LUNDS TEKNISKA HÖGSKOLA
Lunds universitet

Institutionen för
REGLERTEKNIK

FRTN15 Predictive Control

Final Exam October 17, 2011, 8am - 1pm

General Instructions

This is an open book exam. You may use any book you want, but no notes, exercises, exams, or solution manuals are allowed. Solutions and answers to the problems should be well motivated. The exam consists of 7 problems. The credit for each problem is indicated in the problem. The total number of credits is 25 points. Preliminary grade limits:

Grade 3:	12 – 16 points
Grade 4:	17 – 21 points
Grade 5:	22 – 25 points

Results

The results of the exam will be posted at the latest October 24 on the notice board on the first floor of the M-building.

1.

a. Consider the nonlinear model:

$$y(t) + a_1 y(t-1) = b_1 u(t-1) + b_2 u(t-1) y(t-1)$$

Find a linear-in-parameters regression model for estimation of the parameters a_1, b_1 and b_2. (1 p)

b. It is desired to use this regression model to perform online identification of the parameters. In addition, it is known that the parameters may be slowly time-varying. Write down an appropriate algorithm for this estimation task, and explain its operation. (1 p)

c. An indirect adaptive controller is to be designed, using the estimation method from above. Design a control law, incorporating the reference signal $u_c(t)$, such that the closed-loop system has the transfer function:

$$Y(z) = \frac{b_0}{z + a_0} U_c(z)$$

where $1 > a_0 > -1$. The controller may be nonlinear. (1 p)

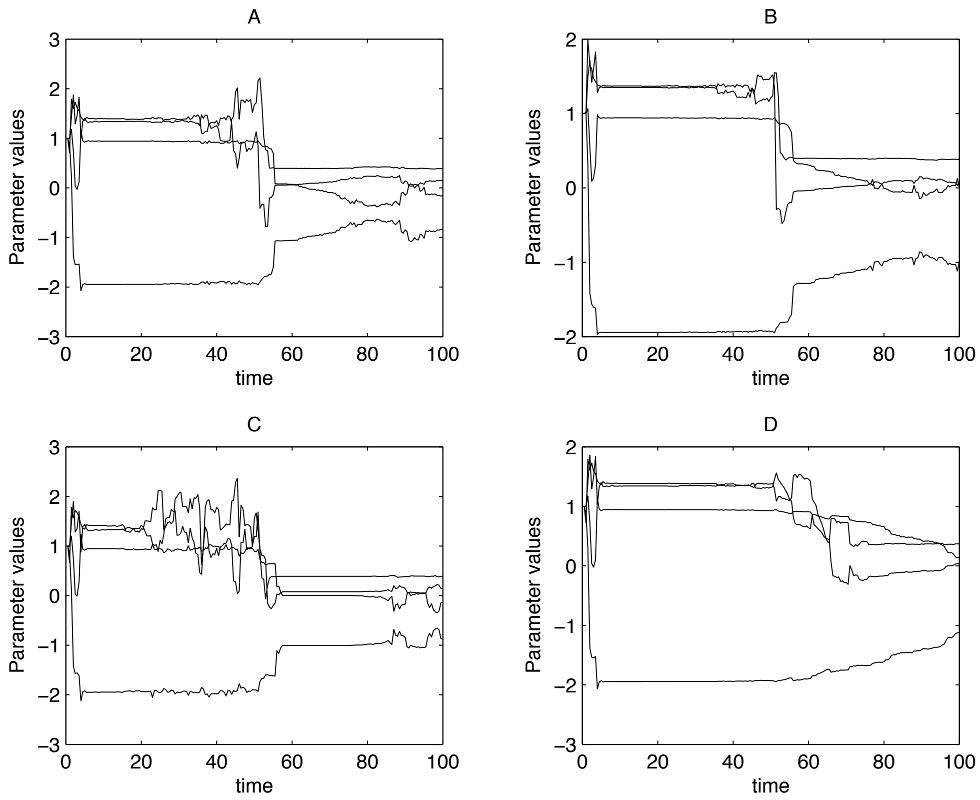
2. A self-tuning regulator using an RLS estimation algorithm has been designed for a second order system with unknown parameters. During testing, simulations have been carried out for different values of the forgetting factor λ and the measurement noise variance σ^2. Figure 1 shows the parameter estimates during a process variation test, where the unknown system's dynamics are changes at $t = 50$. The test was carried out for four different conditions:

1. $\lambda = 0.8,\ \sigma^2 = 0.001$
2. $\lambda = 0.9,\ \sigma^2 = 0.001$
3. $\lambda = 0.9,\ \sigma^2 = 0.0001$
4. $\lambda = 0.95,\ \sigma^2 = 0.001$

Unfortunately, the engineer responsible for the tests was not very methodical and forgot to write down the conditions corresponding to each of the results.

a. Assist the engineer by determining which of the cases 1–4 above correspond to the plots A–D in Figure 1. Clearly state your reasoning. (2 p)

b. The engineer is not pleased with the noise performance of the system with lower values of the forgetting factor. Is it possible to achieve better noise rejection by adjusting the value of the initial covariance matrix P_0? Explain your answer. (1 p)



Figur 1 Parameter plots for Problem 2

3. Consider the process

$$y_k = \frac{1}{z - a_u} u_k + \frac{z}{z - a_e} e_k$$

where the noise sequence $\{e_k\}$ is independent white noise and u_k is the control signal.

a. Design a controller that minimizes $E(y_k^2)$. (2 p)

b. Assume that the parameter a_u is known and that a_e is unknown. Derive a regression model for identification of the parameter a_e. (1 p)

c. Design a controller for the process when a_u is known and a_e is slowly time-varying with values in a large interval. Motivate your choice of controller structure. (2 p)

4.

a. Briefly explain the principle of Model Predictive Control. Use a sketch or a diagram to illustrate the terms *prediction horizon* and *control horizon*. (1 p)

b. What computational problems can arise if the plant is operated near constraints on the outputs, and how can the MPC formulation be modified to limit these problems? (1 p)

5. The following equations describe the problem of Iterative Learning Control (ILC):

$$
\begin{aligned}
y_k(t) &= G_c(q)r(t) + G_c(q)u_k(t) \\
e_k(t) &= r(t) - y_k(t) \\
u_k(t) &= Q(q)[u_{k-1}(t) + L(q)e_{k-1}(t)]
\end{aligned}
$$

where $G_c(q)$ is the closed-loop transfer function of the system and q is the forward time shift operator.

a. Explain the principle of ILC and a draw a block diagram of the system. (1 p)

b. Give two examples of applications where ILC would be a suitable control strategy. (1 p)

c. Assume that $Q(q) = 1$ and that

$$
G_C(q) = \frac{1}{q - 0.9}, \qquad L(q) = (\alpha q + 1)(q - 0.9)
$$

Determine how to choose α in order to assure ILC stability and error convergence. (1 p)

d. It is desired that the error dynamics of the ILC algorithm should be given by:

$$
e_k(t) = H(q)e_{k-1}(t)
$$

Assuming a model of the closed loop system $\hat{G}_c$ is available, and that $Q(q) = 1$, explain how to design the filter $L(q)$ in order to achieve the desired error dynamics. (1 p)

6. Consider the stable, linear system

$$
\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{aligned}
$$

where

$$
x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ k \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \end{pmatrix}
$$

k is an unknown parameter. The control law is given by $u = \theta u_c$, where $\theta = \theta(t)$ is the controller parameter and u_c is the reference signal. The system can be written as $y = kG(s)u$, that is, k is the feedforward gain for the system, where

$$
G(s) = \frac{1}{s^2 + s + 1}
$$

a. We want to determine the control parameter θ, such that the output follows the reference model $y_m = k_0 G(s) u_c$. Introduce the state error $\tilde{x} = x - x_m$, and the output error $e = y - y_m$. Let the desired θ be $\theta_0 = k_0/k$. Then the state-space equations for the the system from $(\theta - \theta_0)u_c$ to e are given by

$$\begin{aligned}\dot{\tilde{x}} &= A\tilde{x} + B(\theta - \theta_0)u_c \\ e &= C\tilde{x}\end{aligned}$$

Use Lyapunov theory to derive a control law which guarantees that $\tilde{x}$ goes to zero. Motivate your choice of the controller. What extra knowledge do we need? (2 p)

Hint: Use the Lyapunov function

$$V = \frac{1}{2}\left(\tilde{x}^T P \tilde{x} + \frac{1}{\gamma}(\theta - \theta_0)^T(\theta - \theta_0)\right)$$

b. We can use the output error e instead of the state error $\tilde{x}$ in the update law for the parameter θ and guarantee that e goes to zero if $G(s)$ is SPR. Assume that $G(s)$ is SPR, and derive such an update law. (2 p)

c. Is the transfer function $G(s)$ SPR? Also check whether

$$G_1(s) = \frac{1}{s+1}$$

is SPR.

(1 p)

7. Determine the Kalman filter and derive the steady-state estimation covariance and filter gain for the system

$$\begin{aligned}x_{k+1} &= 0.4x_k + v_k \\ y_k &= x_k + e_k\end{aligned}$$

where v_k and e_k are zero-mean, uncorrelated white noise processes with variance 1. Compare the steady-state estimation covariance to that of x_k using the direct measurement as an estimate of $\bar{x}_k = y_k$ to predict x_k. Consider the time-invariant case only! (3 p)