# Formulating an Optimization Problem for Minimization of Losses due to Utilities

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**Abstract:** Utilities, such as steam and cooling water, are often shared between several production areas at industrial sites, and the effects of disturbances in utilities could thus be hard to predict. In addition, production areas could be connected because of the product flow at the site. This paper introduces a simple representation of the correlation between utilities and production. This representation can be used to formulate an optimization problem with the objective to minimize the economical effects of disturbances in utilities, by controlling the production of all areas at a site. The formulation of the problem is general, and thus the optimization can be performed for any site with the given structure. The results are useful for investigating the impact of plant-wide disturbances in utilities, and give suggestions on how to control the production at utility disturbances. To enable on-line advise to operators on how to control the production, the posed optimization problem is solved in receding horizon fashion. This gives suggestions for how to control the production, given estimated utility disturbance trajectories.

*Keywords:* Enterprise modelling; Optimization problems; Production control; Utility; Chemical industry; Model-based control.

## 1. INTRODUCTION

Complex chemical plants are often hard to model in detail (Kano and Nakagawa (2008)), and for some applications, a detailed model might not be needed. In this paper, disturbances in utilities, that affect one or more production areas at a site, are studied. The objective is to determine how the production in all areas at a site should be controlled, and buffer tanks should be used, to minimize the economical effects of such disturbances. The key idea is to model the network of areas at a site without including the complex dynamics within each area.

The first problem is to represent how disturbances in utilities affect production, and model how utilities are shared between production areas. This does not concern determining how utilities could be synthesized to satisfy a certain demand (as studied by e.g. Iyer and Grossmann (1998)), but rather to model how utilities affect the production of different areas.

The second problem is to determine the supply of utilities to different areas at a site, such that the loss of revenue due to disturbances is minimized. This could be seen as to determine how to transfer the variability of a process from sensitive locations to locations where it does less damage, as discussed in Qin (1998) and Luyben et al. (1999). To formulate this optimization problem, the previously discussed representation of how utilities affect production is used. In this paper, a model predictive control (MPC) formulation of the optimization problem is used, to enable on-line advice to plant operators, given an estimated disturbance trajectory. A cost function that aims to reduce the revenue loss due to disturbances is designed, with weights that could be chosen to find a good trade-off between keeping the buffer tank levels at the site at desirable levels, and maximizing profit. Aspects of how to use MPC to optimize process economics are studied by Rawlings and Amrit (2009).

## 2. ROLE-BASED EQUIPMENT HIERARCHY

According to the standard ISA-95.00.01 (2009), a site consists of one or more production areas, where each area produces either end products or intermediates. The intermediate products may either be sold on the market or refined to end products in other areas at the site. Buffer tanks could be placed between areas to serve as inventory of the intermediates or as buffer tanks with the purpose to allow independent operation of upstream and downstream areas. Thus, the production at a site may be viewed as a network of areas, with intermediate buffer tanks between some areas. An example is given in Fig. 1.

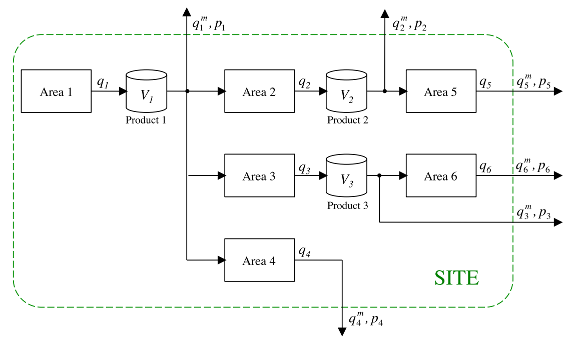


Fig. 1. An example of a site hierarchy.

In this paper, the dynamics of production areas are ignored, i.e. it is assumed that the production of an area is directly proportional to the inflow to the area, i.e.

$$q_j^{in} = q_j y_{ij} \tag{1}$$

where $q_j^{in}$ is the inflow of product $i$ to area $j$, $q_j$ the production of area $j$, and $y_{ij}$ is denoted the conversion factor between product $i$ and product $j$.

## 3. REPRESENTATION OF UTILITIES

Utilities are support processes that are utilized in production, but that are not part of the final product. Common utilities include steam, cooling water, electricity, compressed air, and water treatment. Some of these utilities operate continuously, such as steam, cooling water, feed water and vacuum systems, whereas some utilities have on/off characteristics. Example of such utilities could be electricity and nitrogen.

The measurements related to utilities are often parameters like temperature, flow or pressure of the utility. The mapping from these measurements of utility properties to the constraints it imposes on production is not trivial, and might look different for different utilities. Thus, operation outside its normal limits might give very different effects on the production of the areas that require the utility. Furthermore, a utility might be shared between several production areas. If the effects of disturbances in utilities at an entire site should be studied, this must also be modeled in some way.

The suggestion in this paper is to interpret the utilities as volumes, or power, which all areas that require the utilities have to share. This interpretation makes sense for example for cooling water and steam utilities, where all areas that require these utilities have to split the total cooling or heating power. The amount of a utility an area is assigned is assumed to give a constraint on the production of the area according to

$$q_j \leq c_{ij} u_{ij} + m_{ij} \tag{2}$$

where $q_j \geq 0$ is the production of area $j$, $u_{ij} \geq 0$ the assignment of utility $i$ to area $j$ and $c_{ij} \geq 0$ and $m_{ij} \geq 0$ constants. If $c_{ij} > 0$, this model should correspond quite well to many utilities with continuous characteristics. For example, for cooling water: The cooling water utility produces a certain cooling power, that is shared between production areas that are connected to the cooling water system. If an area is assigned more cooling water power, it should be able to produce at a higher production speed, within the normal range of production rates. The constraint in (2) is presented graphically for $m = 0$ and some $c_{ij} > 0$ in Fig. 2.

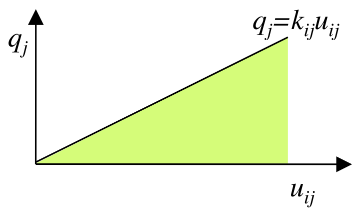


Fig. 2. Representation of a continuous utility.

If $c_{ij} = 0$, (2) corresponds to representation of a utility with on/off characteristics, where the area can produce at some maximum speed if the supply of utility is greater than zero, and not at all when it does not get assigned any amount of the utility. This is represented in Fig. 3.

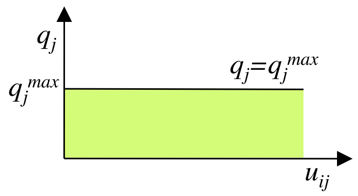


Fig. 3. Representation of an on/off type of utility.

In reality, there could be a minimum amount of a utility that is required for a production area to be able to operate, here denoted $u_{ij}^{min}$. Also, there could be an upper limit, $u_{ij}^{max}$, such that supplying more utility than $u_{ij}^{max}$ does not permit higher production than the maximum possible production if $u_{ij}^{max}$ is assigned to the area. This modification to the constraint in (2) could be captured by setting maximum and minimum constraints on the production rates. If these constraints are taken into account, the representation of the continuous type and on/off type utilities become as in Fig. 4.

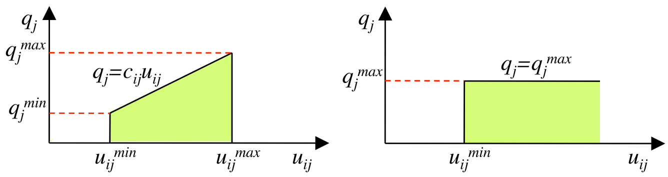


Fig. 4. Utility representations with production constraints.

As mentioned previously, utilities are often shared between several production areas at a site. The volume interpretation of utilities makes it possible to represent this by constraints of the form

$$\sum_j u_{ij} \leq U_i, \quad i = 1, \ldots, N_u \tag{3}$$

where $u_{ij}$ is the amount of utility $i$ that is required by area $j$ at the site, $U_i$ is the total amount of utility $i$, and $N_u$ the number of utilities used at the site. This is used to formulate the optimization problem in Section 4.

## 4. FORMULATING THE OPTIMIZATION PROBLEM

The formulation of the optimization problem for minimizing the economical effects of disturbances in utilities consists of defining the model and the constraints, and shaping the objective function. After having defined the optimization problem, the problem is posed as an MPC problem, to enable online use. The idea is to let the operators at the plant estimate the disturbance trajectory some steps ahead, and use MPC to provide decision support for how to handle the disturbance. Estimating the disturbance trajectory is equivalent to estimating the total available amount of all utilities, $U_i(t)$, over the prediction horizon.

### 4.1 Model

The model of the site is given by the connections of its production areas. An example of what the site structure could look like is given in Fig. 1. The connections of areas are represented by the mass balances at the internal buffer tanks, i.e.

$$V(t+1) = q_i(t) - q_i^m(t) - \sum_j q_j(t) y_{ij}, \quad i = 1, \dots, N_b \quad (4)$$

where $V_i$ is the volume in the buffer tank for product $i$, $q_i$ the production of product $i$, $q_i^m$ the flow to the market of product $i$, and $y_{ij}$ the conversion factor between product $i$ and $j$ (see eq. (1)). $j$ denotes all areas downstream of area $i$, and $N_b$ is the number of internal buffer tanks.

### 4.2 Constraints

Constraints are imposed on buffer tanks and production rates. Disturbances in the supply of utilities give time-varying constraints on production rates.

*Buffer Tanks* The volume of the buffer tanks has to be kept between some high and low limits, i.e.

$$V_i^{min} \leq V_i(t) \leq V_i^{max}, \quad i = 1, \dots, N_b \quad (5)$$

The maximum and minimum limits might correspond to the entire buffer tank, or it could correspond to a part of the tank that is reserved for handling disturbances in utilities. Reference levels $V_{ref}$ for the buffer tanks are also defined.

*Production Rates* Limited capacity of production areas give upper constraints on the production rates. There could also be a minimum rate at which an area could operate. Shutdown and start-up of areas are often very expensive and should be avoided. One way to model this would be to use integer variables and punish shut-down of areas in the objective function. Doing this would result in a Mixed-Integer Linear Program (MILP), which could be hard to solve because of the combinatorial nature of such problems (Grossmann and Biegler (1995)), and the computational cost may increase rapidly with the problem size (Kondili et al. (1993)), i.e. in this case with the number of areas at the site. A way to avoid the integer variables is to impose a soft constraint on the production rates instead. The way this is done is by introducing a slack variable, $s_i$, such that

$$q_i^{min} + s_i \leq q_i(t) \leq q_i^{max} \quad (6)$$
$$-q^{min} \leq s_i(t) \leq 0 \quad (7)$$

The slack variable is punished in the objective function to avoid shutdown of areas, if it is not necessary.

*Utilities* At a disturbance in the supply of a utility, the available amount of the utility might not be enough to supply all areas with the amount they need for maximum production. If utilities are modeled according to Section 3, this constraint is represented by requiring (3) to hold for all times $t$, e.g.

$$\sum_j u_{ij}(t) \leq U_i(t), \quad i = 1, \dots, N_u \quad (8)$$

where $u_{ij}$ is the amount of utility $i$ that is assigned to area $j$, $U_i$ is the total amount of utility $i$, and $N_u$ the number of utilities used at the site. Since (2) holds for all areas $j$ and utilities $i$, this constraint is equivalent to time-varying constraints of the production rates of all areas that share a utility. It can be assumed that equality holds in (8), since it would not be optimal for an area to not use all its assigned utility volume to produce its product. Rewriting (8) using (2) gives

$$\sum_j k_{ij} q_j(t) - d_{ij} \leq U_i(t), \quad i = 1, \dots, N_u \quad (9)$$

where $k_{ij} = 1/c_{ij}$ and $d_{ij} = m_{ij}/c_{ij}$ are positive constants for utility $i$, area $j$.

### 4.3 Objective Function

The optimization is performed in two steps. First, the steady-state production that gives the optimal profit, $p_{ref}$, is determined, assuming that there are no disturbances in utilities, and no buffer tanks between areas. This profit is used as a reference value for the final optimization problem, where the objective is to minimize the economical effects of disturbances in the supply of utilities. This problem is formulated as an MPC problem, such that given an estimated disturbance trajectory some steps ahead, the operators at a site can obtain advice on how to control the production.

*Steady-state Optimization* If there are no disturbances, and no buffer tanks between areas, the optimal profit in each time step can be determined by the linear program

$$p_{ref} = \max_{q, q^m} \sum_{i=1}^{N_a} p_i q_i^m \quad (10)$$
$$\text{subject to } (4) - (9)$$

where $p_i$ is the contribution margin of product $i$, $q_i^m$ the flow to the market of product $i$, and $N_a$ the number of areas at the site. The flows that give the optimal profit are denoted $q^0$, $q^{m0}$.

*Dynamic Optimization* The objective function that is suggested is $J = \sum_t J_t$, where

$$\begin{aligned} J_t = (p^T q^m(t) - p_{ref})^2 Q_p + \Delta V^T(t) Q \Delta V(t) + \\ + \Delta q^T(t) R \Delta q(t) - g^T s(t) + s^T(t) Q_s s(t) \end{aligned} \quad (11)$$

and

$$\Delta V = V - V_{ref}$$
$$\Delta q = q - q^0$$

$Q_p > 0$ is a scalar weight, $g^T$ a positive weighting vector, and $Q$, $R$, and $Q_s$ are positive definite weighting matrices. The optimization problem becomes

$$\min_{q(t),q^m(t),s(t)} J \quad (12)$$
$$\text{subject to } (4) - (9)$$

The objective function $J$ consists of four parts:

(1) $(p^T q^m(t) - p_{ref})^2 Q_p$

To minimize the deviation from the reference profit, i.e. maximize the profit.

(2) $\Delta V^T(t) Q \Delta V(t)$

To minimize deviations from reference buffer tank levels, and avoid solutions where all inventories are sold to maximize profit.

(3) $\Delta q^T(t) R \Delta q(t)$

To minimize deviations from nominal production.

(4) $-g^T s(t) + s^T Q_s s(t)$

To inflict extra cost to area shutdown.

To enable online use of the method, the suggestion is to pose the optimization problem as an MPC problem, where the utility trajectories, $U_i$, are estimated $N$ steps ahead. The MPC problem can be formulated as

$$\min \sum_{\tau=0}^{N-1} J_t \quad (13)$$
$$\text{subject to } (4) - (9)$$

where $N$ is the prediction horizon.

The posed optimization problem has a structure that makes it possible to solve it in a distributed fashion. Therefore, the solution method presented in Giselsson and Rantzer (2010) is used to solve the problem.

## 5. AN EXAMPLE

In this section, an example of how to formulate a specific optimization problem for minimizing the economical loss due to disturbances in utilities is given. The formulation is possible due to the choice of the representation of utilities presented in Section 3. The site that is considered is the site with six production areas given in Fig. 1. Table 1 summarizes the maximum and minimum production rates of all areas, $q^{max}$ and $q^{min}$, the contribution margins of all products, $p$, the maximum and minimum volume of all buffer tanks, $V^{max}$ and $V^{min}$, and the reference volumes for the buffer tanks, $V^{ref}$.

Three utilities are considered at the example site; high pressure (HP) steam, middle pressure (MP) steam, and cooling water. The utilities that are required at each area are given in Table 2. It is assumed that, at maximum production, the utilities are shared equally between the areas that require them.

Table 1. Production data.

| | $q^{min}$ | $q^{max}$ | $p$ | $V^{min}$ | $V^{max}$ | $V^{ref}$ |
|---|---|---|---|---|---|---|
| Product 1 | 0.10 | 1 | 0.4 | 0 | 0.5 | 0.5 |
| Product 2 | 0.05 | 0.5 | 0.7 | 0 | 0.5 | 0.5 |
| Product 3 | 0.02 | 0.2 | 0.1 | 0 | 0.5 | 0.5 |
| Product 4 | 0.01 | 0.1 | 0.5 | - | - | - |
| Product 5 | 0.02 | 0.2 | 0.8 | - | - | - |
| Product 6 | 0.02 | 0.2 | 1.0 | - | - | - |

Table 2. Utilities required at each area.

| Area → | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Steam HP | x | | x | | | |
| Steam MP | | x | | x | | x |
| Cooling water | x | x | x | x | x | x |

### 5.1 Model

The mass balances at all buffer tanks at the site give

$$V_1(t+1) = q_1(t) - q_1^m(t) - q_2(t) - q_3(t) - q_4(t) \quad (14)$$
$$V_2(t+1) = q_2(t) - q_2^m(t) - q_5(t) \quad (15)$$
$$V_3(t+1) = q_3(t) - q_3^m(t) - q_6(t) \quad (16)$$

with the same notation as in Section 4. As can be seen in the equations, all conversion factors are assumed to be equal to one in the example, for simplicity.

### 5.2 Constraints

*Buffer Tanks* Upper and lower level constraints for buffer tanks are given by (5) for the three buffer tanks, $i = 1, \dots, 3$. The limits $V^{max}$ and $V^{min}$ are given in Table 1 for all buffer tanks.

*Production Rates* Minimum and maximum limitations on production rates give constraints according to (6) and (7), where also slack variables are introduced to be able to punish shutdown of areas in the cost function. The limits $q^{max}$ and $q^{min}$ are given in Table 1 for all areas.

*Utilities* It is assumed that the total amount of each utility is equal to one, i.e. $U_1 = U_2 = U_3 = 1$. This could correspond to 100 % available utility. The utilities in the example are of continuous type (see Section 3), and it is assumed that zero assignment of a utility to an area gives zero production in the area, i.e. $c_{ij}, k_{ij} > 0$ and $m_{ij} = d_{ij} = 0$ for all areas $j$ and utilities $i$. The time-varying constraints on the production rates due to shared utilities are obtained from (9) using Table 2. We get

$$k_{11}q_1 + k_{13}q_3 \leq 1 \quad (17)$$
$$k_{22}q_2 + k_{24}q_4 + k_{26}q_6 \leq 1 \quad (18)$$
$$\sum_{i=1}^{6} k_{3i}q_i \leq 1 \quad (19)$$

If the utilities are shared equally at maximum production, we get

$$k_{11} = \frac{1}{2q_1^{max}}, \quad k_{13} = \frac{1}{2q_3^{max}} \tag{20}$$

$$k_{22} = \frac{1}{3q_2^{max}}, \quad k_{24} = \frac{1}{3q_4^{max}}, \quad k_{26} = \frac{1}{3q_6^{max}} \tag{21}$$

$$k_{3i} = \frac{1}{6q_i^{max}}, \quad i = 1, \ldots, 6 \tag{22}$$

### *5.3 Objective Function*

Since the flows to the market in the end product areas are the same as the production in these areas, the flows to the market from end product areas are omitted in the optimization. Merging the production of all areas, and the flows to the market of intermediate products to one decision variable array, we get

$$\bar{q} = [\, q_1 \;\; q_2 \;\; q_3 \;\; q_4 \;\; q_5 \;\; q_6 \;\; q_1^m \;\; q_2^m \;\; q_3^m \,]^T \tag{23}$$

*Steady-state Optimization* The steady-state solution that maximizes (10) becomes

$$\bar{q}_{ref} = [\, 1 \;\; 0.5 \;\; 0.2 \;\; 0.1 \;\; 0.2 \;\; 0.2 \;\; 0.2 \;\; 0.3 \;\; 0 \,]^T \tag{24}$$

with the optimal profit $p_{ref} = 0.7$.

*Dynamic Optimization* The MPC problem formulation for dynamic optimization could be posed as (13), with

$$p^T = [\, p_1 \;\; p_2 \;\; p_3 \;\; p_4 \;\; p_5 \;\; p_6 \,] \tag{25}$$

$$q^m = [\, p_1^m \;\; p_2^m \;\; p_3^m \;\; p_4^m \;\; p_5^m \;\; p_6^m \,]^T \tag{26}$$

$$\Delta V = \left[\, V_1 - V_1^{ref} \;\; V_2 - V_2^{ref} \;\; V_3 - V_3^{ref} \,\right]^T \tag{27}$$

$$\Delta q = \bar{q} - \bar{q}_{ref} \tag{28}$$

$$s = [\, s_1 \;\; s_2 \;\; s_3 \;\; s_4 \;\; s_5 \;\; s_6 \,]^T \tag{29}$$

and the constraints (14)-(22). $s_1$ to $s_7$ are slack variables to prevent unnecessary shutdown of area 1 to 7.

The weights for the optimization are in the example chosen as $Q_p = 100$, $Q = I_3$, $R = [\, 0.1 \; 0.1 \; 0.1 \; 10 \; 10 \; 10 \; 10 \; 10 \; 10 \,]$, $Q_s = I_7$, $g = 100q_{max}$. The prediction horizon was chosen as $N = 10$.

### *5.4 Results*

Estimating the disturbance trajectory ten steps ahead and using the MPC formulation gives the trajectories that suggest how the production should be controlled to minimize the economical effects of the disturbance. The solution trajectories of the example problem for a disturbance in middle pressure steam is given in Fig. 5, and the results for a disturbance in the cooling water utility is given in Fig. 6. In this example, it is assumed that the actual disturbance trajectory is identical to the estimated trajectory. To give a clearer view of how the disturbance is handled, the production and the sales at the time of the disturbance are shown in Fig. 7 for the MP steam disturbance, and in Fig. 8 for the cooling water disturbance.

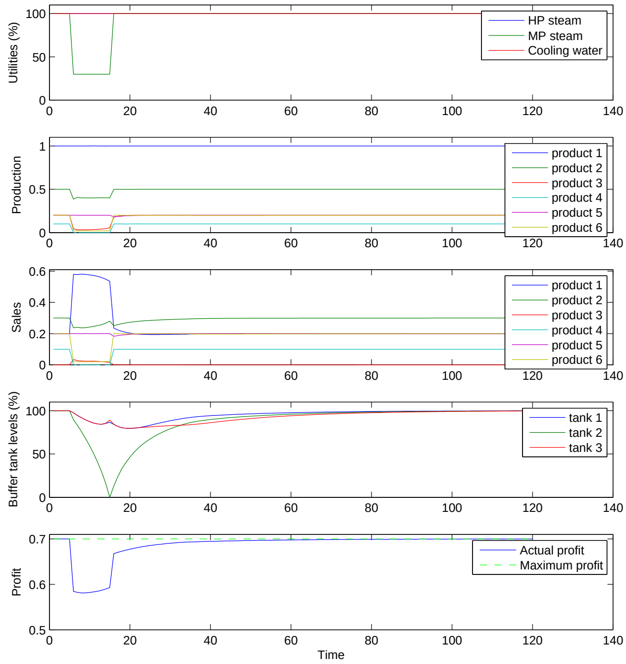


Fig. 5. Optimal trajectories for MP steam disturbance.

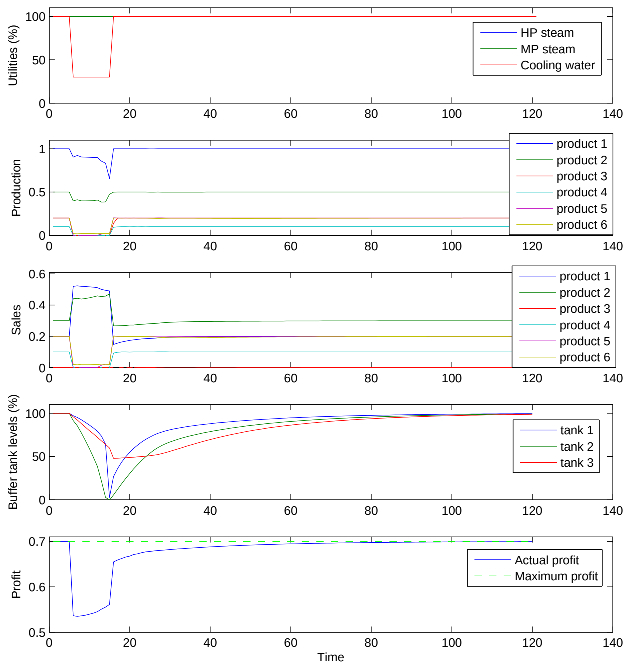


Fig. 6. Optimal trajectories for cooling water disturbance.

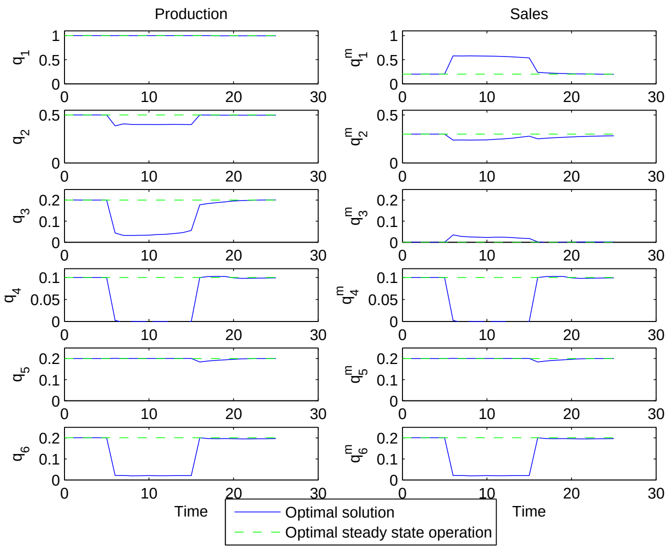


Fig. 7. Optimal production and sales trajectories at MP steam disturbance compared to optimal steady state solution.

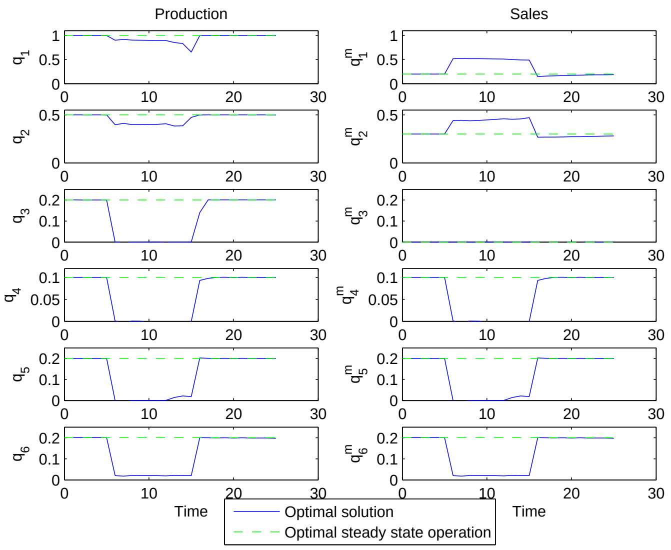


Fig. 8. Optimal production and sales trajectories at cooling water disturbance compared to optimal steady state solution.

## 6. CONCLUSIONS

The representation of utilities that is introduced is a simple way to model both continuous and on/off utilities. The representation allows formulation of an optimization problem that aims to find production trajectories that minimize the revenue loss due to disturbances in utilities, when utilities are shared between one or more production areas. The MPC formulation of the optimization problem allows optimization of an estimated disturbance trajectory that may be updated in each time step. The optimization results can be used to analyze the effects of different plant-wide disturbances in utilities. The optimization could also be used for obtaining advice on how to handle different types of disturbances in utilities, given certain constraints on the production and the buffer tanks at the site. In addition, the trade-off between keeping buffer tank levels at reference levels and maximizing the profit can be studied by manipulating the weights of the cost function for the optimization problem. Something that is not considered in the optimization problem formulation is market constraints. A possible future work direction is to include the supply chain, e.g. market demand and transports, in the problem formulation.

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