Research Statement

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My main research interests lie in modeling and control of large-scale systems. A large-scale system can be considered as a collection of smaller systems connected to each other in some manner. Standard academic techniques both in modeling and control have not yet been fully extended to systems with a structure, which makes it an exciting topic to work on. The implication in various fields of engineering is also considerable, since in practice many decisions are taken and computations are made in a distributed or decentralized fashion.

The topic of modeling general structured systems can potentially answer many important questions. For example, identification of network topology of a system using limited information about the system, design of optimal network topology for specific applications, reducing the network by eliminating or replacing inefficient nodes etc. Surely there are tools to address these problems, however in my understanding improving state of the art in this area is far from being complete.

Other research interests include linear-parameter varying system modeling, frequency domain identification and optimization techniques. These topics were partially investigated during my PhD project.

1. Research During PhD Project

The main topic of my PhD thesis is order reduction of linear time- invariant models with three main research directions highlighted in the thesis. First one is the **scalable $\mathcal{H}_\infty$ model order reduction** algorithm, which was tackled by many researchers and the optimal solution is still not known. However, there exist a vast amount of methods, which solve suboptimal problems and deliver very accurate solutions. Recently more attention of the research community have been devoted to large-scale systems and computationally scalable extensions of existing techniques. Indeed, when the number of equations in the original model is more than a million, the accuracy is not crucial - the speed of the algorithm is of essence. The method developed in my PhD thesis is relatively computationally cheap for a large class of models. Numerical simulations also show that the accuracy is comparable to the best techniques.

For some systems it is much easier to measure frequency response for specific frequencies than actually compute it. Thus the developed method can be also seen as $\mathcal{H}_\infty$ frequency domain identification from exact measurements, although some noise models can be easily included into the framework. A more conventional approach would consist of $\mathcal{H}_2$ model matching with a noise model and, perhaps, $\mathcal{H}_1$ regularization term. Such an objective can be treated using techniques described in my PhD thesis, however, this work has not been done. An intriguing detail about $\mathcal{H}_2$ model matching is that optimal conditions for optimality are known in the literature and evaluation of optimality in theory can be performed.

The second part of the thesis is devoted to **parameterized model order** reduction. The parameterized model is essentially a family of models which depend on a certain parameter. The model reduction goal is to approximate the whole family of models for all values of parameters. Surely these methods are extremely computationally complex, however, they are also required in practical applications. These methods are straightforward extensions of the ones described above. Besides the extension itself theoretical properties of the method were investigated: error bounds and continuity of solution with respect to a parameter.

Linear Parameter Varying models seem like a natural generalization of the parameterized models. As an easiest example of an LPV model we may consider a linear time-varying (LTV) model, where time is a parameter. However the parameter also depends on time and state or frequency variable in the frequency domain. As a treatment the framework described in my PhD thesis may be applied with assumption that the parameter is constant. However this approach is quite conservative for some systems. It is not entirely clear if such conservatism may be reduced by computing in advance additional information and e.g. designing frequency and parameter dependent weights. It may be a subject of future research.

Finally, **structured model reduction** was investigated. A structured model is, in fact, a collection of models, which are connected to each other in a certain manner. The entire model is called in the thesis *a supersystem,* as opposed to *subsystems,* which a supersystem consists of. Typical model reduction tools usually destroy any structure during the approximation procedure. Therefore there is a need in specialized tools to address these problems. The objective of model reduction in this setting is to approximate certain subsystems in such a way that crucial characteristics of a supersystem do not deteriorate too much. The direction taken in the thesis is studying coprime factorizations of a linear fractional transformation (LFT) and using them to develop a model reduction tool. The investigation raised another question: how structures in transfer functions are translated to structures in state-spaces matrices. The structured model reduction problem is tightly related with a decentralized control problem, since both are non-convex optimization problems, where a decision variable has a certain sparsity structure.

As a subclass of structured model I also consider positive models, where all the matrices in the state space form have non-negative entries. Positive matrices are used to models undirected graphs, perhaps it is one of the reason why positive systems naturally model a structure. Another great property is admittance of a, so called, vector Lyapunov function. As a consequence Gramians also have the same amount of freedom and application of balanced truncation algorithm is straight-forward. However initial results indicate that the method is very conservative.

2. Future Research Directions

In every direction investigated during my PhD thesis there was left a few interesting unanswered questions. For example, understanding relationships between structures in frequency and time domains, order reduction of positive systems, LPV modeling etc. In future I would like to address some of these problems due to their significance and possible applications. However, I am open to suggestions and willing to explore new problems and new challenges.