1 Indirect Min-Degree STR Design

In the disturbance free case we start out with:

Process model:	$Ay = Bu,$	(1)
Controller structurel:	$Ru = -Sy + Tu_c,$	(2)
Reference model:	$A_m y_m = B_m u_c.$	(3)

We require causal, proper transfer functions. Combining (1), (2) yields

$$y = \frac{BT}{AR + BS} u_c \tag{4}$$

$$u = \frac{AT}{AR + BS} u_c. \tag{5}$$

The model matching critera from (3), (4) is

$$\frac{BT}{AR + AS} = \frac{B_m}{A_m}. \tag{6}$$

Define

$$A_c = AR + BS \tag{7}$$

and introduce A_o such that

$$A_c = A_o A_m B \tag{8}$$

$$T = A_o B_m. \tag{9}$$

Factor

$$B = B^+ B^-, \tag{10}$$

where B^+ will be cancelled and B^- will not. Hence, it makes sense to put any non-minimum phase zeros (and close to non-minimum phase zeros) of B in B^-. Since B^+ will be cancelled and $\gcd(A, B) = 1$ (since (1) is proper), (6) yields

$$R = R_1 B^+, \tag{11}$$

for some R_1 and

$$A_c = A_o A_m B^+ \tag{12}$$

from (8). Equations (7), (11), (12) now yields

$$AR_1 B^+ + B^+ B^- S = A_o A_m B^+ \Leftrightarrow AR_1 + B^- S = A_o A_m. \tag{13}$$

If R_0, S_0 solve (7), it is also solved by

$$R = R_0 + \Lambda B \tag{14}$$

$$S = S_0 - \Lambda A \tag{15}$$

(since $AR + BS = AR_0 + A\Lambda B + BS_0 - B\Lambda A = A_c$). Hence Λ minimizing $\deg R$ yields

$$\deg R < \deg B. \tag{16}$$

To see this, assume $\deg R_0 \geq \deg B$. We can then choose Λ such that $\deg R = \deg(R_0 + \Lambda B) < \deg B$. Further, properness of (1) imples

$$\deg A > \deg B, \tag{17}$$

whereas causality of (2) implies

$$\deg S \leq \deg R. \tag{18}$$

From (17), (18) we now see that the degree of A_c is given by

$$\deg A_c = \deg(AR + BS) = \deg A + \deg R. \tag{19}$$

And to conclude

$$\deg A + \deg R < 2 \deg A. \tag{20}$$