Market Driven Systems (FRTN20)

Exercise 8

Game Theory

Last updated: 2014

1. Consider the traffic planning example (Braess' paradox) described at the lecture. Consider instead a centralized traffic controller dictating routes for each car and optimizing total travel time in the system. Would the new road help now? How much total traffic time is saved? How many cars will be allocated to the different routes?

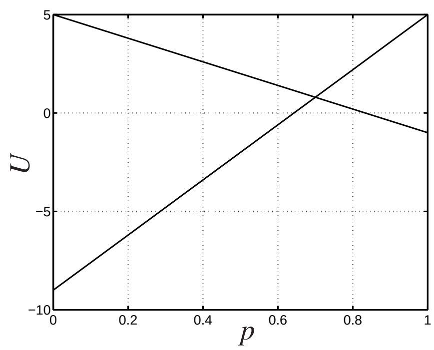
2. Prove the result claimed on the lecture that

$$\min_x(\max_y A_{xy}) \geq \max_y(\min_x A_{xy})$$

3. Let's play the following game: Each of us choose either a red or a black card. If we have chosen different cards I win 5, if we both have chosen red you win 9 and if we both chose black you win 1. Is this a fair game? What are the optimal strategies?

		I: red	I: black
You	red	-9	5
	black	5	-1

The figure below might help



4. Show how the calculation of the zero-sum game value

$$\overline{V}(A) = \min_x(\max_y x^T Ay)$$

can be found by the following LP problem

$$\overline{V}(A) = \min \alpha$$
$$x^T A \leq \alpha 1^T$$
$$1^T x = 1$$
$$x \geq 0$$

where $1^T = \begin{pmatrix} 1 & \cdots & 1 \end{pmatrix}$.

5. Verify that “Bach or Stravinsky” has the mixed Nash Equilibrium

$$p_1(B) = 2/3, p_1(S) = 1/3, \qquad p_2(B) = 1/3, p_2(S) = 2/3$$

with expected outcome $U_1^* = U_2^* = 2/3$.

6. Construct three two-stage two-person games for which

a. Game1: Both players prefer to be the leader

b. Game2: Both players prefer to be the follower

c. Game3: Both players prefer Player 1 to be the leader

7. Solve the optimal “laziness problem” (find Nash-equilibrium) assuming $f > i$ and $f > w$.

		boss: not inspect	boss: inspect
worker	work	-w,g	-w,g-i
worker	shirk	0,0	-f,f-i

How often should the worker shirk and the boss inspect when $w = 1$, $f = 5$, $i = 3$, $g = 1$? What is the expected game value for the boss? For the worker?

8. Consider a slightly changed “optimal laziness problem” where the boss does not receive the fine.

	not inspect	inspect
work	-w,g	-w,g-i
shirk	0,0	-f,-i

a. Show that the Nash-equilibrium is (shirk, not inspect).

b. Assume now instead a changed situation where the boss is required to declare his inspection probability in advance. Show that if $fg > iw$ then a Stackelberg solution is (work, inspect with probability w/f). Notice that this solution is more advantageous for the boss than the Nash equilibrium above.

9. Assume n random valuations are drawn from a uniform $[0,1]$ distribution.

a. Calculate the probability density functions for the highest valuation, and for the 2$^{\text{nd}}$ highest.

b. Show that the expected value of the highest valuation is $\frac{n}{n+1}$ and the 2$^{\text{nd}}$ highest is $\frac{n-1}{n+1}$.

10. Show that everybody bidding $(n-1)/n$ times their valuation gives a Nash equilibrium for the sealed-bid first price auction with uniform random valuations in $[0,1]$.

Calculate $S_i(v)$ and $P_i(v)$.