This vicious circle leads to *turn-off* of the control. This problem mainly occurs for systems with strongly time-varying parameters. An adaptive control scheme is sometimes also denoted *weakly dual* if it uses the model uncertainties when deriving the control signal.

The certainty equivalence and the cautious controllers do not deliberately take any measure to improve the information about the unknown process parameters. They are thus non-dual adaptive controllers. The learning is "accidental" or "passive", i.e. there is no intentional probing signal introduced.

Example Consider an integrator process in which the gain is changing in a stochastic way, i.e. we have the model

$$y(k) - y(k-1) = \theta(k)u(k-1) + e(k)$$

where $e(k)$ is white noise. The gain of the integrator is modeled as

$$\theta(k+1) = \varphi\theta(k) + v(k)$$

where φ is known and $v(k)$ is white noise.

The certainty equivalence controller that minimizes the variance of the output is given by

$$u(k) = -\frac{1}{\hat{\theta}(k+1)}y(k)$$

It is immediately clear that this controller is not good when $\hat{\theta} = 0$. The cautious controller is

$$u(k) = -\frac{\hat{\theta}(k+1)}{\hat{\theta}^2(k+1) + p_\theta(k+1)}y(k)$$

where p_θ is the uncertainty of the estimate $\hat{\theta}$. By including the parameter uncertainty the gain in the controller is decreased when p_θ becomes large. The cautious controller is also less sensitive than the certainty equivalence controller to parameter errors when $\hat{\theta}(k+1)$ is small. The gain in the cautious controller approaches zero when p_θ increases, i.e. there is a possibility that the control action is turned off when the excitation of the process decreases. The cautious controller approaches the certainty equivalence controller when p_θ approaches zero. □

3. Optimal dual controllers

To understand some of the difficulties of calculating the optimal dual controller we will give the functional equation that can be used to calculate the dual controller. For simplicity, consider the quadratic loss function (2). Assuming a model that is linear in the parameters the separation principle is optimal. The optimal controller is decomposed into two parts: an estimator and a feedback regulator. The estimator generates the conditional probability distribution of the state given the measurements $\mathcal{Y}_k$. This distribution is called the *hyperstate* of the process and is denoted $\xi(k)$. There is no distinction between the parameters and the other state variables in the hyperstate. The controller is then able to handle very rapid parameter variations.

The feedback regulator is a nonlinear function mapping the hyperstate into the control signal, see Figure 2. The hyperstate includes the parameter estimates, their accuracy, and old inputs and outputs of the system. Notice the similarity with the self-tuning controller in Figure 1. The structural simplicity of the controller is obtained thanks to the introduction of the hyperstate. The output signal is included in in the hyperstate, but in Figure 2 the ordinary feedback loop is kept to further illustrate the similarity with an ordinary adaptive controller. Unfortunately, the hyperstate will be of very high dimension making the calculations difficult.

The general multi-step optimization problem can be solved using the idea of dynamic programming. Assuming that the properties of the stochastic variables in the process are known we can define

$$V\left(\xi(k),\, k\right) = \min_{u(k-1)\,\ldots\, u(N-1)} E\left\{ \sum_{j=k}^{N} \left(y(j) - y_r(j)\right)^2 \left| \mathcal{Y}_{k-1} \right. \right\}$$

where $V(\xi(k), k)$ can be interpreted as the minimum expected loss for the remaining part of the control horizon given data up to $k-1$, i.e. $\mathcal{Y}_{k-1}$.

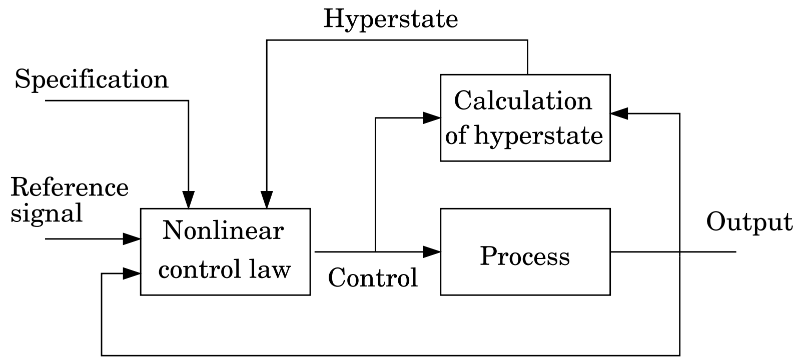

Figure 2 Block diagram of an adaptive controller obtained from stochastic control theory.

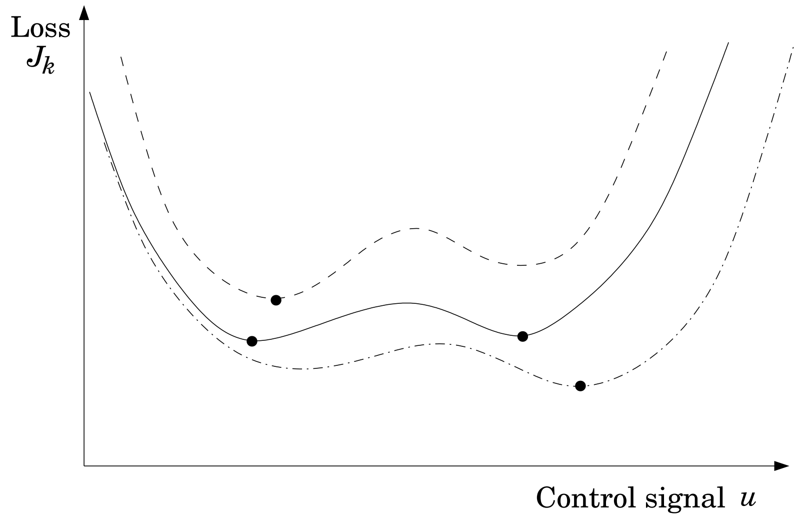

Figure 3 A possible shape of the loss J_k as function of the control signal u and for three different values of a scalar hyperstate ξ. The absolute minima for the three cases are marked by dots.

It can be shown that if the optimal dual controller exists it must satisfy the *Bellman equation*

$$V\left(\xi(k),\, k\right) = \min_{u(k-1)} J_k = \min_{u(k-1)} E\Big\{ \left(y(k) - y_r(k)\right)^2 + V\left(\xi(k+1),\, k+1\right) |\mathcal{Y}_{k-1} \Big\} \tag{3}$$

The difficulty with this equation is the nested minimization and mathematical expectation. The minimization in (3) is done over one variable, $u(k-1)$, but the problem is that the dimension of the hyperstate is very large, which complicates the minimization as well as evaluating the conditional expectation. Also it is difficult to give conditions for when the solution to the dynamic programming solution actually exists.

The choice of $u(k-1)$ influences the immediate loss, the future parameter estimates, their accuracy, and also the the future values of the outputs of the process. The controller will thus have the desired dual feature in contrast to the certainty equivalence and cautious controllers.

Except for very special cases the Bellman equation has to be solved numerically. Since both V and u have to be discretized it follows that the storage requirement increases drastically with decreasing grid size. Optimal dual controllers have thus only been calculated for some very simple and specialized cases. The resulting control law will be nonlinear in the parameter estimates and the covariance matrix of the parameter estimates. The simple examples give, however, some useful indications how suboptimal dual controllers can be constructed.

To illustrate how the optimal dual can switch between probing and regulating consider Figure 3. In the figure the function J_k in (3) is given for three possible values of a scalar hyperstate. J_k has several minima. For the dashed curve local minimum to the left gives the absolute minimum, while for the full line case the two local minima have the same value. Finally, for the dash-dotted curve the local minimum to the right represents the absolute minimum. The control action will thus switch in character when ξ

is changed. This can be interpreted as that the control action is switching between probing and control actions.

The minimization over several steps to obtain the dual controller makes is possible for the controller to introduce probing in the beginning or when the information about the process is poor and still gaining by being able to make a better control towards the end of the control horizon.

4. Suboptimal dual controllers

The difficulties to find the optimal solution have made it interesting to find approximations to the loss function or to find other ways to change the controller such that a dual feature is introduced. The suboptimal adaptive dual controllers can be constructed in essentially two ways. Either by various approximations of the optimal dual control problem or by reformulating the problem such that a simple solution can be calculated that still has some dual features. Some ways to construct suboptimal dual controllers are:

- Adding perturbation signals to the cautious controller
- Constraining the variance of the parameter estimates
- Approximations of the loss function
- Modifications of the loss function
- Finite parameter sets

4.1 Perturbation signals

The turn-off phenomenon is due to lack of excitation. The intentional addition of a perturbation signal is one way to increase the excitation of the process and to increase the accuracy of the estimates. Typical added signals are pseudo-random binary sequences, square-waves, and white noise signals. The perturbation can be added all the time or only when the uncertainty of a process parameter is exceeding some limit. The controller may have the form

$$u_{perturb}(k) = u_{cautious}(k) + u_p(k)$$

where u_p is the intentional perturbation signal. The addition of the extra signal will naturally increase the probing loss but may make it possible to improve the total performance by decreasing the control loss in future steps. A drawback with the introduction of the perturbation signal is that there is no systematic way of deciding when to add the signal and how large the signal should be.

4.2 Constrained one-step-ahead minimization

Another class of suboptimal dual controllers is obtained by minimizing the loss function one-step-ahead under certain constraints. The constraints are used to guarantee a certain level of accuracy of the parameter estimates. Suggested constraints are, for instance, to limit the minimum value of the control signal or to limit the variance of the parameter estimates by intentionally increasing the gain in the controller.

4.3 Approximations of the loss function

One approach to obtain a suboptimal dual controller is to make a serial expansion of the loss function in the Bellman equation. The expansion can be done around the certainty equivalence or the cautious controllers. The numerical computations are, however, still quite complex and the approach has mainly been used when the control horizon is short.

Another approximation is to solve the two-step minimization problem ($N = 2$). A suboptimal dual controller with a two-step horizon is determined. The suboptimal dual control modifies the cautious controller design by numerator and denominator correction terms which depend upon the sensitivity functions of the expected future cost and avoids the turn-off and slow convergence. The two step problem gives clues how to make sensible approximations that retain the dual features.

A third way to make an approximation of the loss function is to modify the available information in the evaluation of the loss function. One such modification is to assume that no further information will be available when evaluating the mean value in the Bellman equation.

4.4 Modifications of the loss function

An approach that is similar to constrained minimization is to extend the one-step-ahead loss function. The idea is to add terms in the loss function that are reflecting the quality of the parameter estimates. This will prevent the cautious controller from turning off the control. It is important to add as simple terms as possible to make it easy to numerically be able to find the resulting controller.

One possibility is to add terms depending on the covariance matrix of the parameter estimates. This leads to a loss function of the form

$$E\left\{(y(k+1) - y_r(k+1))^2 \,|\mathcal{Y}_k\right\} + \lambda f(P(k+2))$$

where λ is a weighting factor and $P(k+2)$ is the first time at which the covariance is influenced by $u(k)$.

Another possibility is to extend the quadratic loss function with a term that reflects the need to gather as much information as possible about the unknown parameters. This gives the loss function

$$E\Big\{\,(y(k+1) - y_r(k+1))^2 - \lambda(k+1)\nu^2(k+1)|\mathcal{Y}_k\Big\}$$

where ν is the innovation or prediction error, i.e.

$$\nu(k+1) = y(k+1) - \hat{y}(k+1, \hat{\theta}(k+1))$$

For this loss function it is possible to find a closed form solution for the control signal.

4.5 Finite parameter sets

When the parameter set contains a finite number of elements it is easier to numerically solve the dual control problem. The number of possible combinations will be considerably reduced since the mathematical expectation in the Bellman equation is then replaced by a summation.

5. When to use dual control?

Non-dual adaptive controllers are successfully used today in many applications. When may it be advantageous to use a controller with dual features? One obvious situation is when the time horizon is short and when the initial estimates are poor. It is then necessary to rapidly find good estimates before reaching the end of the control horizon. It has been suggested that dual controllers are suitable for economic systems. The reason is the short time horizon and the highly stochastic parameters in the processes.

Another situation when to use dual control is when the parameters of the process are changing very rapidly. This is a situation that is not very common in practice. There are, however, processes where the parameters are changing fairly rapidly and the gain is also changing sign. This is the situation when the process has an even nonlinearity and it is desired to operate the process close to the extremum point. The gain of the linearized model will then change sign and at the same time some of the parameters may be small. One successful example reported in the literature is grinding processes in the pulp industry. This application is probably the first true application of suboptimal dual control to process control. The controller is an active adaptive controller, which consists of a constrained certainty equivalence approach coupled with an extended output horizon and a cost function modification to get probing.

6. Summary

The solution to the optimal adaptive control problem over an extended time horizon leads to a controller that have dual features. It uses control actions as well as probing actions. The solution of the dual control problem is intractable from a computational point of view. Approximations to obtain simpler suboptimal dual controllers are thus used instead.

There are many ways to obtain suboptimal dual controllers. Many of the approximations use the cautious controller as a starting point and introduce different active probing features. This can be done by including a term in the loss function that reflects the quality of the estimates. To introduce a dual feature this term must be a function of the control signal that is going to be determined and it should also contain information about the quality of the parameter estimate. The suboptimal controllers should also be such that they easily can be used for higher-order systems.

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