Welcome to
FRTN15 Predictive Control
Lecture 1

Lund University
Dept Automatic Control
2012

http://www.control.lth.se/course/FRTN15/

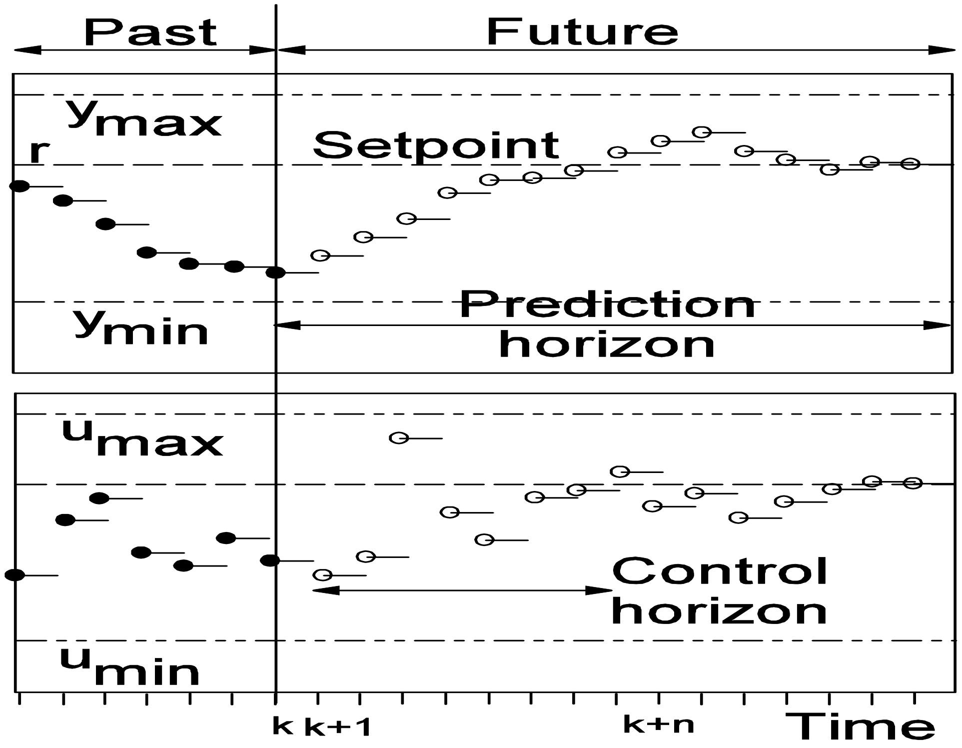
Why Prediction and Adaptation?

- Limitations of linear feedback
- Model-based control
- Optimization
  - Minimization of tracking errors
  - Covariance minimization
  - Minimization of control actions
  - Minimization of tracking errors subject to control constraints
- Time-varying Exosystems & Disturbances
- Adaptation and Learning

Predictive Control

Predictive Control

Predictive Control

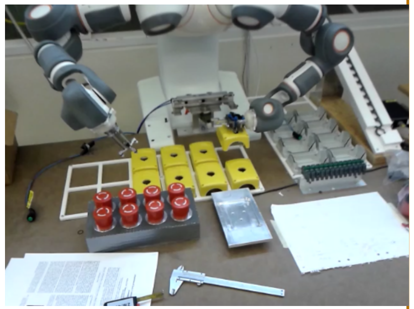


Predictive Control

Predictive Control

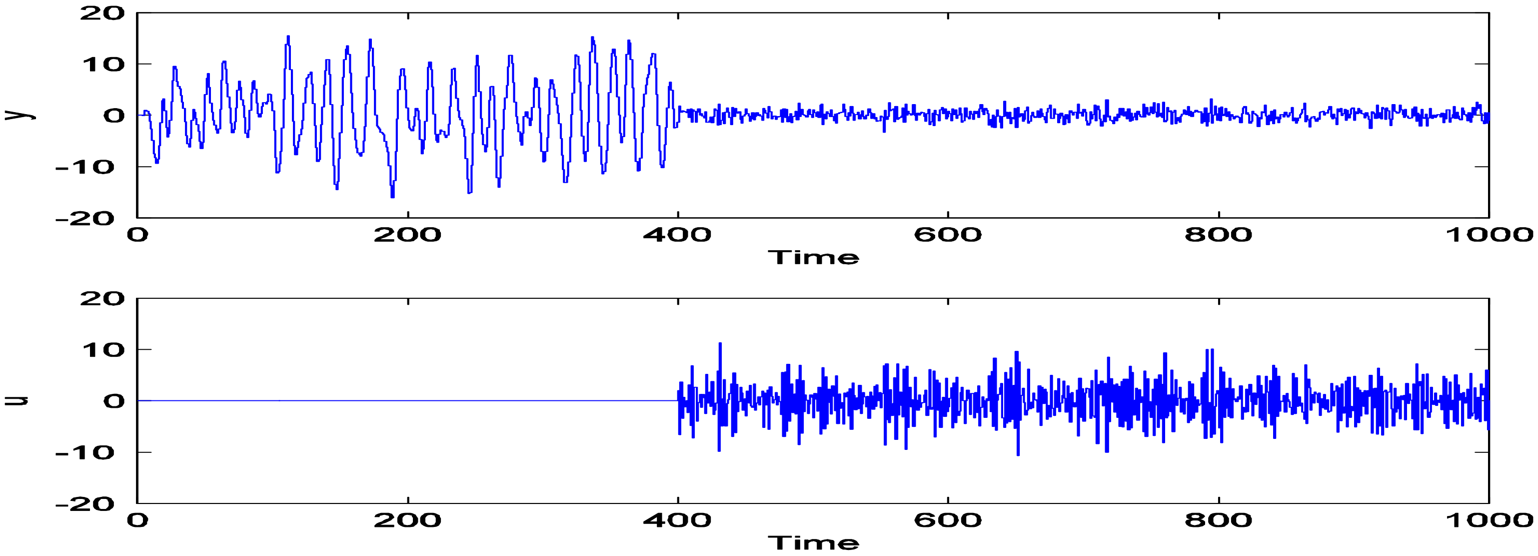
Predictive Control

Frida Robot

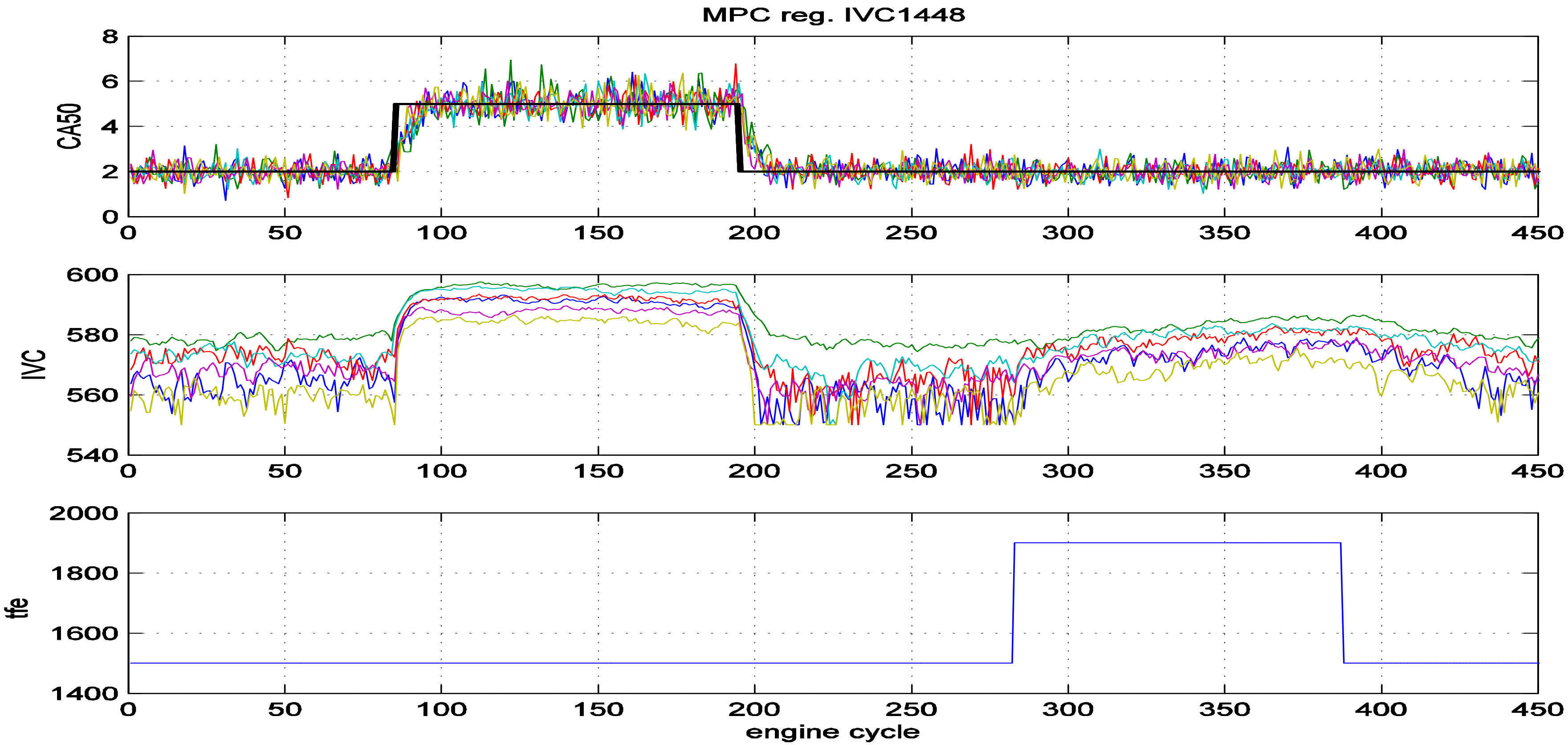


Frida Robot

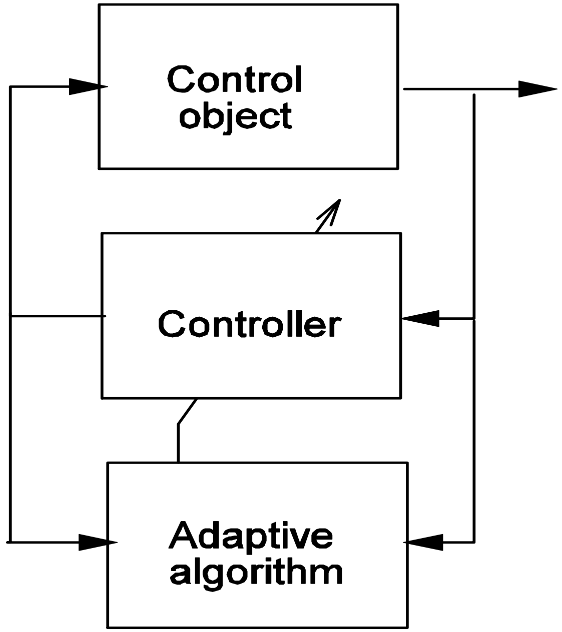
Minimum Variance Control



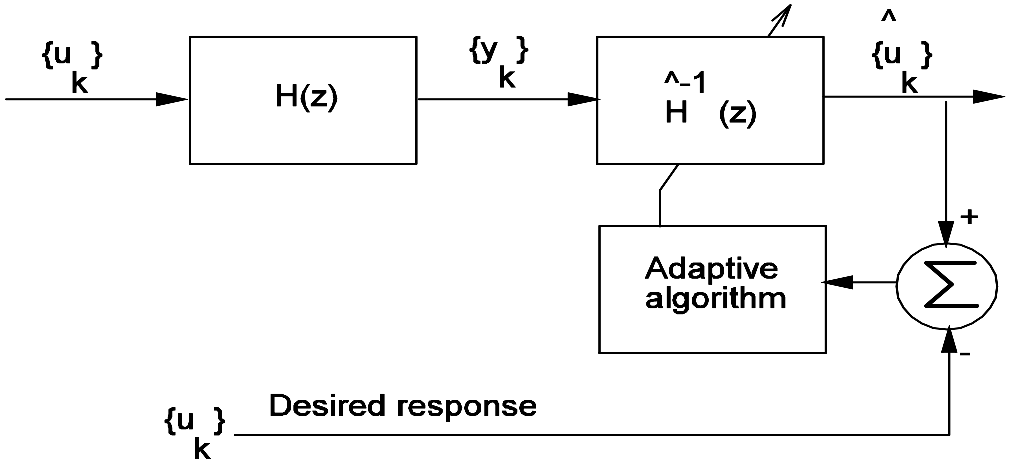
HCCI MPC Engine Control



Adaptation



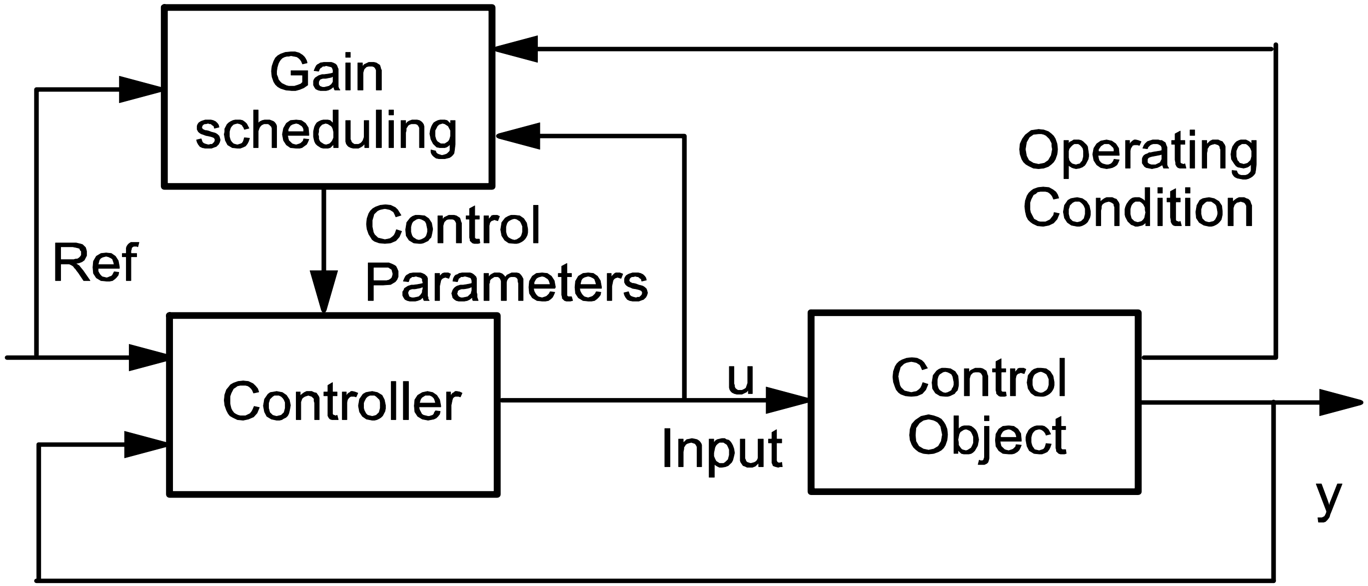
Equalization



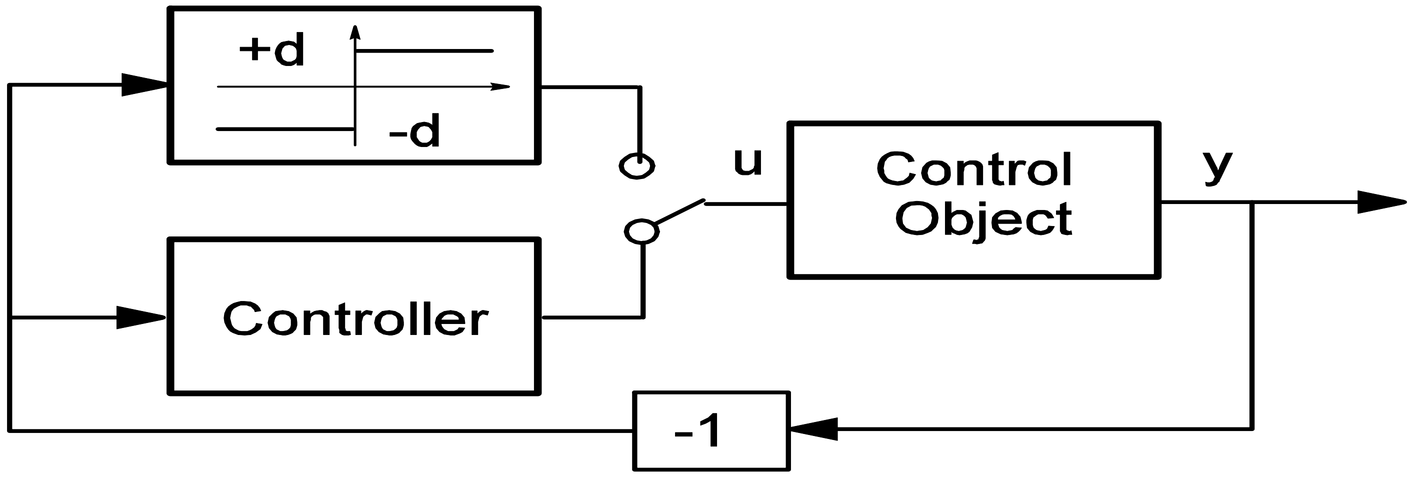
Adaptive Schemes

- **Gain Scheduling**
- **Autotuning**
- **Model Reference Adaptive Control**
- **Self-Tuning Control**
- **Extremum Control**
- **Iterative Learning Control (ILC)**

Gain Scheduling



Autotuning



Autotuning

$$G_0(s) = \frac{K_0}{1+s\tau} e^{-sL}$$

where k is the static gain, L is the latency (or apparent time delay) and τ a time constant.

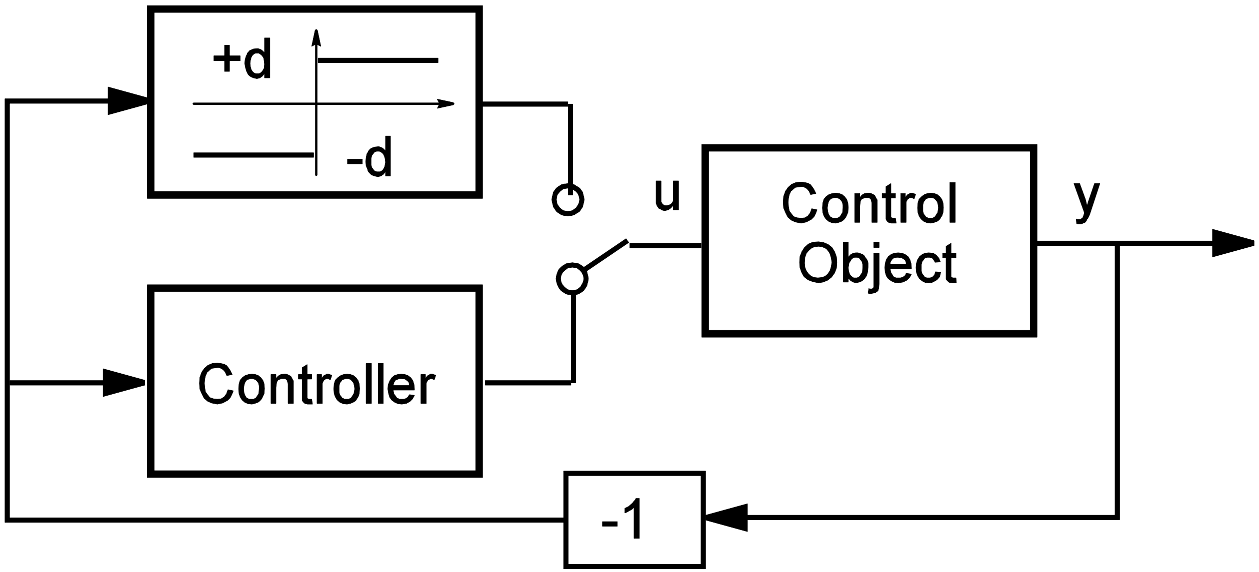
Tuning of a PID regulator

$$u(t) = K_c(e(t) + \frac{1}{T_I}\int_0^t e(s)ds + T_D \frac{de}{dt})$$

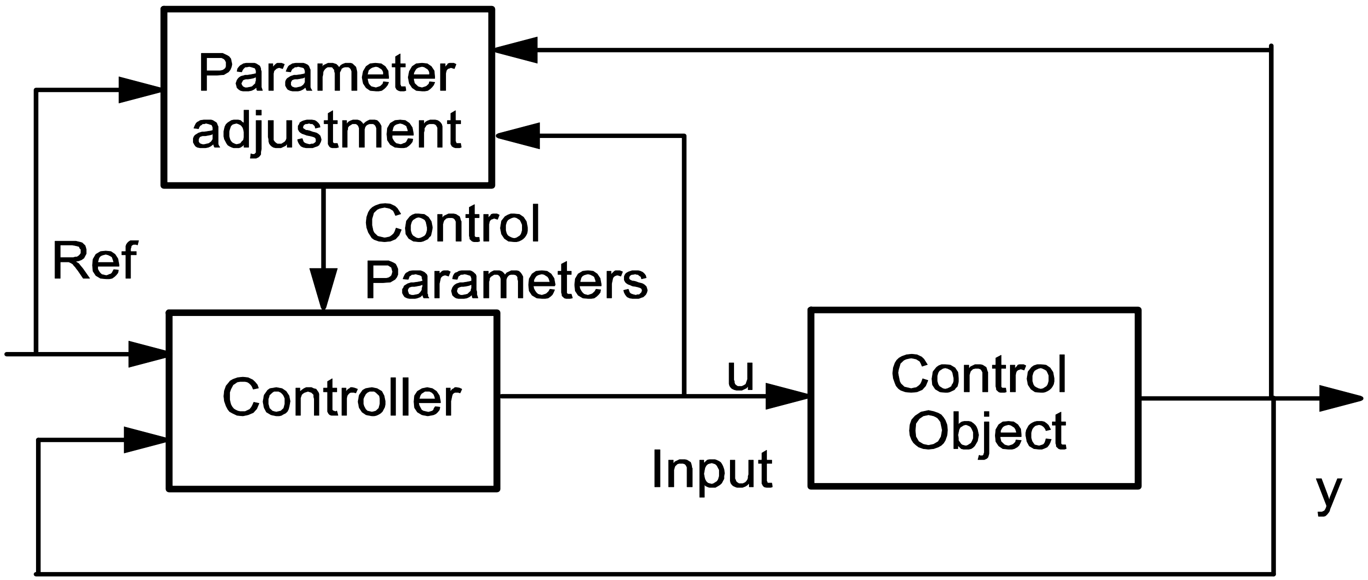
or more elaborate

$$\begin{aligned}
u(t) &= f(v(t)) \\
v(t) &= P(t) + I(t) + D(t) \\
P(t) &= K_c(\beta u_c(t) - y(t)) \\
\frac{dI}{dt} &= \frac{K_c}{T_I}(u_c(t) - y(t)) + \frac{1}{T_I}(v(t) - u(t)) \\
\frac{T_D}{N}\frac{dD}{dt} &= -D - K_c T_D \frac{dy}{dt}
\end{aligned}$$

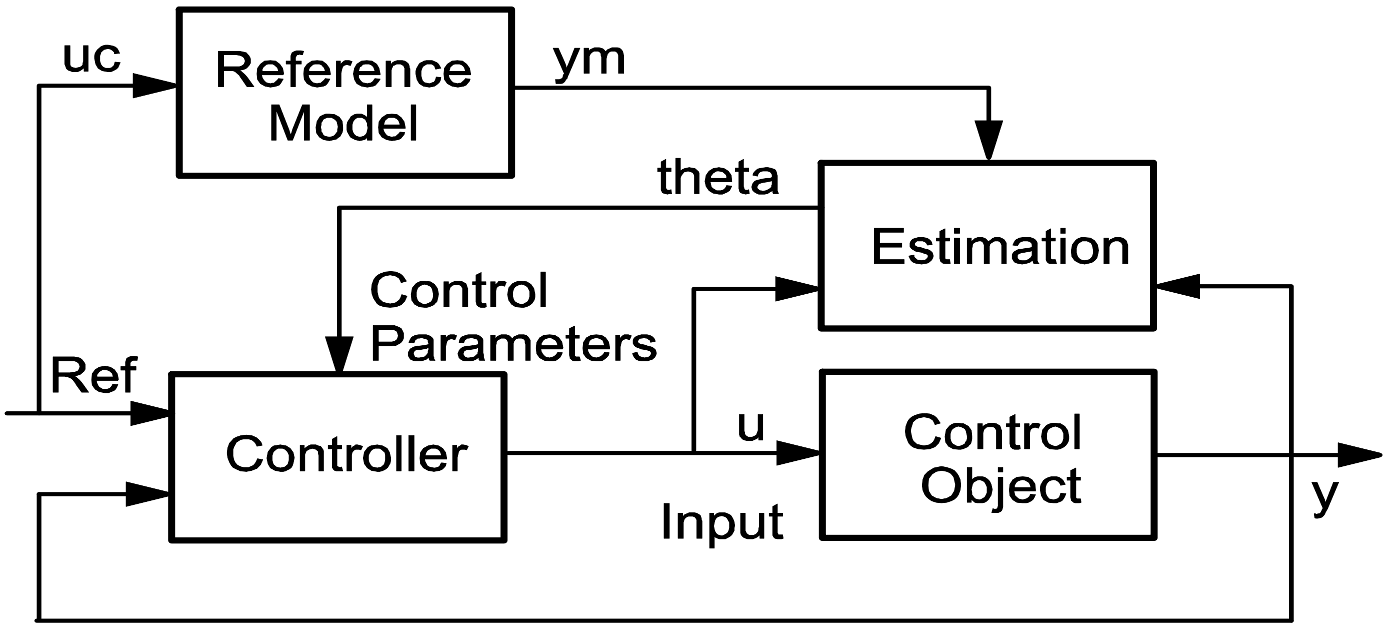
Autotuning



Adaptive Control



Model Reference Adaptive Control



Adaptive Control

Let the system

$$\frac{Y(z)}{U(z)} = H(z) = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2}$$

be controlled by means of output feedback control designed for pole assignment with the denominator polynomial

$$P(z) = z^3 + p_1 z^2 + p_2 z + p_3$$

Adaptive Control

The polynomial $T(z^{-1})$ can, for instance, simply be chosen such that the controlled system has static gain equal to 1—*i.e.*,

$$T(z^{-1}) = \frac{P(1)}{B(1)} = \frac{1 + \sum_{i=1} 3p_i}{\sum_{i=1} 2b_i}$$

These choices suggest the controller

$$u_k = -r_1 u_{k-1} + s_o y_k + s_1 y_{k-1} + t_0 r_k$$

Adaptive Control

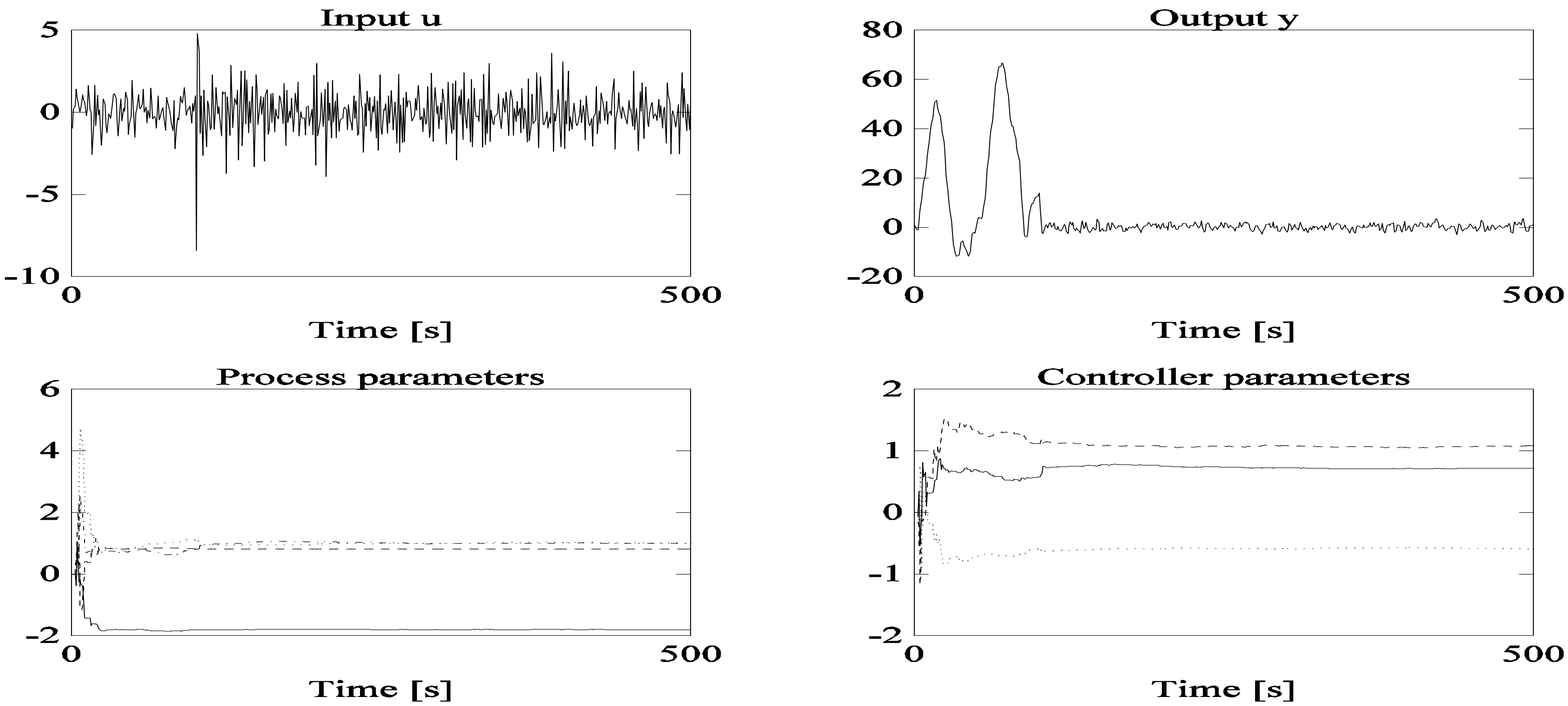
Equation including a Sylvester matrix is then reduced to the linear equation

$$\begin{pmatrix} 1 & b_1 & 0 \\ a_1 & b_2 & b_1 \\ a_2 & 0 & b_2 \end{pmatrix} \begin{pmatrix} r_1 \\ s_0 \\ s_1 \end{pmatrix} = \begin{pmatrix} p_1 - a_1 \\ p_2 - a_2 \\ p_3 \end{pmatrix}$$

or

$$A(\theta)\vartheta = b(\theta), \text{ with } \vartheta = \begin{pmatrix} r_1 \\ s_0 \\ s_1 \end{pmatrix}$$

Adaptive Control



Indirect adaptive control. The second-order process is controlled by an output feedback control law with pole assignment to the origin z=0. The histories of the input u, output y, and the estimated process parameters and the controller parameters are displayed.

Discrete-time Signals

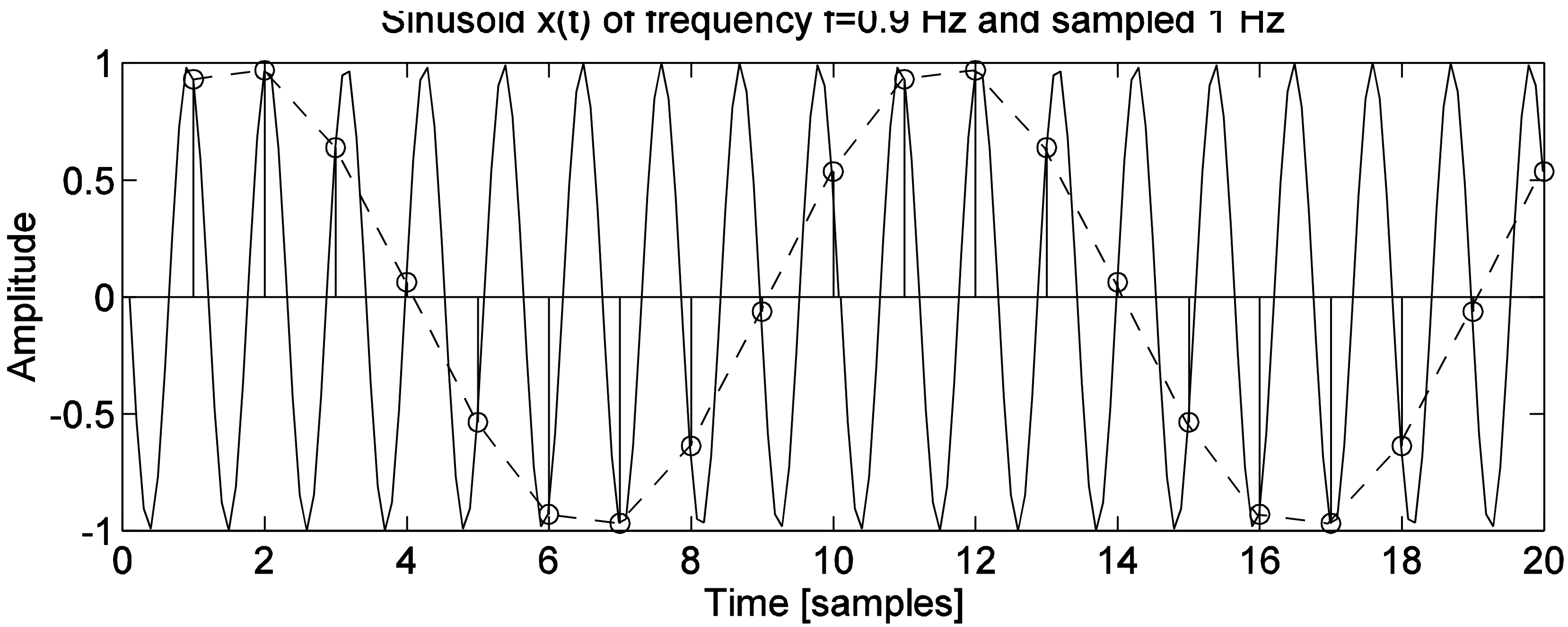
Periodic uniform sampling described by

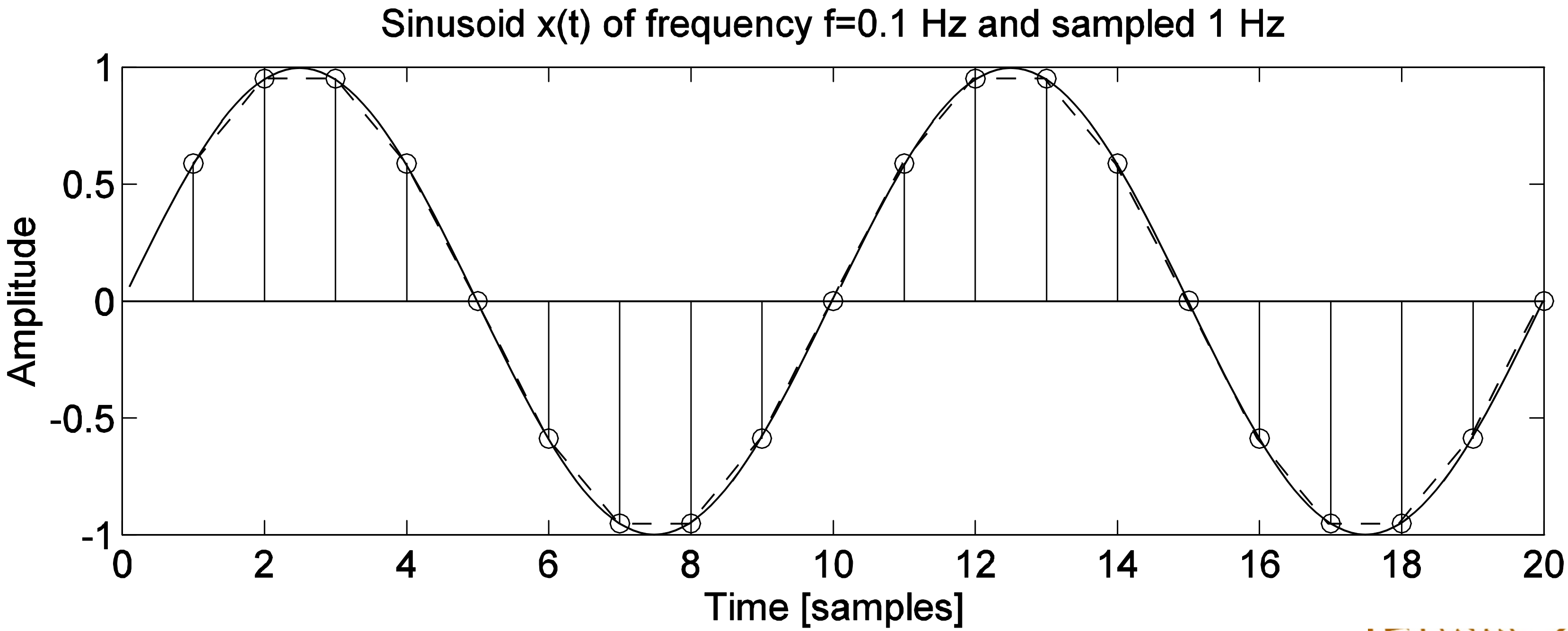
- **sampling frequency** ω_s
- **sampling period** ***h***
- **Reconstruction of a continuous-time signal from a sequence of samples can be made provided that there is no signal energy beyond the *Nyquist frequency***

$$\omega_N = \frac{1}{2}\omega_s$$

Note: Imperfections in uniform sampling

- Jitter
- Ethernet


Sinusoid x(t) of frequency f=0.9 Hz and sampled 1 Hz
Amplitude
Time [samples]
1
0.5
0
-0.5
-1
0
2
4
6
8
10
12
14
16
18
20


Sinusoid x(t) of frequency f=0.1 Hz and sampled 1 Hz
Amplitude
Time [samples]
1
0.5
0
-0.5
-1
0
2
4
6
8
10
12
14
16
18
20

Discretized Linear Systems

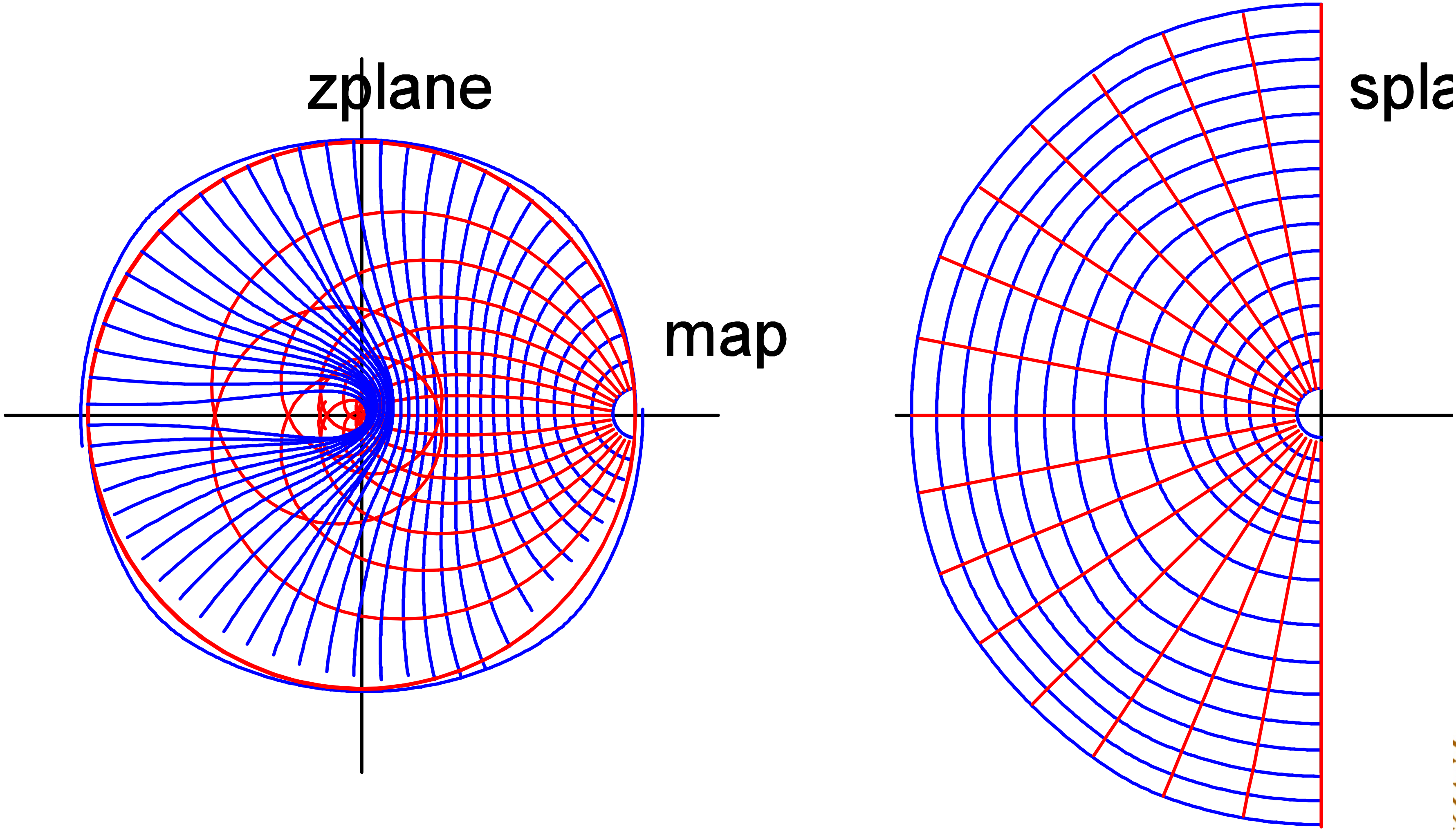
Under an additional assumption of zero-order-hold properties of the input u, sampled linear systems may be discretized as

$$\begin{cases} \dfrac{dx}{dt} = A_c x + B_c u \\ y = Cx \end{cases} \qquad \begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k \end{cases}$$

with

$$A = e^{A_c h}, \qquad B = \int_0^h e^{A_c t} B_c dt$$

Note that the discretized system is a linear time-invariant system.


zplane
map
spla

Example: Double Integrator

The continuous-time system

$$
\begin{aligned}
\frac{dx}{dt} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x
\end{aligned}
$$

is discretized as

$$
\begin{aligned}
x_{k+1} &= \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} h^2 \\ h \end{bmatrix} u_k \\
y_k &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_k
\end{aligned}
$$

under assumptions of uniform sampling with sampling period h and zero-order-hold input u.

The z-Transform

The one-sided z-transform $\mathcal{Z}\{z_k\}$ of a sequence $\{x_k\}_{k=0}^{\infty}$

$$X(z) = \mathcal{Z}\{z_k\} = \sum_{k=0}^{\infty} x_k z^{-k}$$

For further mathematical detail, see Appendix

Example: The z-Transform

A step function

$$u_k = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

has the z-transform

$$U(z) = 1 + z^{-1} + z^{-2} + z^{-3} + \cdots = \frac{z}{z-1}$$

z-transform

	Time function		z transform
Linearity	$Z\{af + bg\}$	$=$	$aZ\{f\} + bZ\{g\}$
Convolution	$Z\{f * g\}$	$=$	$Z\{f\} \cdot Z\{g\}$
	$Z\{f \cdot g\}$	$=$	$Z\{f\} * Z\{g\}$
Time translation	$Z\{f((k-d)h)\}$	$=$	$z^{-d} Z\{f(kh)\}$
Multiplication	$Z\{a^k f(k)\}$	$=$	$F_z(a^{-1}z)$
Final-value theorem	$f(\infty)$	$=$	$\lim_{z\to 1}(1 - z^{-1})F(z)$
Initial-value theorem	$f(0)$	$=$	$\lim_{z\to\infty} F_z(z)$

Table 2.1. Properties of the z-transform

State-Space Systems

Consider the discrete-time state-space system

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k \end{aligned}$$

Applications of the z-transform to the state-space system gives

$$\begin{aligned} zX(z) &= AX(z) + BU(z), \qquad X(z) = \mathcal{Z}\{z_k\} = \sum_{k=0}^{\infty} x_k z^{-k} \\ Y(z) &= CX(z) \end{aligned}$$

Elimination of $X(z)$ provides the transfer function $G(z)$ obtained as

$$\begin{aligned} X(z) &= (zI - A)^{-1} BU(z) \\ Y(z) &= CX(z) = C(zI - A)^{-1} BU(z) = G(z)U(z), \\ G(z) &= \frac{Y(z)}{U(z)} = C(zI - A)^{-1} B \end{aligned}$$

Impulse Response

The impulse response or the pulse response—*i.e.*, the output response to the unit pulse input—is

$$\begin{aligned} y_k &= CA^{k-1}B, \\ \text{for } u_k &= \delta_k = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases} \qquad k = 1, 2, 3, \ldots \end{aligned}$$

The components of the pulse response

$$g_k = CA^{k-1}B, \qquad k = 1, 2, 3, \ldots$$

are known as the *Markov parameters*

Impulse Response (cont'd)

In response to an arbitrary input sequence $\{u_k\}_{k=0}^{\infty}$ and with non-zero initial condition x_0, the output is

$$
\begin{aligned}
y_k &= CA^k x_0 + \sum_{j=0}^{k} h_{k-j} u_j \\
&= CA^k x_0 + \sum_{j=0}^{k} CA^{k-j-1} B u_j, \qquad k = 1, 2, 3, \ldots
\end{aligned}
$$

Transfer Function

The transfer function $G(z)$ of the linear system is defined by

$$
\begin{aligned}
Y(z) &= G(z)U(z), \\
G(z) &= C(zI - A)^{-1}B \\
&= \sum_{k=0}^{\infty} g_k z^{-k} = \mathcal{Z}\{g_k\}
\end{aligned}
$$

The *transfer operator* is

$$
G(q) = C(qI - A)^{-1}B
$$

Spectra

The z-spectra are

$$\begin{aligned} S_{yu}(z) &= G(z)S_{uu}(z) \\ &= \text{ input-output cross z-spectrum} \\ S_{yy}(z) &= G(z)S_{uu}(z)G^T(z^{-1}) \\ &= \text{ output z-autospectrum} \end{aligned}$$

Evaluated the spectra on the unit circle

$$\begin{aligned} S_{yu}(z) &= G(z)S_{uu}(z) \text{ for } z = e^{i\omega h} \\ S_{yy}(z) &= G(z)S_{uu}(z)G^T(z^{-1}) \text{ for } z = e^{i\omega h} \end{aligned}$$

Difference Equations

Input-output models may take on the form of *difference equations*

$$\begin{aligned} y_k &+ a_1 y_{k-1} + \cdots + a_n y_{k-n} \\ &= b_1 u_{k-1} + b_2 u_{k-2} + \cdots + b_n u_{k-n} \end{aligned}$$

or

$$\begin{aligned} y_k &= -a_1 y_{k-1} - \cdots - a_n y_{k-n} \\ &+ b_1 u_{k-1} + b_2 u_{k-2} + \cdots + b_n u_{k-n} \end{aligned}$$

Difference Equations (cont'd)

A difference equation may be converted to a state-space systems by introducing

$$\begin{aligned} \xi_k &= -a_1 \xi_{k-1} - \cdots - a_n \xi_{k-n} + u_k \\ x_k &= \begin{bmatrix} \xi_{k-1} & \xi_{k-2} & \cdots & \xi_{k-n} \end{bmatrix}^T \end{aligned}$$

Then, the following state-space system—the *controllable canonical form*—will reproduce the difference equation output

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} -a_1 & -a_2 & \dots & -a_n \\ 1 & 0 & \dots & 0 \\ & \ddots & & \vdots \\ & & 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_k \\ y_k &= \begin{bmatrix} b_1 & b_2 & \dots & b_n \end{bmatrix} x_k \end{aligned}$$

Difference Equations (cont'd)

A variety of difference equations are used such as the *autoregressive (AR) models*

$$y_k = -a_1 y_{k-1} - \cdots - a_n y_{k-n}$$

and *moving average (MA) models*

$$y_k = b_1 u_{k-1} + b_2 u_{k-2} + \cdots + b_n u_{k-n}$$

Linear models of colored noise are often formulated by means of *autoregressive moving average (ARMA) models*

$$y_k + a_1 y_{k-1} + \cdots + a_n y_{k-n} =$$
$$= w_k + c_1 w_{k-1} + c_2 w_{k-2} + \cdots + c_n w_{k-n}$$

where $\{y_k\}$ is a filtered $\{w_k\}$

ARMA models

The ARMA models obey

$$\begin{aligned} Y(z) = & - a_1 z^{-1} Y(z) - \cdots - a_n z^{-n} Y(z) \\ & + W(z) + c_1 z^{-1} W(z) + \cdots + c_n z^{-n} W(z) \end{aligned}$$

in z-transformed signals $Y(z)$, $W(z)$ and generating polynomials $A(z^{-1})$, $C(z^{-1})$

$$\begin{aligned} A(z^{-1}) Y(z) &= C(z^{-1}) W(z) \\ A(z^{-1}) &= 1 + a_1 z^{-1} + \cdots + a_n z^{-n} \\ C(z^{-1}) &= 1 + c_1 z^{-1} + \cdots + c_n z^{-n} \end{aligned}$$

ARMAX Models

$$
\begin{aligned}
y_k = & - a_1 y_{k-1} - \cdots - a_n y_{k-n} \\
& + b_1 u_{k-1} + b_2 u_{k-2} + \cdots + b_n u_{k-n} \\
& + w_k + c_1 w_{k-1} + c_2 w_{k-2} + \cdots + c_n w_{k-n}
\end{aligned}
$$

or

$$
\begin{aligned}
A(z^{-1})Y(z) &= B(z^{-1})U(z) + C(z^{-1})W(z) \\
A(z^{-1}) &= 1 + a_1 z^{-1} + \cdots + a_n z^{-n} \\
B(z^{-1}) &= b_1 z^{-1} + \cdots + b_n z^{-n} \\
C(z^{-1}) &= c_0 + c_1 z^{-1} + \cdots + c_n z^{-n}
\end{aligned}
$$

The transfer function relationships are

$$
Y(z) = \frac{B(z^{-1})}{A(z^{-1})}U(z) + \frac{C(z^{-1})}{A(z^{-1})}W(z)
$$

Forward vs backward shift form

$$A(z) = z^n + a_1 z^{n-1} + \ldots + a_n$$

$$A^*(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}$$

$$A(z) = z^n A^*(z^{-1}) \text{ where } n = \deg(A)$$

Example:

$$A(z)y = B(z)u \quad \Leftrightarrow \quad A^*(z^{-1})y = z^{-d} B^*(z^{-1})u$$

$$\text{where } d = \deg(A) - \deg(B)$$

System Interconnections

Series interconnection

$$\begin{cases} Y_1(z) = H_1(z)U_1(z) \\ Y_2(z) = H_2(z)U_2(z) \end{cases}$$

$$\begin{aligned} Y_2(z) &= H_2(z)U_2(z) = H_2(z)Y_1(z) \\ &= H_2(z)H_1(z)U_1(z) \end{aligned}$$

System Interconnections

Parallel interconnection

$$Y(z) = Y_1(z) + Y_2(z)$$

$$\text{with} \quad \begin{cases} Y_1(z) = H_1(z)U(z) \\ Y_2(z) = H_2(z)U(z) \end{cases}$$

with

$$Y(z) = H(z)U(z),$$

$$H(z) = H_1(z) + H_2(z)$$

System Interconnections

Feedback interconnection

$$\begin{cases} Y_1(z) = H_1(z)U_1(z) \\ Y_2(z) = H_2(z)U_2(z) \end{cases}$$

with $\begin{cases} U_2(z) = Y_1(z) \\ U_1(z) = -Y_2(z) + U_0(z) \end{cases}$

with

$$\begin{aligned} Y_1(z) &= H_1(z)U_1(z) = H_1(z)(-Y_2(z) + U_0(z)) \\ &= H_1(z)(-H_2(z)Y_1(z) + U_0(z)) \\ Y_1(z) &= (I + H_2(z)H_1(z))^{-1}U_0(z) \\ Y_2(z) &= H_2(z)(I + H_2(z)H_1(z))^{-1}U_0(z) \end{aligned}$$

Stability

The set of eigenvalues $\{\lambda_i\}$ of the matrix A of the discrete-time state-space system

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k \end{aligned}$$

will determine the stability properties of the system.

Stability

For $z = \lambda_i \in \mathbb{C}$, the corresponding transfer function

$$G(z) = C(zI - A)^{-1}B$$

will assume infinite value and these points are called the *poles* of the transfer function.

The values of z for which $G(z) = 0$ are called the *zeros* of the transfer function.

Stability

If all poles $z = \lambda_i \in \mathbb{C}$ are such that $|z| = |\lambda_i| < 1$, then the system is asymptotically stable.

Hence, the stability condition for z requires that all poles be within the unit circle $|z| = 1$ in the complex plane.

Example: Double Integrator

The discrete-time double integrator

$$
\begin{aligned}
x_{k+1} &= \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} h^2 \\ h \end{bmatrix} u_k \\
y_k &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_k
\end{aligned}
$$

has a transfer function

$$
\begin{aligned}
G(z) &= C(zI - A)^{-1} B = \\
&= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z-1 & -h \\ 0 & z-1 \end{bmatrix}^{-1} \begin{bmatrix} h^2 \\ h \end{bmatrix} \\
&= \frac{h^2 z}{(z-1)^2}
\end{aligned}
$$

with two poles at $z = 1$—*i.e.*, the system is not asymptotically stable.

Example: Double Integrator

The state feedback control

$$u_k = -Kx_k = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} x_k$$

results in the closed-looop system

$$\begin{aligned} x_{k+1} &= (A - BK)x_k \\ &= \begin{bmatrix} 1 - k_1 h^2/2 & h - k_2 h^2/2 \\ -k_1 h & 1 - k_2 h \end{bmatrix} x_k \end{aligned}$$

with the characteristic equation

$$z^2 + (k_1 \frac{h^2}{2} + k_2 h - 2)z + (k_1 \frac{h^2}{2} - k_2 h + 1) = 0$$

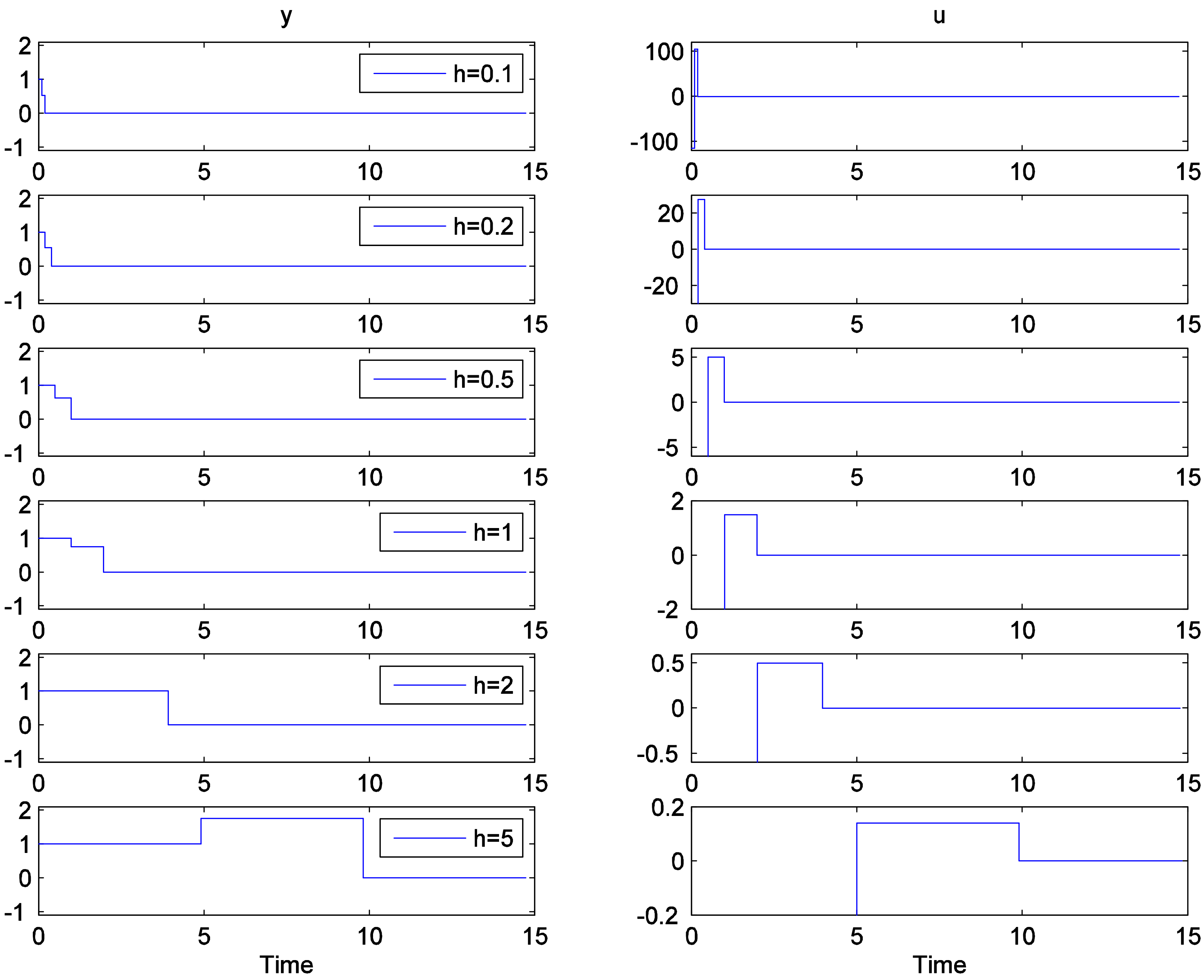
Example: Double Integrator

Stabilizing pole assignment to $z = 0$ is accomplished by choosing k_1, k_2 such that the characteristic equation become $z^2 = 0$ or

$$\begin{bmatrix} \frac{1}{2}h^2 & h \\ \frac{1}{2}h^2 & -h \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

that is

$$K = \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{h^2} & \dfrac{3}{2h} \end{bmatrix}$$



y
u
h=0.1
h=0.2
h=0.5
h=1
h=2
h=5
Time
Time

Linear Disturbance Models

Discrete-time impulses are modeled with a duration of one sampling period and with unit amplitude

$$\delta(kh) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}, \qquad \mathcal{Z}\{\delta(kh)\} = 1 \quad \text{Impulse}$$

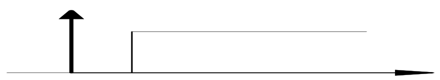
For impulses in continuous-time systems, the Dirac impulse $\delta(t)$ is the most commonly used signal model.

$$\delta(t) = \begin{cases} \infty, & t = 0, \\ 0, & t \neq 0 \end{cases} \qquad \text{with } \int_{-\infty}^{\infty} \delta(t)dt = 1,$$

Steps and Ramps

$$\mathbf{1}(kh) = f(kh) = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

$$F(z) = \frac{z}{z-1}$$



$$f(kh) = \begin{cases} kh, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

$$F(z) = \frac{hz}{(z-1)^2}$$

Periodic Disturbances

Sinusoidal disturbances are often used to model periodic disturbances or loads.

$$f(kh) = \begin{cases} \sin \omega kh, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

$$F(z) = \frac{z \sin \omega h}{z^2 - 2z \cos \omega h + 1}$$

Periodic Disturbances

Disturbances with sinusoidals are often used to model periodic disturbances or loads.

$$f(kh) = \begin{cases} \sum_m \sin \omega m k h, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

$$F(z) = \sum_m \frac{z \sin m\omega h}{z^2 - 2z \cos m\omega h + 1}$$

Useful Matlab Commands

- s=tf('s');   % define s-operator
- z=tf('z',h); % define z (h sample time)
- Gc=1/s^2; % define system
- Gd=c2d(G,0.1); % sampling system
- ss,tf,zpk   % system representations
- acker, place   % pole placement
- help control

What to do now

- Get book
- Read chap 1-2. (Reading notes available on web)
- Go to Exercise 1 (computer simulation of adaptive system)
- Check out the matlab functions
- Next lecture: Parameter estimation (chap 3-4)