



**LUND**  
UNIVERSITY

Department of  
**AUTOMATIC CONTROL**

## **FRTN20 – Market-driven Systems**

**Exam, 2015-06-01, 14.00-19.00**

### **Points and grades**

All answers must include a clear motivation and a well-formulated answer. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each subproblem.

### **Accepted aid**

Standard mathematical tables like TEFYMA, an authorized “Formelsamling i Reglerteknik”/”Collection of Formulas” and a pocket calculator.

### **Results**

The results will be entered into LADOK and the solutions will be posted on the course home page:

<http://www.control.lth.se/Education/EngineeringProgram/FRTN20.html>

1. Let  $x$ ,  $y$ , and  $z$  be boolean variables.

a. Prove that

$$xyz + xy\bar{z} + x\bar{y}z = x(y + \bar{y}z).$$

(1 p)

b. A process is run by setting the boolean variable  $u = 1$ . The expression for  $u$  is given from

$$u = x(y + \bar{y}z).$$

Draw a truth table for all the possible combinations of  $x$ ,  $y$ ,  $z$ , that shows when the system will be running. (1 p)

*Solution*

a.

$$xyz + xy\bar{z} + x\bar{y}z = x(yz + y\bar{z} + \bar{y}z) = x(y(z + \bar{z}) + \bar{y}z) = x(y + \bar{y}z).$$

b. The truth table is given by

<b>x</b>	<b>y</b>	<b>z</b>	<b>u</b>
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	0
0	1	1	0
0	0	1	0
0	0	0	0

2. In the projects done in the course, each group worked with the ISO22400 standard.

a. Describe what a standard is. (1 p)

b. Give your view on when it is suitable to follow the recommendations in a standard and when it is suitable to “go your own way”. (1 p)

c. Select one of the KPIs that you worked with in your project and describe how the use of it can help a company improve production. (1 p)

*Solution*

a. A standard is a recommendation of how something could be done/managed. A standard is not a regulation nor a guideline. A standard has been created with input from many experts and industry specialists with experience and knowledge in the specific field. A standard makes it easier to understand and implement a certain issue.

- b. No solution provided.
  - c. Individual answers based on the projects.
3. Summer is arriving and you would like to do some blackcurrant juice for the warm days. A recipe is given below:

***Ingredients***

- 500g (5 cups) blackcurrants
- 275g (1 1/8 cups) caster sugar
- 250ml (1 cup) water
- 1/2 tsp citric acid

***Method***

- Simmer the sugar, blackcurrants and water in a heavy based pan gently for 5 minutes.
- Using a potato masher, break up the fruit to release as much juice as possible. Add the citric acid and simmer for another 2 minutes.
- Strain the liquid through a piece of muslin and pour it into a sterilised bottle and keep it in the fridge. If you can't find muslin anywhere, a fine sieve will strain the largest of the remaining pulp but the liquid wont be as clear.

- a. Which type of production process best describes the juice making process? Motivate! (1 p)
- b. Give one example of each of the following groups for this process
  - Raw material
  - Equipment
  - Utilities

(1 p)

***Solution***

- a. Batch process
  - b.
    - Raw material – e.g. blackcurrants, sugar.
    - Equipment – e.g. pan, potato masher.
    - Utilities – e.g. electricity to the stove.
4. The ISA95-Part 3 is focused on describing the Activities of Manufacturing Operations Management (MOM).
- a. How does MOM relate to Business and Logistics and how does MOM relate to Process control? (1 p)
  - b. Select two (2) of the eight (8) activities of MOM, see Figure 1, and explain their functionality. (1 p)

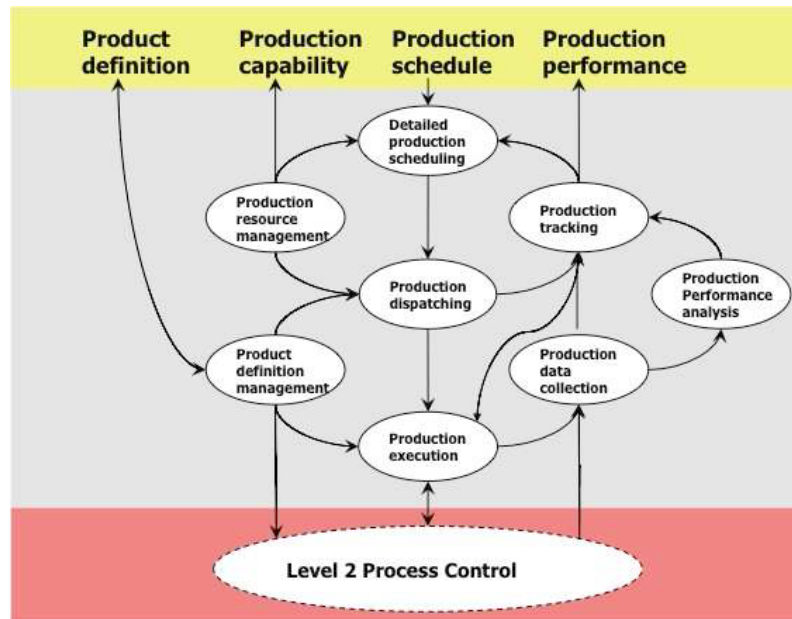


Figure 1 The eight (8) activities of MOM.

*Solution*

- a. MOM connects the business and logistics (level 4) to the process control (level 2), hence MOM resides on level 3.

ISA95-Part3 defines MOM as: "Manufacturing Operation Management is those activities of a manufacturing facility that coordinate the personnel, equipment, material, and energy in the conversion of raw materials and/or parts into products. Manufacturing operations management includes activities that may be performed by physical equipment, human effort, and information systems."

- b. The 8 activities are described below:

- **Production Resource Management** Management of Personnel, Material, Equipment and Process Segments., Providing Resource Definitions, Generating Production Capability, Assuring Certifications, etc, etc...
- **Product Definition Management** Management of production rules, bill of material, bill of resources, manage exchange with business and control level, manage versions and modifications, managing rules for cleaning, start-up and shutdown, etc, etc.
- **Detailed Production Scheduling** Generation of a schedule at a level lower than corporate planning, Finite capacity scheduling, exact timing, Possibility to split/merge one/several production schedule(s), Determines optimal use of resources (based on availability), Comparison of actual and planned production schedule.
- **Production Dispatching** Send start request for production to equipment or personnel (at a time decided by the scheduler); Starting batches, sending work orders, Starting production runs, Assigning resources (work to equipment and personnel, material to work), Control the amount of work in progress.

- **Production Execution** Execute the dispatched work; Give work instructions to personnel and assure they are carried out, Assures visibility into the status of an order in the plant, Dynamic Resource Allocation (if not in scheduler).
- **Production Tracking** Provides feedback to ERP; Splitting/merging production records so that the feedback relates to a production schedule, Track material status changes (e.g., from wet to dry), Track start and end time of segments, Reports about performance analysis results, costs, resources, genealogy, etc.
- **Production Performance Analysis** Product Analysis (e.g. in-process testing of product quality), Production Analysis (e.g. analysis of resource utilization, equipment utilization, procedure efficiency. This data can be used to optimize production and resources), Process Analysis (i.e, analysis across multiple production runs)
- **Production Data Collection** Collecting and Maintaining Data such as: Sensors readings, equipment states, event data, properties (temperatures, rates, etc), operator entered data, operator actions etc

5. A heating, ventilation and air condition (HVAC) unit is used to control the air temperature in a room. The unit has three different actions, heating could be turned on by setting  $Heat = 1$ , the ventilation is on if  $Vent = 1$ , and cooling is turned on by setting the variable  $AC = 1$ . The room temperature is measured by the sensor  $Temp$ . The control strategy for each action is the following.

- The heating should be turned on if the measured room temperature is below  $19^{\circ}\text{C}$  and should stay on until the temperature rises above  $20^{\circ}\text{C}$ .
- The cooling should be turned on if the room temperature is above  $22^{\circ}\text{C}$  and should be turned off if the temperature is below  $20^{\circ}\text{C}$ .
- The ventilation should be switched on at all times that the HVAC unit is running, and should only be turned off if a smoke detection warning, indicated by the sensor  $Smoke$ , is turned to  $Smoke = 1$ . The heating and air condition functionalities should continue working even if the smoke detection warning has been turned on. The smoke detection warning will only be active for a short while, but the ventilation should not be turned on again until the entire HVAC system is restarted.

Draw a Grafchart model explaining this system. (3 p)

*Hint: The Exception Transition used in the first lab could be of use. "An exception transition can be connected to the left side of a macro step and is used to abort the macro step. It has priority over ordinary transitions."*

*Solution*

A suggestion of a Grafchart implementation of the HVAC system described is shown in Figure 2.

6. This exercise is about utility disturbance management. Some formulas that might be of use in this exercise is given on the last page of the exam. Subproblems a,b,c can be solved independently, while subproblem d requires information from the previous tasks.

A production site contains the production areas shown in Figure 3.

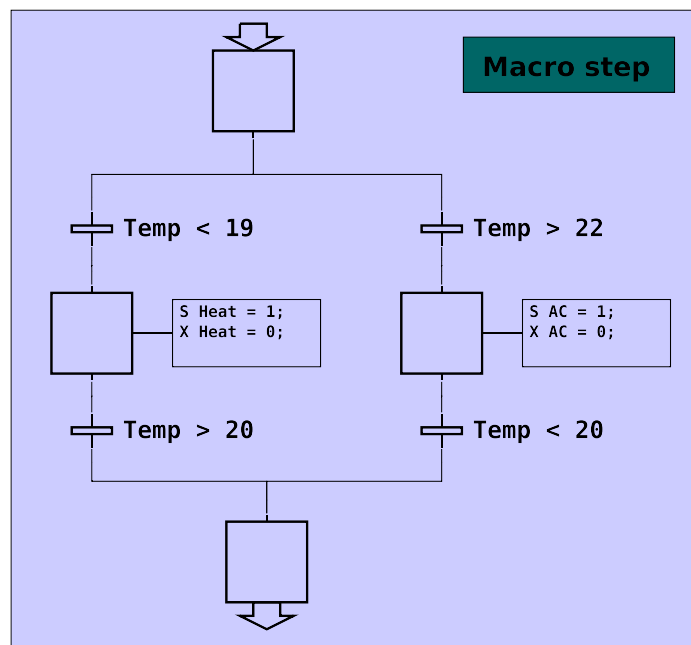
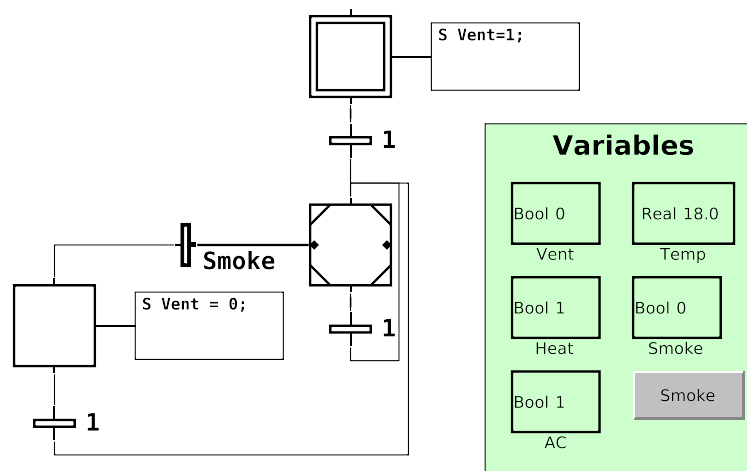


Figure 2 Grafchart implementation of the HVAC system in Problem 5

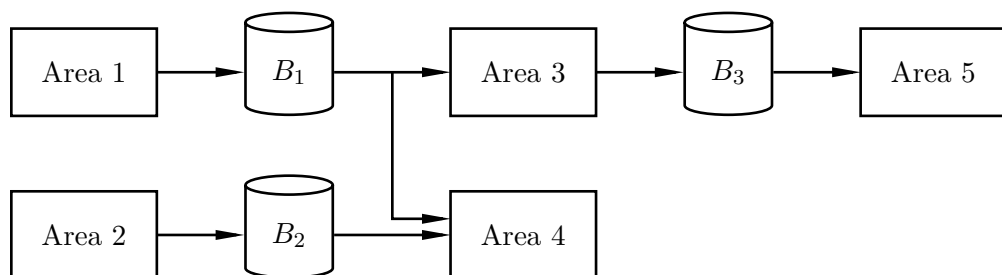


Figure 3 The interconnection of production areas in Problem 6.

- a. Write the *Area Dependence Matrix*  $A_d$ . (1 p)
- b. All the areas need electricity. In area 2 and area 3 steam is required, and area 4 needs cooling water. Write the *Area-Utility Matrix*  $A_u$ . (1 p)
- c. The utility measurement data for the site, retrieved for  $n_s = 7$  samples where the sample time  $t_s = 1$  time unit, is shown below

$$\begin{aligned}\mathbf{Electricity} &= (1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1) \\ \mathbf{Steam} &= (40 \ 38 \ 40 \ 37 \ 38 \ 33 \ 41) \\ \mathbf{Cooling \ Water} &= (25 \ 23 \ 27 \ 30 \ 28 \ 22 \ 20)\end{aligned}$$

In order to work as planned the electricity needs to be on, the steam need to have a pressure  $> 35$  bar, and the temperature of the cooling water needs to be  $\leq 27^\circ\text{C}$ . Write the *Utility Operation Matrix*  $U$ . (1 p)

- d. Removing the utility dependence, shown in Figure 4, from  $U$  does not change anything, so  $U_{ud} = U$ . Assume that the production rate in each area is given by

$$q^m = (7 \ 3 \ 4 \ 6 \ 3),$$

and that the contribution margins are given by

$$p = (1 \ 2 \ 2 \ 5 \ 7).$$

Estimate the total revenue loss due to each utility  $J_u^{tot}$ . (1 p)

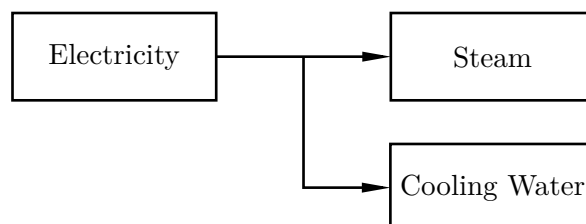
*Solution*

a.

$$A_d = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

- b. With  $u_1 = \text{Electricity}$ ,  $u_2 = \text{Steam}$ , and  $u_3 = \text{Cooling Water}$ , we get the following area-utility matrix.

$$A_u = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$



**Figure 4** The interdependence of the utilities in Problem 6.

c.

$$U = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

d. The total revenue loss due to each utility is given by

$$J_u^{tot} = \text{diag}(\mathbf{1} - U_{av}^{ud}) \cdot \text{sign}(A_d A_u)^T (q^m \cdot * p) n_s t_s,$$

where everything is given from the exercise text or previous tasks, except for the utility availability  $U_{av}$ . This is given from  $U_{av} = U \cdot \mathbf{1} / n_s = (6/7 \ 6/7 \ 5/7)^T$ . Entering all information into the formula we get

$$\begin{aligned} J_u^{tot} &= \text{diag}(\mathbf{1} - U_{av}^{ud}) \cdot \text{sign}(A_d A_u)^T (q^m \cdot * p) n_s t_s = \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \cdot \text{sign} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \\ 3 & 1 & 0 \end{pmatrix}^T \begin{pmatrix} 7 \\ 6 \\ 8 \\ 30 \\ 21 \end{pmatrix} = \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 7 + 6 + 8 + 30 + 21 \\ 6 + 8 + 30 + 21 \\ 30 \end{pmatrix} = \begin{pmatrix} 72 \\ 65 \\ 60 \end{pmatrix}. \end{aligned}$$

7. Answer the following two questions.

a. In 1990 it was reported that the closing of 42nd street in New York City reduced the amount of congestion in the area.

The reported observation seems strange. Explain it with something that you have learned in the course.

Also give an example of a traffic network where the closing of a road reduces the transportation delays. (1 p)

b. You would like to sell a valuable object at an auction. Which type of auction would you prefer in order to maximize your revenue (assuming that the bidders are rational)? Explain why.

1. An English auction where all bids are open and everyone is free to raise their bids at any time. The person with the highest bid gets to buy the object at the bid price.

2. A first price, sealed bid auction where everyone leaves a bid in a sealed envelope and the bidder with the highest bid gets to buy the object at the bid price.

(1 p)

*Solution*

a. The observed phenomenon does indeed seem counter-intuitive, but situations like that do arise due to that individuals act out of self interest. The phenomenon is known as Braess' paradox.

The example is taken from Wikipedia.

A traffic network with the required property may be found in the lecture notes or on Wikipedia.



- b. By the *Revenue Equivalence Theorem* the two auction mechanisms gives the seller the same revenue. See the lecture slides for further details.
8. Assume that it currently is the month of April and that you are about to move to a beach town where you will work with manufacturing and selling ice cream from May to August. You will work 200 hours/month. In one hour you may manufacture 20 ice creams or you may sell 10 ice creams.
- The cost of manufacturing one ice cream is 5 SEK.
- The base-price that you will sell the ice cream for is 25 SEK in May, 35 SEK in June and August and 40 SEK in July.
- If you sell many ice creams one month people will tire of them and you will have to reduce the price by 5 SEK after that you have sold 1000 ice creams in order to keep up the demand. Every month new people will arrive to the beach town so in the beginning of every month there is a full demand of ice cream.
- You do have a freezer so it is possible to store ice creams between consecutive months.
- In preparation for the summer you spend the last days of April to manufacture 300 ice creams that you store in the freezer.

- a. You would like to compute the optimal production plan that maximizes your profit for two month ahead (May and June). Formulate the problem as a linear program and write down the matrices that should be plugged in to the following expression

$$x = \text{linprog}(f, A, b, Aeq, beq, LB, UB) \quad (3 \text{ p})$$

- b. Is it a good idea to follow the production plan resulting from subproblem a? (1 p)

*Solution*

- a. For month  $i$  ( $i = 1$  corresponds to May,  $i = 2$  corresponds to June), let  $J_i$  be the income from selling ice cream, let  $s_i$  be the number of sold ice creams,  $m_i$  the number of manufactured ice creams and  $V_i$  the number of ice creams in the freezer after the month is over.

It is clear that all variables must be positive. Some of the variables ( $s_i$  and  $m_i$ ) will be limited from above, but this will be ensured by the equality constraint and the positivity constraint so it is not necessary to include those constraints explicitly.

The total profit is given by

$$c = J_1 + J_2 - 5m_1 - 5m_2$$

Let  $p(i)$  be the nominal price of ice cream in month  $i$ , then it is clear that

$$\begin{aligned} J_i &\leq p(i)s_i \\ J_i &\leq 1000p(i) + 0.8p(i)(s_i - 1000) \end{aligned}$$

There will be two equality constraints for each month, one for the total work time and one for the ice cream balance count,

$$s_i/10 + m_i/20 = 200$$

$$V_i = V_{i-1} + m_i - s_i.$$

Summing it up as a linear program gives

$$\text{minimize } - \underbrace{\begin{pmatrix} 1 & 0 & -5 & 0 & 1 & 0 & -5 & 0 \end{pmatrix}}_f x$$

subject to

$$\underbrace{\begin{pmatrix} 1 & -25 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -20 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -35 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -28 & 0 & 0 \end{pmatrix}}_A x \preceq \underbrace{\begin{pmatrix} 0 \\ 5000 \\ 0 \\ 5000 \end{pmatrix}}_b$$

$$\underbrace{\begin{pmatrix} 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{10} & \frac{1}{20} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{10} & \frac{1}{20} & 0 \end{pmatrix}}_{A_{eq}} x = \underbrace{\begin{pmatrix} 300 \\ 200 \\ 0 \\ 200 \end{pmatrix}}_{b_{eq}}$$

$x \succeq 0$  (corresponds to `LB=zeros(8,1)` `UB=inf*ones(8,1)`).

Solving the linear program in MatLab gives the plan below

	Income	# sold	# made	# in store
May	19167	767	2467	2000
June	61000	2000	0	0

- b.** Only planning for two months ahead does not take into account that the ice-cream price will soar in June, July and August. The two-month plan opts to reduce the number of ice creams in store after June to 0, not anticipating the increased demand in July. The too short prediction horizon leads to that significant profits are missed out on.

Compare with the following plan for all the 4 months.

	Income	# sold	# made	# in store
May	0	0	4000	4300
June	50108	1540	921	3681
July	72000	2000	0	1681
Aug	60025	1894	212	0

9. Solve the following game-theoretic problems.

- a. Two students, Alice and Bengt are working on a project. They plan to work on it for  $x_1$  and  $x_2$  days respectively.

The quality  $Q$  of the finished project depends on the work that is done as

$$Q = \log(1 + x_1 + x_2).$$

Alice values the quality of the finished project to  $5Q$ .

Bengt on the other hand will receive some money from his parents if he does well on the project. He therefore values the quality of the finished project as  $10Q$ .

Both Alice and Bengt put an equal value  $c = 1$  on their time.

The pay-off functions for the two students are thus

$$\begin{aligned} u_A(x_1, x_2) &= 5 \log(1 + x_1 + x_2) - x_1 \\ u_B(x_1, x_2) &= 10 \log(1 + x_1 + x_2) - x_2 \end{aligned}$$

Find all Nash equilibria for when the two students try to maximize their respective pay-off functions (you may restrict yourself to finding pure Nash equilibria).

(1.5 p)

- b. Consider the table in figure 5. You and your friend are playing a game where *you* choose a row of the table and your friend chooses a column. You both make your choices without knowing the choice of the other person.

Then *you* pay your friend the amount stated in the cell corresponding to the choices of row and column. If the amount is negative you receive money from your friend.

Formulate the problem of finding a mixed strategy that maximizes your expected profit as a linear program.

	C1	C2	C3	C4
R1	0	-4	23	4
R2	7	2	-6	-1
R3	-20	10	4	-3

Figure 5 Pay-off matrix for the zero sum game in problem 9b.

(1.5 p)

*Solution*

- a. Alice would like to maximize

$$u_A(x_1, x_2) = 5 \log(1 + x_1 + x_2) - x_1$$

with respect to  $x_1$  (subject to that  $x_1 \geq 0$ ).

It is easily seen that the function is concave and thus has a maximum. The maximum can be found by solving for stationary points of the function  $u_A(x_1, x_2)$ ,

$$0 = \frac{du_A}{dx_1} = \frac{5}{1 + x_1 + x_2} - 1$$

which gives that Alice, knowing the choice of Bengt will choose

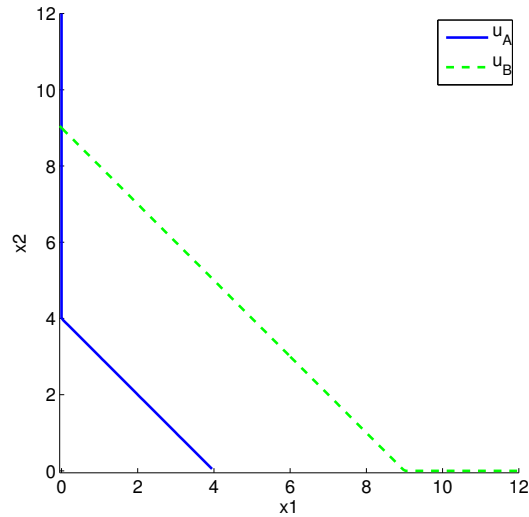
$$x_1 = B_1(x_2) = \max(0, 4 - x_2)$$

Similarly Bengt's best response becomes

$$x_2 = B_2(x_1) = \max(0, 9 - x_1)$$

Plotting the two best response functions give the situation in figure 6. Pure Nash equilibria correspond to points where the graphs of the best response functions intersect.

It is seen that the only such intersection is  $(x_1, x_2) = (0, 9)$  which corresponds to that Bengt work on the project for 9 days and Alice does not do any work at all.



**Figure 6** The best response functions for Alice and Bengt in problem 9a.

- b.** Denote your strategy  $p = (p_1, p_2, p_3)$  where  $p_i$  corresponds to your probability of selecting row  $i$ . Since your strategy is a probability distribution it must hold that  $p_1 + p_2 + p_3 = 1$  and  $p_i$  are positive.

Denote the strategy of your opponent  $q$  for which analogous conditions hold.

To maximize your profit you would like to find the solution to

$$\begin{aligned} & \underset{p}{\text{minimize}} && \max_q p^T A q \\ & \text{subject to} && \mathbf{1}^T p = 1 \\ & && p \geq 0 \\ & && \mathbf{1}^T q = 1 \\ & && q \geq 0 \end{aligned}$$

This problem can be reformulated as a linear program as you did in exercise 8.4,

$$\begin{aligned} & \underset{p}{\text{minimize}} && \alpha \\ & \text{subject to} && p^T A \leq \alpha \mathbf{1}^T \\ & && \mathbf{1}^T p = 1 \\ & && p \geq 0 \end{aligned}$$

which may be solved with the Matlab command `linprog` as you did in lab 2.

## Formulas

$$U_{ud} = \text{sign} \left( U + \text{sign} \left( (I - U_d)(U - \mathbf{1}\mathbf{1}^T) \right) \right)$$

$$U_{av} = U \cdot \mathbf{1}/n_s$$

$$A_{dir} = \mathbf{1}\mathbf{1}^T + \text{sign}(A_u(U - \mathbf{1}\mathbf{1}^T))$$

$$A_{av}^{dir} = A_{dir} \cdot \mathbf{1}/n_s$$

$$A_{tot} = \mathbf{1}\mathbf{1}^T + \text{sign}(A_d(A_{dir} - \mathbf{1}\mathbf{1}^T))$$

$$A_{av}^{tot} = A_{tot} \cdot \mathbf{1}/n_s$$

$$J_p^{dir} = (\mathbf{1} - A_{av}^{dir}) \cdot * q^m \cdot * p n_s t_s$$

$$J_p^{tot} = (\mathbf{1} - A_{av}^{tot}) \cdot * q^m \cdot * p n_s t_s$$

$$J_u^{dir} = \text{diag}(\mathbf{1} - U_{av}^{ud}) \cdot A_u^T (q^m \cdot * p) n_s t_s$$

$$J_u^{tot} = \text{diag}(\mathbf{1} - U_{av}^{ud}) \cdot \text{sign}(A_d A_u)^T (q^m \cdot * p) n_s t_s$$