FRTN15 Predictive Control—Exercise Session 6

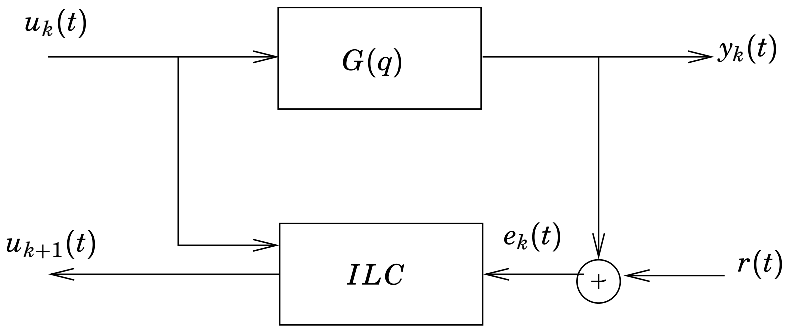
1. Consider the system

$$G(q) = \frac{0.09516}{q - 0.9048}.$$

It is controlled using ILC (see Figure 1) such that the control signal at an iteration k is given by:

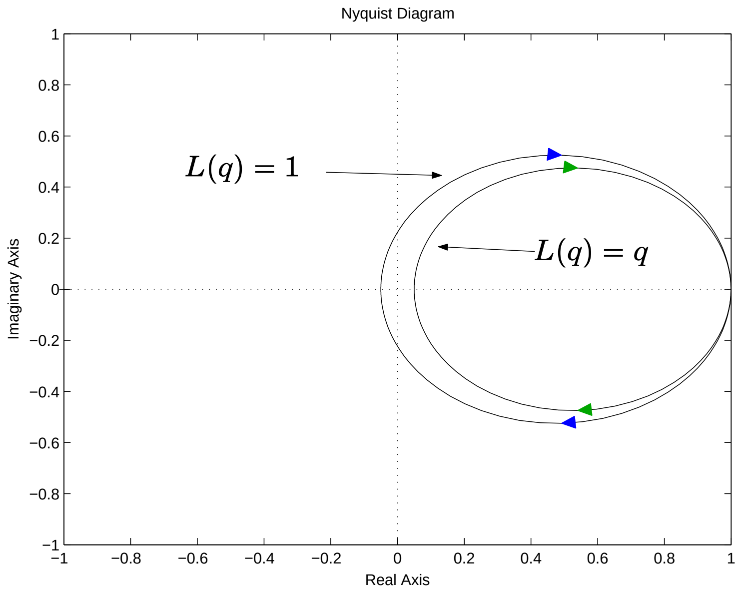
$$u_{k+1}(t) = u_k(t) + L(q)e_k(t)$$

where $e_k(t) = r(t) - y_k(t)$.



Figur 1 AN ILC feedback system.

Study the convergence of the ILC iterations for $L(q) = 1$ and $L(q) = q$.
Hint: The Nyquist plots of $G(q)L(q)$ for the two chosen L are shown in Figure 2.



Figur 2 Nyquist plots for $G(q)L(q)$.

2.

a. Show that the system

$$\begin{aligned} \dot{x} &= -x + u, \quad x(0) = x_0, & (1) \\ y &= x & (2) \end{aligned}$$

with transfer function

$$G_1(s) = \frac{1}{(s+1)} \tag{3}$$

is strictly positive real (SPR) and that the storage function

$$V(x) = \frac{1}{2} x^T x$$

fulfills the passivity property

$$V(x(t)) = V(x(0)) + \int_0^t y^T(\tau) u(\tau) dt - \int_0^t x^T(\tau) x(\tau) d\tau \tag{4}$$

What is the interpretation of all the three terms on the right-hand side of Eq. (4)?

b. Show that the transfer function

$$G_2(s) = \frac{1}{(s+1)^2} \tag{5}$$

is not positive real.

3. Consider the second order system:

$$\begin{aligned} \dot{x}_1 &= -x_2 + \theta x_1^2 \\ \dot{x}_2 &= u \end{aligned}$$

a. Assume that the parameter θ is known. Design a controller that stabilizes the system using the backstepping method.

b. Assume that the parameter θ is unknown, and may be time-varying. Design a controller and a parameter adjustment mechanism such that the resulting system is stable. Use the adaptive backstepping design method.