Predictive Control Exercise 5
Model Predictive Control (MPC)

1. Explain the 'receding horizon' principle used in Model Predictive Control. What is meant by the terms the terms *prediction horizon* and *control horizon*?

2. Consider the state-space system:

$$\begin{aligned} \hat{x}_{k+1|k} &= 2\hat{x}_{k|k-1} + u_k \\ \hat{y}_{k|k-1} &= 3\hat{x}_{k|k-1} \end{aligned}$$

with the output constraint:

$$-1 \leq y_k \leq 2, \quad \forall\ k$$

If the current state estimate $\hat{x}_{k|k-1} = 3$ and the previous control input $u_{k-1} = -1$, show that the corresponding constraint on the control signal change Δu_k is given by:

$$-\frac{16}{3} \leq \Delta u_k \leq -\frac{13}{3}$$

3. The state-space system:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k \end{aligned}$$

is to be controlled using a Model Predictive Controller. The equations describing the predicted outputs can be written on vector form as:

$$\mathcal{Y} = S_x x_k + S_{u-1} u_{k-1} + S_u \Delta \mathcal{U}$$

where:

$$\mathcal{Y} = \begin{pmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \\ \vdots \\ \hat{y}_{k+p} \end{pmatrix}, \quad \Delta\mathcal{U} = \begin{pmatrix} \Delta u_k \\ \Delta u_{k+1} \\ \vdots \\ \Delta u_{k+p-1} \end{pmatrix}$$

are the vectors of predicted outputs and control changes respectively.

a. What are the structures of the matrices S_x, S_{u-1} and S_u?

b. Give some interpretations of S_x, S_{u-1} and S_u in terms of linear systems theory.

4. A linear plant described by the model:

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k$$

is subject to a constant output disturbance d_k, such that:

$$y_k = Cx_k + d_k$$

A model predictive controller is to be used to control the plant. The specifications state that the controller should be capable of removing the effects of constant output disturbances.

a. To remove the effects of the output disturbance, the plant model can be augmented to contain a model of the disturbance. What does this augmented model look like? (Hint: consider the extended state vector $\begin{pmatrix} x_k \\ d_k \end{pmatrix}$) .

b. Recalling that only the plant output y is measurable, what else is required for this method to work in practice?

5. Now consider a plant with a constant input disturbance:

$$x_{k+1} = Ax_k + B(u_k + d_k)$$
$$y_k = Cx_k$$

What is the augmented plant model in this case?