LUND UNIVERSITY

Department of
AUTOMATIC CONTROL

FRTN20 – Market-driven Systems

Exam, 2016-06-02, 14.00-19.00

Points and grades

All answers must include a clear motivation and a well-formulated answer. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each problem.

Accepted aid

Standard mathematical tables like TEFYMA, an authorized “Formelsamling i Reglerteknik”/”Collection of Formulas” and a pocket calculator.

Results

The results will be entered into LADOK and the solutions will be posted on the course home page:
http://www.control.lth.se/Education/EngineeringProgram/FRTN20.html

Formulas

$$
\begin{aligned}
U_{ud} &= \operatorname{sign}\left(U + \operatorname{sign}\left((I - U_d)(U - \mathbf{1}\mathbf{1}^T)\right)\right) \\
U_{av} &= U \cdot \mathbf{1}/n_s \\
A_{dir} &= \mathbf{1}\mathbf{1}^T + \operatorname{sign}(A_u(U - \mathbf{1}\mathbf{1}^T)) \\
A_{av}^{dir} &= A_{dir} \cdot \mathbf{1}/n_s \\
A_{tot} &= \mathbf{1}\mathbf{1}^T + \operatorname{sign}(A_d(A_{dir} - \mathbf{1}\mathbf{1}^T)) \\
A_{av}^{tot} &= A_{tot} \cdot \mathbf{1}/n_s \\
J_p^{dir} &= (\mathbf{1} - A_{av}^{dir}). * q^m. * p n_s t_s \\
J_p^{tot} &= (\mathbf{1} - A_{av}^{tot}). * q^m. * p n_s t_s \\
J_u^{dir} &= \operatorname{diag}(\mathbf{1} - U_{av}^{ud}) \cdot A_u^T (q^m. * p) n_s t_s \\
J_u^{tot} &= \operatorname{diag}(\mathbf{1} - U_{av}^{ud}) \cdot \operatorname{sign}(A_d A_u)^T (q^m. * p) n_s t_s
\end{aligned}
$$

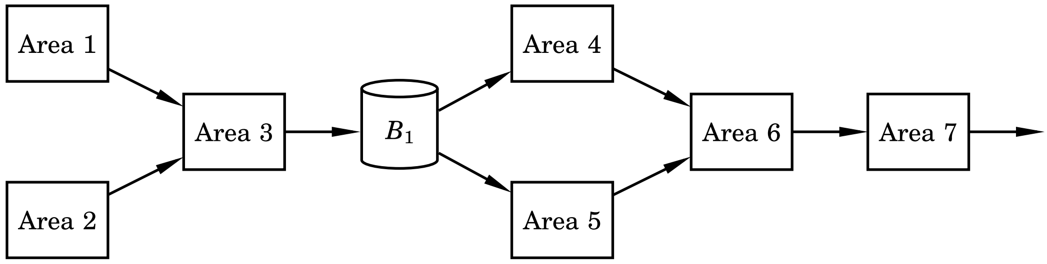

Figure 1 Interconnection of production areas for *Cookie Corner*, used in problems 1-5.

The first 5 problems will be about the small factory *Cookie Corner* that is a part of the *Big Bakery Brand*. In Cookie Corner two different products are made, cinnamon rolls and mudcake.

The Cookie Corner factory can be represented by the interconnection of production areas shown in Figure 1. Area 1 and 2 are machines that mixes the ingredients, Area 3 is a big table for baking out the products. They are then put on baking sheets and placed in Buffer 1. Area 4 and 5 are two different ovens. Area 6 is a cooling area where the products are stored until they are cold enough to go to the packaging step in Area 7.

The three main utilities considered in this factory are electricity, baking sheets and manpower. *Remark: Note that some of these are not usually considered as utilities.* Electricity is needed in the mixing machines, the ovens and the cooling area. Baking sheets are needed in Buffer 1, the ovens and the cooling area. Manpower is needed at the baking table and in the packaging area.

1.

a. Write the area dependence matrix A_d for the factory. (1 p)

b. Write the area-utility matrix A_u for the factory. (1 p)

c. Calculate the average total area availability matrix A_{av}^{tot}, given the utility matrix

$$U = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

where the utilities are in the order electricity, baking sheets, manpower. (1.5 p)

d. Is the result in c) reasonable? Motivate. (0.5 p)

Solution

a.

$$A_d = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

b.

$$A_u = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

c. To calculate A_{av}^{tot} we need A_{tot} which needs A_{dir}. So first we calculate

$$\begin{aligned} A_{dir} &= \mathbf{1}\mathbf{1}^T + \text{sign}(A_u(U - \mathbf{1}\mathbf{1}^T)) = \\ &= \mathbf{1}\mathbf{1}^T + \text{sign}\left[\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{pmatrix}\right] = \\ &= \mathbf{1}\mathbf{1}^T + \begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 \\ -1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{pmatrix} \end{aligned}$$

This result is then entered into the expression for A_{tot} which gives

$$\begin{aligned} A_{tot} &= \mathbf{1}\mathbf{1}^T + \text{sign}(A_d(A_{dir} - \mathbf{1}\mathbf{1}^T)) = \\ &= \mathbf{1}\mathbf{1}^T + \text{sign}\left[\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 \\ -1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{pmatrix}\right] = \\ &= \mathbf{1}\mathbf{1}^T + \begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

Then we get the result by taking the average

$$A_{av}^{tot} = A_{tot} \cdot \mathbf{1}/n_s = \begin{pmatrix} 4/5 & 4/5 & 2/5 & 1/5 & 1/5 & 1/5 & 1/5 \end{pmatrix}^T$$

d. Yes it is reasonable. Since electricity is off 1/5 of the samples the first two areas that are only affected by that utility will have availability 4/5. Since all the other areas depend on these areas they will be affected in the same way. The third area is also affected by the two losses of manpower and hence only has an availability of 2/5. And the loss of baking sheets affects all areas from 4 and onwards which gives them an averaga total availability of 1/5.

2.

a. The production in Cookie Corner is of batch type, but batch processes can be classified as single-product/multi-grade/multi-product and single-path/multi-path/network-structure. Which of these classifications best describes the production? Motivate! (1 p)

b. The Big Bakery Brand has a *general recipe* for the products that Cookie Corner has to follow. What additional information does Cookie Corner need to make its own *control recipe*? (1 p)

Solution

a. Cookie Corner is multi-product since we make two different sorts of pastries. It is network-structured since we do some things in series and some things in parallel.

b. To make the control recipe we need to add site specific information like for instance scheduling, operational and equipment information.

3. The cinnamon roll paste (Swe: deg) are made in any of the two mixing machines. The procedure for preparing the cinnamon bun paste is listed below. When adding an ingredient it must be mixed in properly. Mixing should also be active while heating. Butter can be assumed completely melted when its temperature is at least 50°C.

1. Add butter
2. Melt the butter
3. Add milk
4. Heat to 37°C
5. Add yeast (mix 1 minute)
6. Add salt, sugar, and cardamon (mix 20 seconds)
7. Add wheat flour (mix 3 minutes)

The following inputs and outputs are available:

Analog Input	`temperature`
Digital Input	`start`
Digital Outputs	`heat`, `mix`, `addButter`, `addMilk`, `addYeast`, `addSalt`, `addSugar`, `addCardamon`, `addWheatFlour`

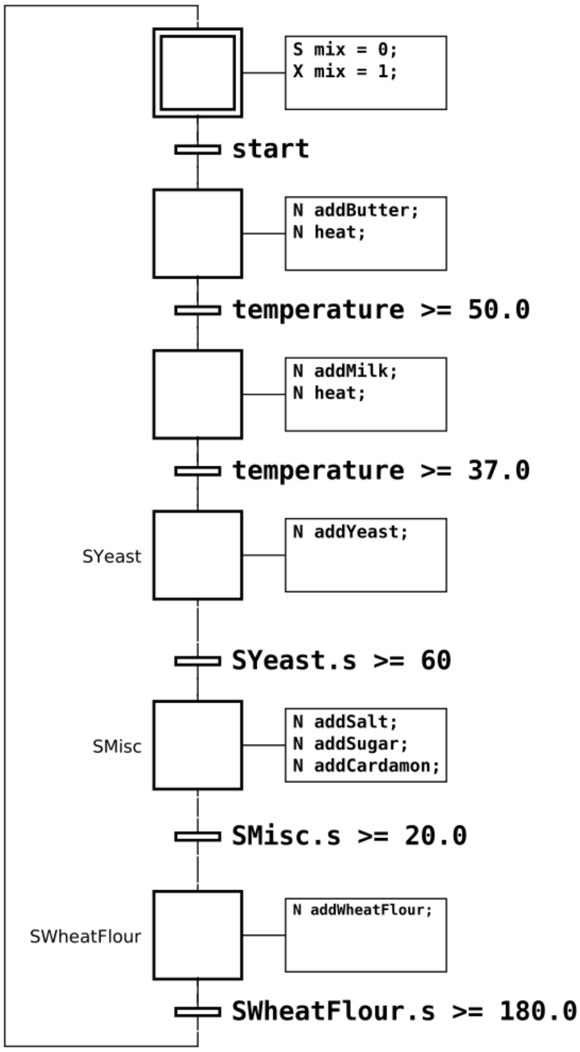

Figure 2 A Grafchart sequence for preparing cinnamon bun paste. It is assumed that all add outputs are 0 initially.

Implement the procedure as a Grafchart sequence. The production of a batch of paste should start when `start` becomes true. The proper amount of an ingredient is added when its corresponding add output is changed from 0 to 1.

Hint: The .s property of a step describes how many seconds it has been active. For example `S1.s` is how many seconds the step named `S1` has been active.

(2 p)

Solution

See Figure 2.

4. The factory has an alarm system that goes of if either the temperature in any of the ovens is too high, this event is denoted x, or if the buffer is full, denoted by y. However, if there is a worker in area 3 when the buffer is full the alarm will not be activated since the worker will then stop the inflow to the buffer. A worker in area 3 is denoted z.

a. Write a truth table for the alarm with all possible combinations of x, y, z. (1 p)

b. Simplify the expression

$$\bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z} + x\bar{y}z + xyz$$

as far as possible by using boolean algebra rules. (1 p)

Solution

a. The truth table is

x	y	z	Alarm
0	0	0	0
1	0	0	1
0	1	0	1
0	0	1	0
1	1	0	1
1	0	1	1
0	1	1	0
1	1	1	1

b.

$$\begin{aligned}\bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z} + x\bar{y}z + xyz &= (\bar{x}y\bar{z} + xy\bar{z}) + (x\bar{y}\bar{z} + xy\bar{z} + x\bar{y}z + xyz) = \\ &= (x + \bar{x})y\bar{z} + x(\bar{y}\bar{z} + y\bar{z} + \bar{y}z + yz) = \\ &= y\bar{z} + x((\bar{y} + y)(\bar{z} + z)) = y\bar{z} + x\end{aligned}$$

5. The aim of the factory is of course to maximize its profit, while making awesome pastries, and you get the task to make a plan for how much of each product you should produce. Cookie Corner has one employee and the working day at the bakery is 6 hours excluding cleaning up. The two ovens can be used during all 6 hours, but the two mixing machines can only be used for the first five hours to guarantee that everything that is started will also be finished. The factory has 35 baking sheets, that can only be used once a day since they need to be cleaned in the cleaning-up phase before used again. Some production data is listed in Table 1. Assume that everything can be scheduled to make the total times spent in different areas the only time restrictions.

a. Write up the optimization problem of maximizing the daily profit due to the given constraints. Denote the number of sheets with mudcake X and the number of sheets with cinnamon rolls Y. (2 p)

b. Plot the feasible region. (1 p)

c. Solve the optimization problem. What is the optimal point and the optimal profit? You can only use integers of baking sheets. (1 p)

Table 1 Production data, times are given per baking sheet.

	Mudcake	Cinnamon rolls
Profit per piece	6 SEK	7 SEK
Pieces per sheet	30	24
Time in oven	20 min	12 min
Total manpower time	12 min	5 min
Time in mixing machines	5 min	12 min

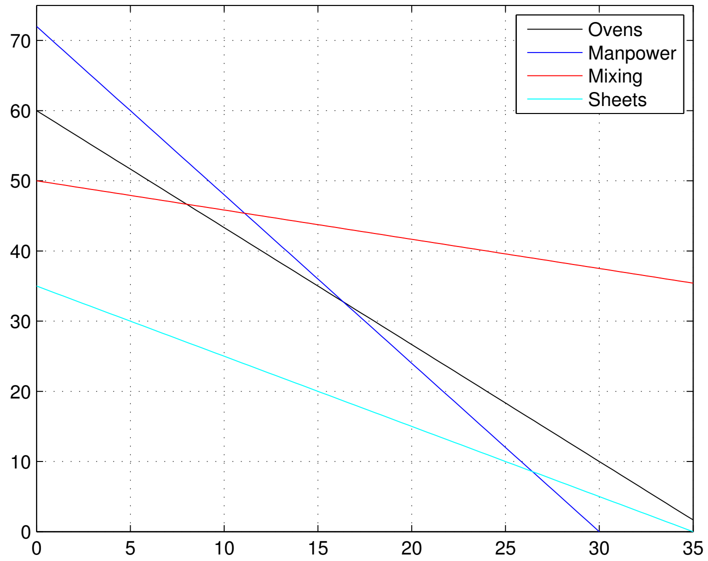

Figure 3 The constraints for problem 5. All constraints should be below the lines which gives the feasible region in the lower left corner.

d. Which of the constraints are the limiting factor(s)? Should the factory consider any new investments. (1 p)

Solution

a.

$$\max_{X,Y} \quad 180X + 168Y$$

$$\begin{aligned}
\text{subj. to} \quad & X, Y \geq 0 \\
& 20X + 12Y \leq 2 \cdot 6 \cdot 60 = 720 && \text{(Oven constraint)} \\
& 12X + 5Y \leq 1 \cdot 6 \cdot 60 = 360 && \text{(Manpower constraint)} \\
& 5X + 12Y \leq 2 \cdot 5 \cdot 60 = 600 && \text{(Mixing constraint)} \\
& X + Y \leq 35 && \text{(Baking sheets)}
\end{aligned}$$

b. The feasible region is shown in Figure 3

c. The optimal point will be in one of the corners of the feasible region. That is either the point (0,0), (0,35), (30,0) or the intersection between the manpower and sheet lines. This intersection is calculated from the two equations

$$y_1 = 35 - x$$
$$y_2 = (360 - 12x)/5$$

which gives that

$$y_1 = y_2 \Rightarrow 35-x = (360-12x)/5 \Rightarrow 175-5x = 360-12x \Rightarrow x = 185/7 \approx 26.4$$

Since we can only have integer number of baking sheets this is rounded off to $X = 26$ and $Y = 9$ which is a feasible point.

The profit of the different corner points are

$$\begin{aligned} J(0,0) &= 0 \\ J(0,35) &= 35 \cdot 168 = 5880 \\ J(30,0) &= 30 \cdot 180 = 5400 \\ J(26,9) &= 26 \cdot 180 + 9 \cdot 168 = 6192. \end{aligned}$$

Hence the optimal point is (26,9) and the optimal daily profit 6192 SEK.

d. The active constraints in the optimal point are the baking sheets and the manpower. By increasing the manpower the factory could reach the new optimal point (35,0) which would give a daily profit of 6300 SEK. The big increase in profit would be to buy some more baking sheets (or make it possible to reuse them during the day). If for example 5 new baking sheets would be added, which is a small investment, for instance the (suboptimal) working point (20,20) could be used that would give a daily profit of 6960 SEK.

6. During the course of the project, three project groups were formed, each one working on a specific topic

- Group A: MOM Maturity Capability Model
- Group B: Work center KPIs
- Group C: KPIs for PID fleet

Below are three questions, answer the one that applies to your project. (2 p)

a. (Question for Group A) Explain the idea with the MOM Maturity Capability level, also describe two aspects of relevance that vendor companies can have on its applicability.

b. (Question for Group B) Explain two concerns if using ISO 22400 KPI formulas for calculating KPIs for production lines, also present one reason for why the ISO 22400 is of interest for vendor companies.

c. (Question for Group C) Describe two reasons given by end-user companies for why the ISO 22400 standard is difficult to use in the continuous industry. Also present (with name and formula) one KPI (not currently included in ISO 22400) that could be useful to use in the continuous industry.

Solution

No solution provided. The answers must be seen in perspective of the project reports of the groups.

7. Assume that you are the production manager of a company producing vitamin pills. The vitamin pills have become extremely popular over the last 3 months and there is now pressure on the production department to deliver more vitamin pills. You, as the production manager, are responsible for presenting ideas of how the production can be increased. You have heard about the ISO 22400 standard, in which 34 different KPIs are defined. You believe it could be a good idea to use some of the KPIs in order to better analyze the performance of production.

a. Select 4 KPIs from the ISO 22400 standard that you believe would be of interest for you and explain why you have selected these 4. (1 p)

b. Further assume that you would be able to get numerical values on the above selected 4 KPIs. In what way could you use that information? (1 p)

Solution

a. The 4 KPIs given in the answer should come from 34 KPIs in the ISO standard, and the reason given should be presented in a reasonable form.

b. A numerical number indicates the current state on that KPI. By knowing the current state, progress can be visualized and measured.

8. Industry 4.0 and Smart Manufacturing is often presented as the next industrial revolution. Digitalization is seen as the main driver, and everything that can be digitalized will be digitalized (i.e. taken care of by software and IT instead of manual work).

In the company in which you are the production manager, you have started to digitalize the MOM functionalities and you have already installed software for taking care of Detailed Production Scheduling, Production Dispatching and Production Execution, se Figure 4.

a. Point out which of the remaining 5 activities that you, as the production manager, would like to digitalize next. Also explain the selected activity itself, as well as the reason for why you selected it. (1 p)

b. The activity model shown in Figure 4 is defined in the ISA95 standard. The standard also includes the definition of other models, such as the equipment hierarchy model. Explain what the equipment model is by making a picture and add a description. (1 p)

Solution

a. Select one (1) of the five remaining activities; production resource management, product definition management, production tracking, production data collection, production performance analysis. Describe, in a reasonable way, why that activity was selected.

b. The equipment model is described by Figure 5. In the physical model, the highest level is the Enterprise, an enterprise consists of one or several Sites, a site consists of one or several Areas. The terminology used for the lower levels varies depending upon the type of industry they apply to (batch, continuous, or

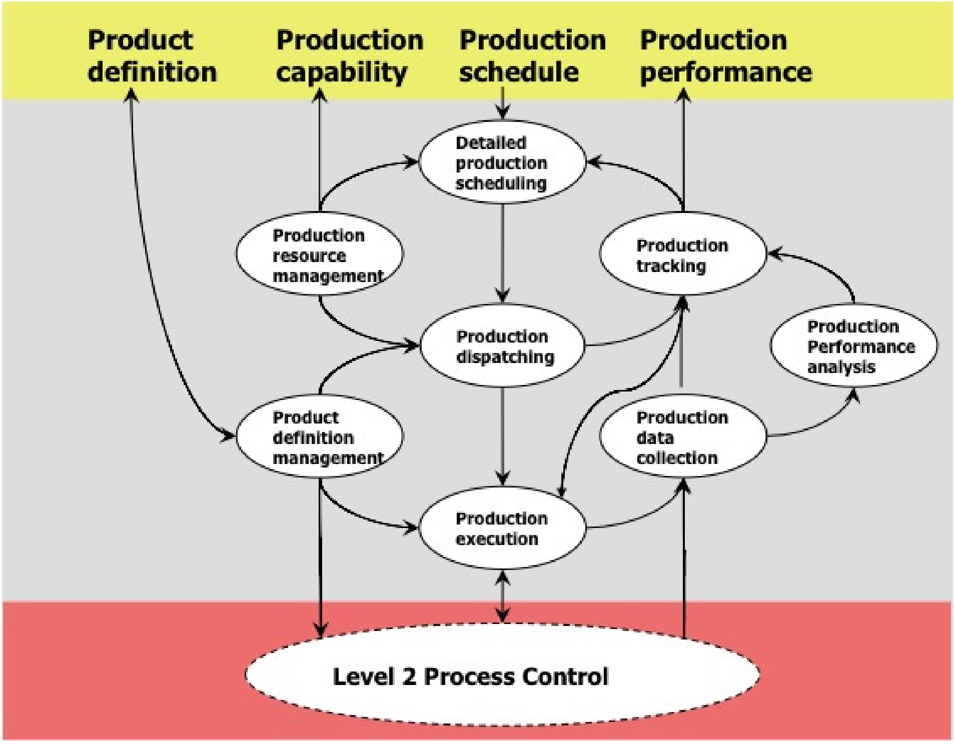

Figure 4 Activity model according to the ISA95 standard.

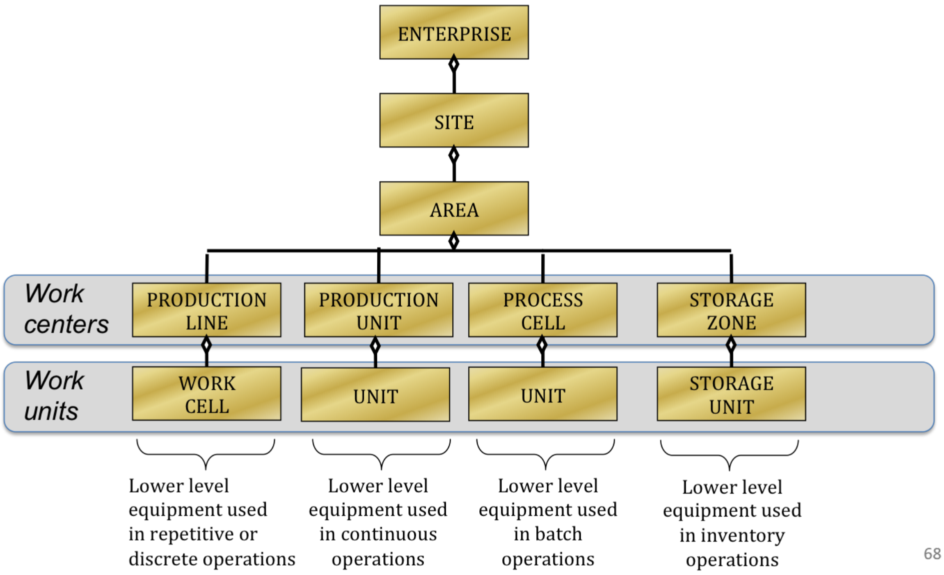

Figure 5 Physical model of an enterprise.

discrete). Below the Area we have Process Cells and Units (batch), Production Units and Units (continuous) and Production Line and Work Cells (discrete). This hierarchy is shown in Figure 5.

9. (2 p)

Two persons share the same lunch room, but have different opinions on how nice it is with a tidy room and how much work it is to put away the garbage.

X thinks that a clean room is worth +1, and a messy room is 0. To clean up the own mess reduces happiness by -1, and cleaning up after another person would never be an option. X is not a very nice person.

Y, who sometimes is more helpful, considers a clean room worth +2, and a messy room -1. To clean up Y's own mess takes away -1 in happiness, and to clean up also after X, if X doesn't clean, takes an additional -1 in happiness.

The outcome of this strategic game is hence given by the following matrix where the first number is the utility for X and the second for Y.

		Y		
		leave mess	clean own	clean all
X	leave mess	0,-1	0,-2	1,0
	clean own	-1,-1	0,1	0,1

a. Find the two pure Nash equilibria of the game.

b. Find the Stackelberg outcome when X is the leader (makes the first decision that can not be changed) and Y is the follower.

c. A third agent *H* can change the utility functions for the players by closing the lunch room. This would change the happiness of both players to -10. *H* would like to avoid this outcome but would like a clean lunch room. If *H* announces that the lunch room will be closed if found messy the following game situation arises

		Y		
		leave mess	clean own	clean all
X	leave mess	-10,-10	-10,-10	1,0
	clean own	-10,-10	0,1	0,1

The new game has a mixed Nash equilibrium where X always chooses "clean own" and Y uses a probabilistic strategy with probabilities (leave mess, clean own, clean all) = $(0, p, 1-p)$. For what values of $p \in [0, 1]$ is this a Nash equilibrium?

Solution

a. When X chooses "leave mess" the best-response for Y is "clean all". This gives a Nash equilbrium, since X or Y can then not improve by deviating. When X chooses "clean own" the best-response for Y is both "clean own" and "clean all". Of these alternatives only the "clean own" strategy gives a Nash equlibrium. So the two pure Nash equilibria are (leave mess, clean all) and (clean own,clean own).

b. Following the reasoning above we see that the Stackelberg solution when X is the leader is (leave mess, clean all).

c. With the described strategies for X and Y the utilities are $(u_X, u_Y) = (0, 1)$. If X changes strategy to "leave mess" the mean expected utility would be $-10 \cdot p + 1 \cdot (1-p) = 1 - 11p$. If $1 - 11p \leq 0$, i.e. $p \geq 1/11$ this would not be an improvement for X. One also sees that Y can not improve utility by deviating, since $u_Y = 1$ is already the largest possible outcome. Hence the described situation is a Nash equilibrium for $p \in [1/11, 1]$.

10. (2 p)

Two firms compete on a common market as modeled in the Cournot game. The price vs production function is given by

$$P(Q) = 100 - Q$$

where $Q = q_1 + q_2$ and q_i is the production of firm i. The utility for firm i is

$$u_i(q_1, q_2) = q_i(P(Q) - c_i)$$

a. Calculate the Nash equilibrium production (q_1, q_2) when the unit production costs for both firms are $c_1 = c_2 = 40$.

b. Firm 1 is considering investing in new equipment reducing production cost to $c_1 = 10$. Calculate the new Nash equilibrium and the change in profit for firm 1, i.e. the break-even point for the investment.

Solution

a. The equations $\dfrac{\partial u_i}{\partial q_i} = 0$ for $i = 1, 2$ gives the following two equations for Nash equibrium

$$-2q_1 + 100 - q_2 - c_1 = 0$$
$$-q_1 + 100 - 2q_2 - c_2 = 0.$$

When $c_1 = c_2 = 40$ the solution is $q_1 = q_2 = (100 - c_2)/3 = 20$. The price is $P(Q) = 100 - 20 - 20 = 60$ and the profits are $u_1 = u_2 = 400$.

b. The solution to the linear equations for general c_1 and c_2 gives the Nash equilibrium

$$q_1 = \frac{100 - 2c_1 + c_2}{3} = \frac{100 - 2 \cdot 10 + 40}{3} = 40$$
$$q_2 = \frac{100 - 2c_2 + c_1}{3} = \frac{100 - 2 \cdot 40 + 10}{3} = 10.$$

The price becomes $P(Q) = 100 - 40 - 10 = 50$, and has hence dropped 10 units when one of the producers lowered its production price by 30 units. The profit for firm 1 is $u_1 = 40(100 - 40 - 10 - 10) = 1600$, an increase of 1200.