Market Driven Systems (FRTN20)

Exercise 9

Last updated: April 2012

1. We will consider a company that makes two products out off plywood. In order to make these products, the material needs to be sawed into the correct sizes. After this, the products are made by gluing together the sawed pieces.

- The company sells the first product for 21 kSEK per unit and the second for 18 kSEK per unit.
- Each unit of the first product require usage of a saw 7 times and usage of a gluing machine 16 times.
- Each unit of the second product require usage of a saw 10 times and usage of a gluing machine 12 times.
- There are 3 saws. The cost of using each saw is shown in Figure 1.

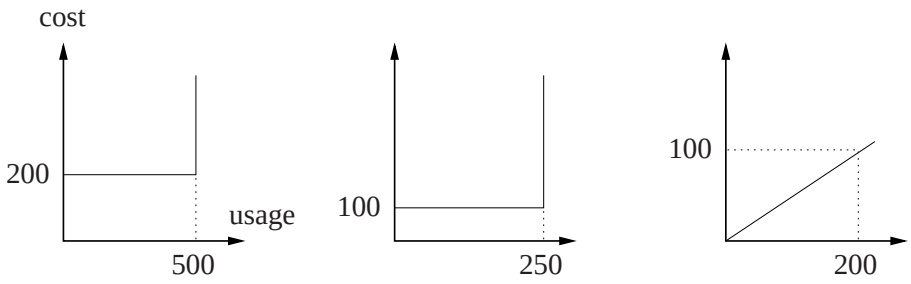

Figure 1 Diagram of the cost of the saws relative to their usage. The costs are in kSEK.

- There are 2 gluing machines. The cost of using each machine is shown in Figure 2.

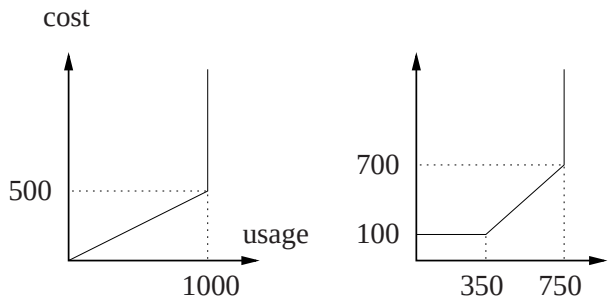

Figure 2 Diagram of the cost of the glue machines relative to their usage. The costs are in kSEK.

The objective of the company is of course to make as much money as possible, when satisfying the constraints given by what the machines can handle.

a. The cost of the second gluing machine seems to have the most complicated structure. Let's start of by viewing the separate problem of minimizing the cost of using this

machine. This can be done by solving

$$\begin{aligned} &\underset{x}{\text{minimize}}\ \varphi(x) \\ &\text{subject to: } x \geq 0 \end{aligned}$$

where $\varphi(x)$ is the function in the graph. Reformulate this problem to an LP by introducing a new variable t which should be larger than the two lines in figure 3.

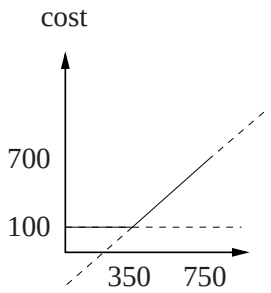

Figure 3 Redrawn graph of the second gluing machine.

b. State the complete production problem as an LP.

c. Show that producing $(\frac{1125}{19}, \frac{1275}{38})$ while using the saws $(500, 250, 0)$ and the gluing machines $(1000, 350)$ is the optimal way for the company to produce the products. (Hint: Use the KKT conditions).

d. Give a price interpretation of the dual variables. For example, what happens to the demand and supply in the sawing constraint when the price is changed from the optimal value?

2. Now assume that instead of doing the sawing and gluing themselves, the company buys the service of sawing and gluing from other companies. Let's assume that the sawing and gluing companies have the same properties as the saws and gluing machines in the previous exercises.

a. By introducing prices show how to separate the problem, i.e. show how the problem can be restated to represent that a company does not have to know what the others are doing. (We must also introduce a "saw-market" and a "glue-market").

b. There is a problem with the current setup. Examine the separated optimization problem of the production company when the dual variables are not equal to the optimal values. Give possible strategies to fix the problem.