1.

1. Batch, cookies are made according to a recipe.
2. Discrete, discrete pieces are put together.
3. Continuous, paper is continuously coming out of the paper machine.
4. Continuous, electricity is continuously produced.
5. Batch, medicine is produced according to a recipe.
6. Batch, beverages are produced according to a recipe.

2. See figure 1.

3.

a. Each inequality corresponds to a line in figure 2. The area in the bottom left of the figure is the feasible area.

b. The optimal value is always in one of the corners of the feasible area, i.e. $v_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, v_3 = \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 1.5 \end{pmatrix}$.

The values in the corners are $v_0 : 0, v_1 : 3, v_2 : 2, v_3 : 3$, meaning that e.g. v_1 or v_3 is optimal.

Alternatively one can note that the boundary $3x_1 + x_2 \leq 3$ is included in the feasible area, and thus the optimal value is 3 for any (x_1, x_2) on this line in the first quadrant.

4.

a) single-path, single product - only one predefined sequential path from raw materials to the only product A

b) multi-path, multi-grade - three parallel branches without interaction from raw materials to the two variants of product A

c) network, single product - interacting parallel branches only producing product A

d) multi-path, multi-product - parallel branches without interaction, more than one product

e) network, single product - interacting parallel branches only producing product A

f) multi-path, multi-product - parallel branches without interaction, more than one product

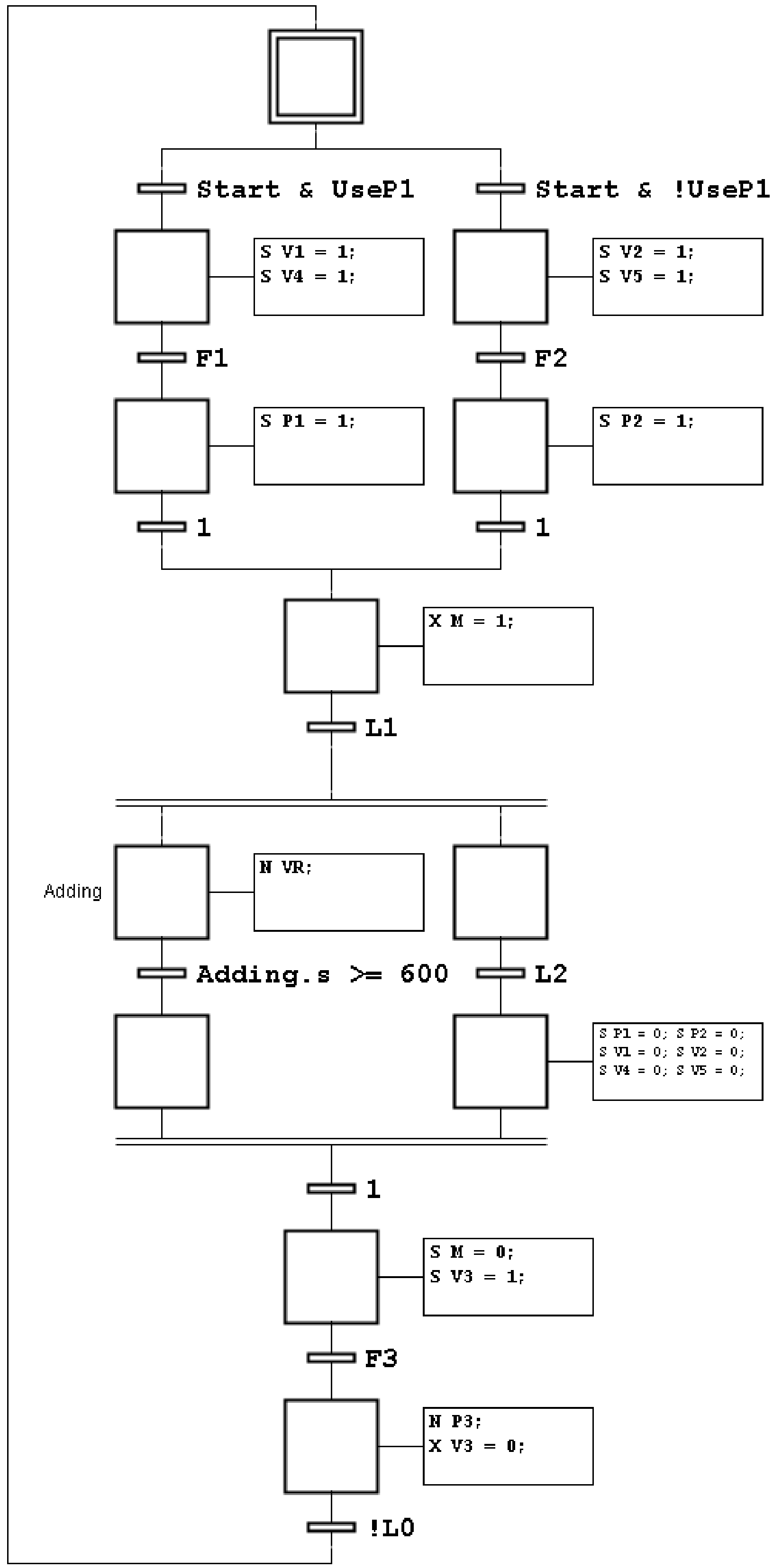

Figure 1 Grafcet for solution preparation process.

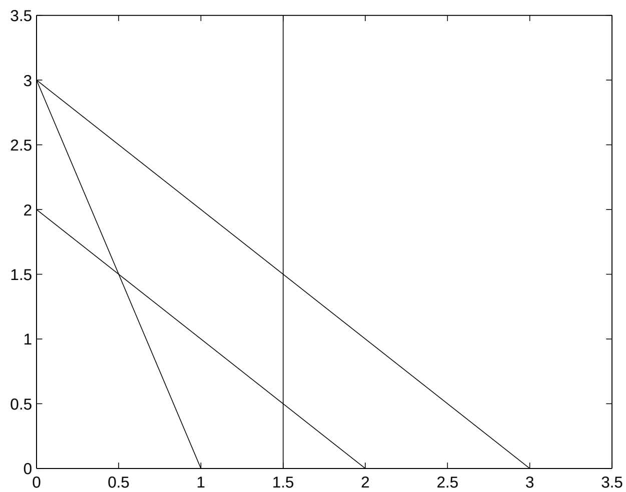

Figure 2 Feasible area.

5. No solution

6.

a. The area dependence matrix A_d gives a representation of the site structure and can be calculated from the flowchart of the product flow.

$$A_d = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

The area-utility matrix A_u gives a representation of which utilities are required in each area. Considering that the utilities are ordered: electricity, instrument air, and vacuum system; the area-utility matrix A_u becomes:

$$A_u = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

The utility dependence matrix U_d defines the interdependence between the different utilities. Considering the same ordering for the utilities as for A_u, then:

$$U_d = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

The utility operation matrix U describes which utilities have operated correctly at each sample point.

$$U = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

b. The total revenue loss due to each utility can be calculated using:

$$\begin{aligned} J_u^{tot} &= \text{diag}(\mathbf{1} - U_{av}^{ud}) \cdot \text{sign}(A_d A_u)^T (q^m \cdot *p) n_s t_s \\ &= \text{diag}\begin{pmatrix} 1/6 \\ 1/6 \\ 0 \end{pmatrix} \text{sign}\begin{pmatrix} 1 & 1 & 3 & 2 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & 2 & 1 \end{pmatrix} (q^m \cdot *p) n_s t_s \\ &= \begin{pmatrix} 1/6 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \cdot 3 \\ 0 \cdot 1 \\ 4 \cdot 6 \\ 5 \cdot 2 \end{pmatrix} \frac{\text{m}^3}{\text{h}} \cdot \frac{\text{kr}}{\text{m}^3} \cdot 6 \cdot \frac{1}{5}\text{h} \\ &= \begin{pmatrix} 8 & 6 & 0 \end{pmatrix}^T \text{kr} \end{aligned}$$

The utility that causes the greatest losses at the site is electricity.

7. Denote the number of rings and necklaces to produce x_1 and x_2 respectively and the utilization of bender 1 and 2 by x_3 and x_4 respectively.
The optimization problem is then

$$\begin{array}{ll} \text{maximize} & 15x_1 + 103x_2 - 100 - \frac{1}{2}x_4 \\ \text{subject to} & x_3 \leq 200 \\ & x_4 \leq 400 \\ & 4x_1 + 21x_2 \leq x_3 + x_4 \\ & x \succeq 0 \end{array}$$

8.

a. In this solution we consider the more general case where we have N companies.
We have the profit function for company i: $u_i = q_i(a - c - Q)$ where $Q = \sum_i q_i$.
We find the Nash Equilibrium by differentiating: $\frac{\partial u_i}{\partial q_i} = a - c - Q - q_i = 0$
Summing these equations we get

$$\begin{aligned} 0 &= \sum_i a - c - Q - q_i \\ 0 &= N(a - c - Q) - Q \\ (N+1)Q &= N(a - c) \\ Q &= \frac{N}{N+1}(a - c) \end{aligned}$$

Using $q_1^* = q_2^* = ... = q_N^*$ we get the Nash Equilibrium $q_i^* = \frac{Q}{N} = \frac{1}{N+1}(a - c)$.
Thus for $N = 3$ we have $q_1^* = q_2^* = q_3^* = \frac{1}{4}(a - c)$.

b. The profit for company i is
$u_i^* = \frac{1}{N+1}(a - c)\left(a - c - \frac{N}{N+1}(a - c)\right) = \frac{1}{N+1}(a - c)\left((a - c)\frac{1}{N+1}\right) = \frac{(a-c)^2}{(N+1)^2}$.
The total profit for all companies is $\frac{(a-c)^2}{(N+1)^2}N$, i.e. $(a - c)^2\frac{2}{9}$ and $(a - c)^2\frac{3}{16}$ for $N = 2$ and $N = 3$ respectively. Thus the total profit is smaller in the triopoly market.

9.

a. The following table describes the strategies of Player 1 and Player 2:

		Player 2: 1	Player 2: 2
Player 1	1	-2	+3
	2	+3	-4

b. Analysing the game from the point of view of Player 1, suppose he calls "one" 3/5ths of the time and "two" 2/5ths of the time at random. In this case:

- If Player 2 calls "one", Player 1 loses 2 SEK 3/5ths of the time and wins 3 SEK 2/5ths of the time, that is, on the average, he wins: $-2(3/5) + 3(2/5) = 0$ SEK (he breaks even in the long run).
- If Player 2 calls 'two', Player 1 wins 3 SEK 3/5ths of the time and loses 4 SEK 2/5ths of the time, that is, on the average he wins: $3(3/5) - 4(2/5) = 1/5$ SEK.

Therefore if Player 1 mixes his choices in the given way, the game is even every time Player 2 calls "one", but Player 1 wins 1/5 SEK on the average every time Player 2 calls "two". In this way Player 1 is assured of at least breaking even on the average no matter what Player 2 does.

c. Considering "p" as the proportion of times that Player 1 calls "one", the idea is to choose "p" such that Player 1 wins the same amount on the average whether Player 2 calls "one" or "two". Then:

			Player 2	
			1	2
Player 1	p	1	-2	+3
	(1-p)	2	+3	-4

The payoff of Player 1 if Player 2 calls "one" is: $-2p + 3(1-p)$

The payoff of Player 1 if Player 2 calls "two" is: $3p - 4(1-p)$

Therefore Player 1 should choose "p" so that:

$$-2p + 3(1-p) = 3p - 4(1-p)$$
$$p = 7/12$$

Thus, Player 1 should call "one" with probability 7/12, and "two" with probability 5/12. Using this strategy Player 1 wins 1/12 SEK every time he plays the game, no matter what Player 2 does.