Predictive Control - Exercise Session 4
Adaptive Control: Self Tuning Regulators and Model Reference Adaptive Systems

1. Indirect Self Tuning Regulator: Consider the system

$$G(s) = G_1(s)G_2(s)$$

where

$$G_1(s) = \frac{a}{s+b}$$
$$G_2(s) = \frac{c}{s+d}$$

Here a and b are unknown parameters and c and d are known. This could for example represent a system where the plant is known but where certain sensor dynamics are unknown. The system is to be controlled in such a way that the closed loop system is given by:

$$G_m(s) = \frac{\omega^2}{s^2 + 2\omega\zeta s + \omega^2}$$

a. Construct a discrete time indirect self tuning regulator *without* zero cancellation.

b. Construct a discrete time indirect self tuning regulator *with* zero cancellation.

Solution

We begin by sampling both the plant and the desired closed loop:

$$H(z) = \frac{b_0 z + b_1}{z^2 + a_1 z + a_2}$$
$$H_m(z) = \frac{b_{m0} z + b_{m1}}{z^2 + a_{m1} z + a_{m2}}$$

Since the sampled plant's discrete time parameters may depend on both the known and unkown continuous time parameters, we will proceed as though all parameters were unkown.

Since we are designing an indirect STR, we require a linear regression model for the plant's parameters. This is given by:

$$\theta = (\, b_o \quad b_1 \quad a_1 \quad a_2\,)^T$$
$$\phi(t) = (\, u(t-1) \quad u(t-2) \quad -y(t-1) \quad -y(t-2)\,)$$

a. First we will design the controller without cancelling the process zero (i.e. $B^+ = 1$). Using the causality condition $\deg A_c \geq 2\deg A - 1$ we find that A_c must have at least degree 3. Since we know that the degree of A is 2, the degree of R must therefore be at least 1. The minimum degree design is achieved by choosing the smallest possible degrees of the design polynomials, so we get $\deg A_c = 3$ and $\deg R = \deg S = 1$. The Diophantine equation for this problem is:

$$(z^2 + a_1 z + a_2)(z + r_1) + (b_0 z + b_1)(s_0 z + s_1) = (z^2 + a_{m1} z + a_{m2})(z + a_{o1})$$

Identification of coefficients of equal powers of z gives:

$$\begin{aligned}
z^2 &: a_1 + r_1 + b_0 s_0 = a_{m1} + a_{o1} \\
z^1 &: a_2 + a_1 r_1 + b_1 s_0 + b_0 s_1 = a_{m1} a_{o1} + a_{m2} \\
z^0 &: a_2 r_1 + b_1 s_1 = a_{m2} a_{o1}
\end{aligned}$$

The solution to these linear equations is:

$$\begin{aligned}
r_1 &= \frac{b_1^2 n_1 - b_0 b_1 n_2 + a_{o1} a_{m2} b_0^2}{b_0^2 a_2 - a_1 b_0 b_1 + b_1^2} \\
s_0 &= \frac{n_1 - r_1}{b_0} \\
s_1 &= \frac{b_0 n_2 - b_1 n_1 - r_1 (a_1 b_0 - b_1)}{b_0^2}
\end{aligned}$$

where:

$$\begin{aligned}
n_1 &= a_{m1} + a_{o1} - a_1 \\
n_2 &= a_{m1} a_{o1} + a_{m2} - a_2
\end{aligned}$$

b. The controller will now be designed using zero cancellation ($B^+ = z + b_1/b_0$). The orders of the polynomials are chosen to be the same as above, but now a factor of B^+ may be cancelled from the Diophantine equation. In this case the Diophantine equation gives:

$$(z^2 + a_1 z + a_2)1 + b_0(s_0 z + s_1) = z^2 + a_{m1} z + a_{m2}$$

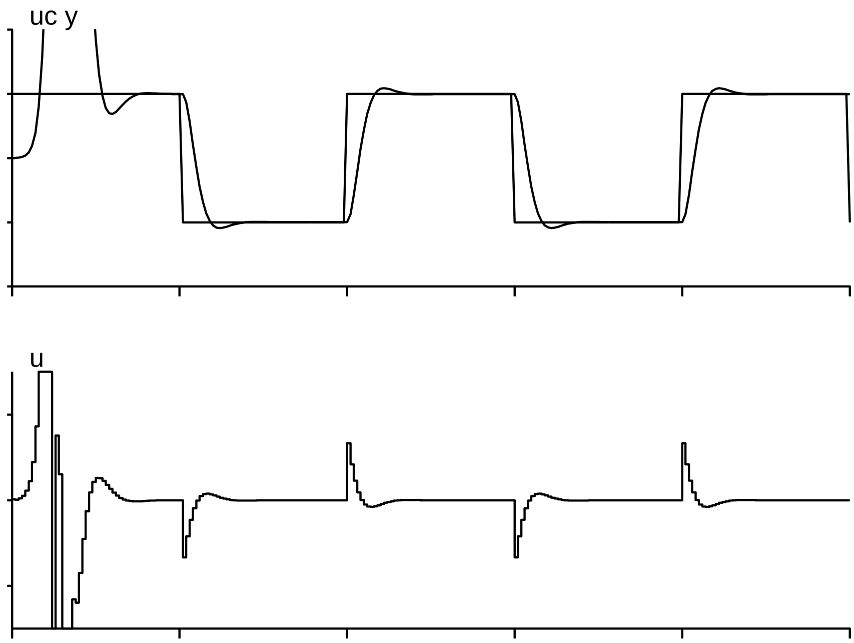
Identification of coefficients of equal powers of z gives:

$$\begin{aligned}
z^1 &: a_1 + b_0 s_0 = a_{m1} \qquad & s_0 &= \frac{a_{m1} - a_1}{b_0} \\
z^0 &: a_2 + b_0 s_1 = a_{m2} \qquad & s_1 &= \frac{a_{m2} - a_2}{b_0}
\end{aligned}$$

The controller is thus given by:

$$\begin{aligned}
R(z) &= z + b_1/b_0 \\
S(z) &= s_0 z + s_1 \\
T(z) &= t_0 z \qquad \text{where} \qquad t_0 = \frac{1 + a_{m1} + a_{m2}}{b_0}
\end{aligned}$$

To examine the behaviour of the systems we have designed, we may use simulations. Simulation of the system without zero cancellation is shown in Figure 1. Simulation results for the system desinged with cancellation are shown in Figure 2. It can be seen that the control signal in the design



Figur 1 Simulation of Problem 1a. Process output and control signal are shown for the indirect self-tuning regulator when the process zero is not cancelled.

with cancellation exhibits 'ringing'. This is the result of cancelling a poorly damped process zero. In this case the process is given by:

$$H(z) = \frac{0.0187(z + 0.936)}{(z - 1)(z - 0.819)}$$

which has a zero in $z = -0.936$.

2. Direct Self Tuning Regulator: Using the same plant and specification as in Problem 1, design:

a. A direct self tuning regulator *without* zero cancellation.

b. A direct self tuning regulator *with* zero cancellation.

Solution

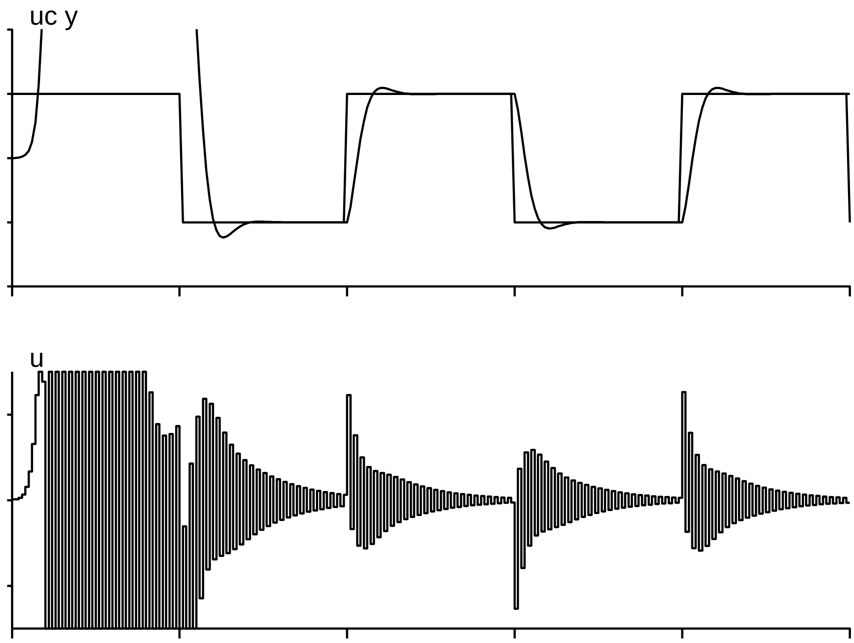
To obtain a direct self-tuning regulator we start with the Diophantine design equation:

$$AR + BS = A_m A_o B^+$$

Let the design equation operate on y:

$$B^+ A_m A_o y = ARy + BSy = BRu + BSy$$

$$y = R \underbrace{\left(\frac{B^-}{A_o A_m} u \right)}_{u_f} + S \underbrace{\left(\frac{B^-}{A_o A_m} y \right)}_{y_f}$$



Figur 2 Simulation of Problem 1b. Process output and control signal are shown for the indirect self-tuning regulator when the process zero is cancelled.

Here, A_m is the desired characteristic polynomial and A_o is the observer polynomial. We now have a regression model in which the R and S polynomials are parameters and the filtered input and output signals u_f and y_f are regressors. Thus we may estimate R and S.

The T polynomial is given by:

$$T = \frac{t_o A_o B_m}{B^-}$$

where t_o is chosen to give the correct steady state gain.

a. We will begin by designing without cancellation. We then have $B^+ = 1$ and $B^- = b_0 q + b_1$. From the analysis is of the indirect STR we know that a first order observer is required, i.e. $A_0 = q + a_{o1}$. We have as before:

$$y = R \underbrace{\left(\frac{B^-}{A_o A_m} u \right)}_{u_f} + S \underbrace{\left(\frac{B^-}{A_o A_m} y \right)}_{y_f} \tag{1}$$

Since B^- is not known we cannot calculate u_f and y_f. One possibility is to rewrite Equation 1 as:

$$y = \underbrace{RB^-}_{R'} \left(\frac{1}{A_o A_m} u \right) + \underbrace{SB^-}_{S'} \left(\frac{1}{A_o A_m} y \right)$$

and to estimate R' and S' as second order polynomials and to cancel the common factor B^- from the estimated polynomials. This is difficult because there will not be an exact cancellation. Another possibility is to use some estimate of B^-. A third possibility is to try to estimate $B^- R$ and $B^- S$ as a bilinear problem.

b. If the polynomial B is canceled we have $B^+ = z + b_1/b_0$, $B^- = b_0$. From the analysis of the indirect STR we know that no observer is needed in this case and that the controller has the structure $\deg R = \deg S = 1$. Hence:

$$y(t) = R\left(\frac{b_o}{A_m}\,u(t)\right) + S\left(\frac{b_o}{A_m}\,y(t)\right)$$

Since b_o is not known we include it in the polynomial R and S and estimate it. The polynomial R then is not monic. We have:

$$y(t) = (r_0 q + r_1)\left(\frac{1}{A_m}\,u(t)\right) + (s_0 q + s_1)\left(\frac{1}{A_m}\,y(t)\right)$$

To obtain a direct STR we thus estimate:

$$\theta = \begin{pmatrix} r_0 & r_1 & s_0 & s_1 \end{pmatrix}^T$$

by RLS. The case $r_0 = 0$ must be taken care of separately. Furthermore T has the form $T(q) = t_0 q$ where:

$$\frac{BT}{AR+BS} = \frac{Bt_o q}{\underbrace{B^+ b_o}_{B} A_m} = \frac{t_o q}{A_m}$$

To get unit steady state gain choose:

$$t_o = A_m(1)$$

In Figures 3–6 we show simulation when the model in Equation 1 is used with:

$$\begin{aligned} B^- &= 1 \\ B^- &= q \\ B^- &= \frac{q+0.4}{1.4} \\ B^- &= \frac{q-0.4}{0.6} \end{aligned}$$

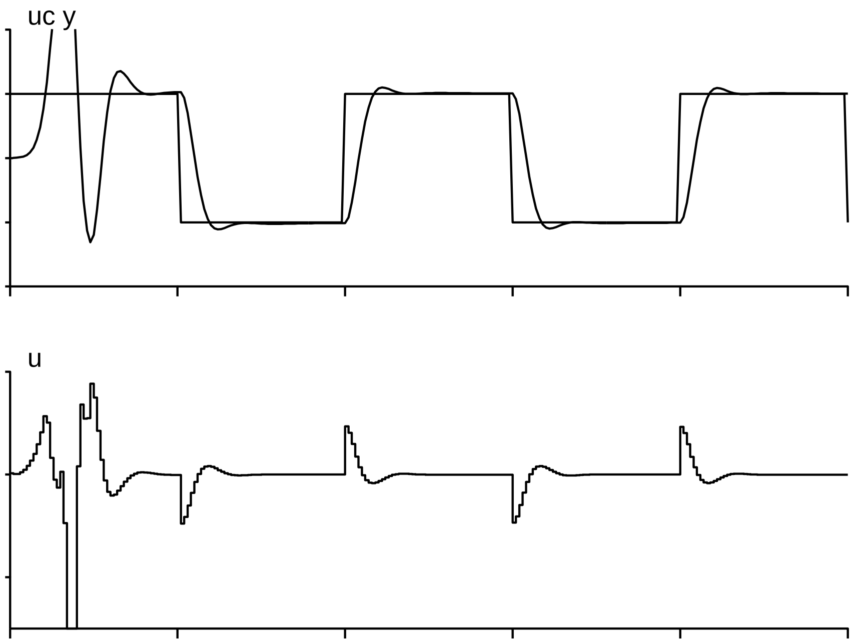
The simulation results for the case when the process zero is cancelled are shown in Figure 7.

Again we see that cancellation of the process zero gives a ‘ringing’ control signal. However, it is also noted that the choice of B^- (which is unknown) is critical for performance in the case where no zero is cancelled.

3. Model Reference Adaptive Control using MIT Rule: In this problem we consider a linear process with the transfer function $kG(s)$, where $G(s)$ is known and k is an unknown parameter. Find a feedforward controller that gives a system with the transfer function $G_m(s) = k_0 G(s)$ where k_0 is a given constant. Use the controller structure

$$u = \theta u_c$$

where u is the control signal and u_c the command signal. Use the MIT rule to update the parameter θ, and draw a block diagram of the resulting adaptive system.



Figur 3 Simulation in Problem 2a. Process output and control signal are shown for the direct self-tuning regulator when the process zero is not canceled and when $B^- = 1$.

Solution

With the controller structure

$$u = \theta u_c$$

the transfer function from command signal to the output becomes $\theta k G(s)$. This transfer function is equal to $G_m(s)$ if the parameter θ is chosen as

$$\theta = \frac{k_0}{k}$$

We will now use the MIT-rule to obtain a method for adjusting the parameter θ when k is not known. The error is

$$e = y - y_m = kG(p)\theta u_c - k_0 G(p) u_c$$

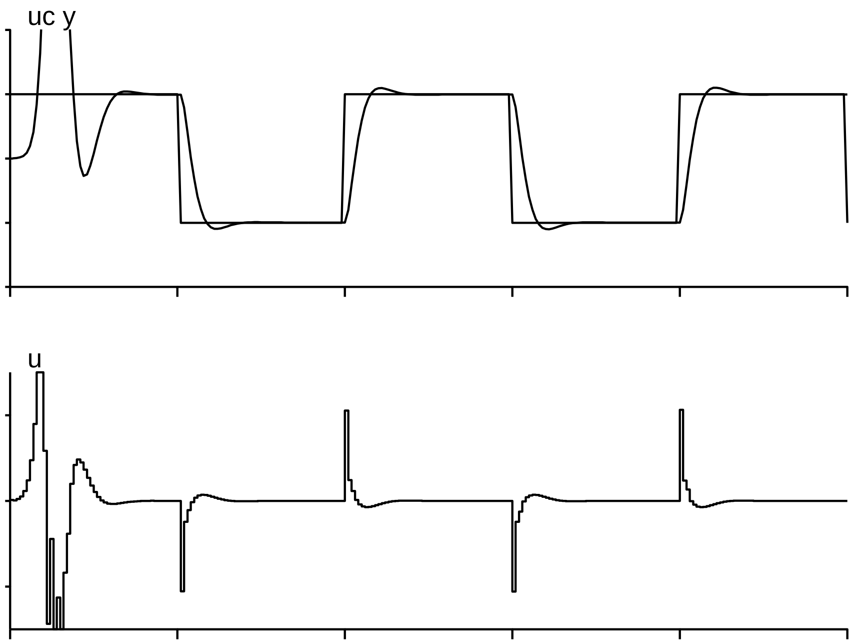
where u_c is the command signal, y_m the model output, y the process output, θ the adjustable parameter, and $p = d/dt$ the differential operator. The sensitivity derivative is given by

$$\frac{\partial e}{\partial \theta} = kG(p)u_c = \frac{k}{k_0} y_m$$

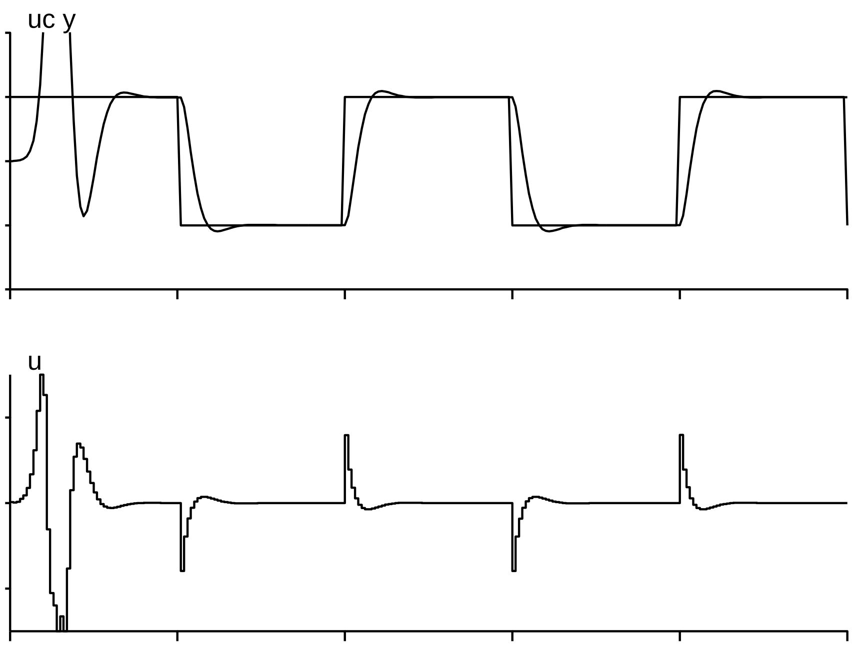
The MIT rule then gives the following adaptation law

$$\frac{d\theta}{dt} = -\gamma' \frac{k}{k_0} y_m e = -\gamma y_m e \tag{2}$$

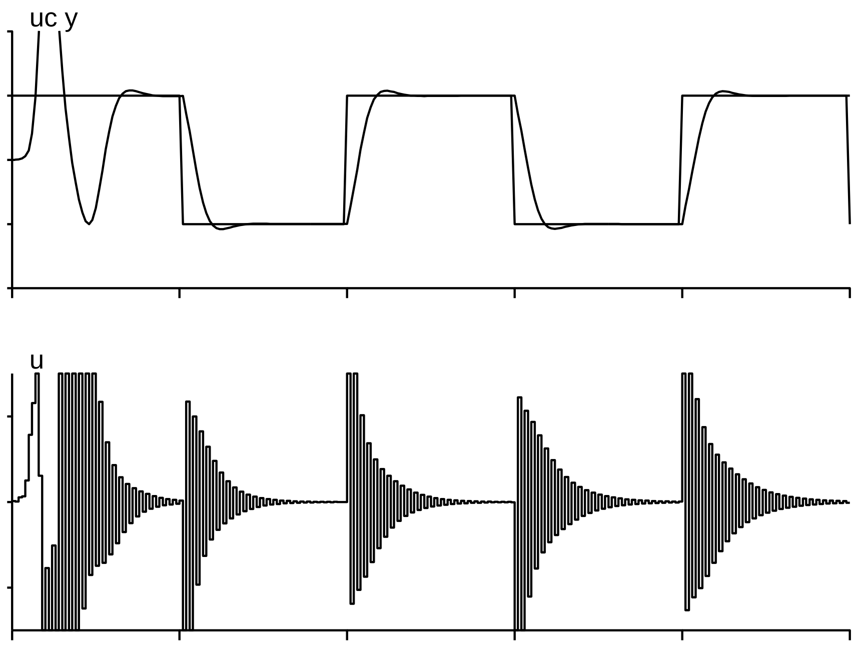
where $\gamma = \gamma' k/k_0$ has been introduced instead of γ'. Notice that in order to have the correct sign of γ it is necessary to know the sign of k. Equation 2 gives the law for adjusting the parameter. A block diagram of the system is shown in Figure 8.



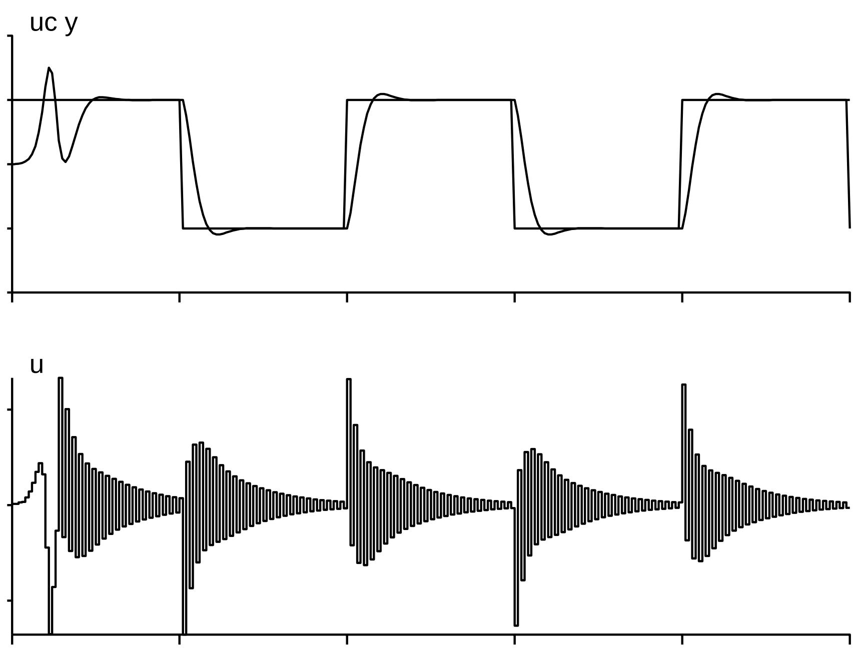
Figur 4 Simulation in Problem 2a. Process output and control signal are shown for the direct self-tuning regulator when the process zero is not canceled and when $B^- = q$.



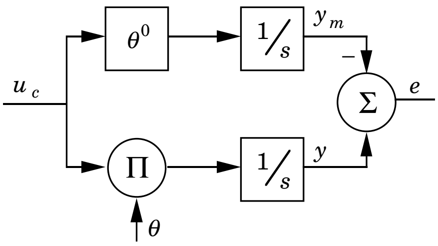
Figur 5 Simulation in Problem 2a. Process output and control signal are shown for the direct self-tuning regulator when the process zero is not canceled and when $B^- = (q + 0.4)/1.4$.



Figur 6 Simulation in Problem 2a. Process output and control signal are shown for the direct self-tuning regulator when the process zero is not canceled and when $B^- = (q - 0.4)/0.6$.



Figur 7 Simulation in Problem 2b. Process output and control signal are shown for the direct self-tuning regulator when the process zero is canceled.



Figur 8 Block diagram of an MRAS for adjustment of a feedforward gain based on the MIT rule.