LUNDS TEKNISKA HÖGSKOLA
Lunds universitet

Institutionen för
REGLERTEKNIK

FRTN15 Predictive Control

Final Exam October 19, 2010, 14-19

General Instructions

This is an open book exam. You may use any book you want. However, no previous exam sheets or solution manuals are allowed. The exam consists of 6 problems to be solved. Your solutions and answers to the problems should be well motivated. The credit for each problem is indicated in the problem. The total number of credits is 25 points. Preliminary grade limits are:

Grade 3:	12 points
Grade 4:	17 points
Grade 5:	22 points

Results

The results of the exam will be posted at the latest October 28 on the notice board on the first floor of the M-building and they will also be available on the course home page.

Do you accept publication of your grading result on our local web page? (Godkänner du publicering av resultatet på vår lokala hemsida?)

1.

a. What is the basic principle of Model Reference Control? What design choices have to be made? (2 p)

b. Consider the process $G(z)$ given by:

$$G(z) = \frac{b_0 z}{z^2 + a_1 z + a_2}$$

Design a Model Reference Controller for the process $G(z)$, which includes integral action, so that the closed-loop system is given by the reference model:

$$G_m(z) = \frac{b_{m0} z + b_{m1}}{z^2 + a_{m1} z + a_{m2}}$$

(2 p)

c. What can be done if the structure of the process $G(z)$ is known, but not its parameters? (1 p)

2. Consider the a process described by:

$$A(z^{-1}) y_k = B(z^{-1}) u_k + e_k$$

where e_k is Gaussian white noise with variance σ^2 and

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} \\ B(z^{-1}) &= b_0 z^{-1} + b_1 z^{-2} \end{aligned}$$

a. Determine the regressor and parameter vector for least-squares identification of the unknown parameters b_0, b_1, a_1 and a_2. Also, derive the normal equations to calculate these parameters. (2 p)

b. How does the algorithm from a) need to be changed to provide new parameter estimates continuously in a real-time setting?
What problem can arise when the parameters are time-variant? How can the algorithm be modified to improve the performance in case of time-varying parameters? (2 p)

3. Consider the 1-step-ahead prediction of a process given as:

$$y_{k+1} = -0.5 y_k + 2.5 y_{k-1} + u_k + 0.8 u_{k-1} + w_{k+1} + 0.1 w_k$$

where w_k is white noise with $E\{w_k w_j^T\} = \sigma_w^2 \cdot \delta_{kj}$.

a. Calculate the 1-step-ahead predictor, its prediction error covariance and the 1-step-ahead minimum-variance controller for this process. (3 p)

b. Show that the controller calculated in a) minimizes the variance of the 1-step-ahead output. (2 p)

4. The following system is to be controlled using Model Predictive Control

$$\begin{aligned} x_{k+1} &= x_k + u_k \\ y_k &= 2x_k \end{aligned}$$

There are constraints on the output and the rate of change of the control signal, $\Delta u_k = u_k - u_{k-1}$ according to

$$\begin{aligned} y^{min} \leq& y_k \leq y^{max} \\ \Delta u^{min} \leq& \Delta u_k \leq \Delta u^{max} \end{aligned}$$

where $y^{min} = -5$, $y^{max} = 5$, $\Delta u^{min} = -2$, $\Delta u^{max} = 2$.

a. Determine what values of u_k that are feasible when $x_k = 1$, and $u_{k-1} = 2$. (2 p)

b. Assume a measurement $y_k = 10$ is received when $u_{k-1} = 0$. Show that there are no feasible control moves Δu_k. (1 p)

c. Suggest a change in how the controller is implemented to avoid the situation in **b**. (1 p)

d. Show how the model can be modified to achieve integral action through the use of a disturbance observer, i.e. by assuming a constant disturbance acting on the input. (1 p)

5. Iterative Learning control for a system described by

$$y_k(t) = G(q)u_k(t)$$

can be implemented as

$$\begin{aligned} e_k(t) &= r_k(t) - y_k(t) \\ u_{k+1}(t) &= u_k(t) + L(q)e_k(t) \end{aligned}$$

a. Give an interpretation of the controller equations. What is the role of $L(q)$? (1 p)

b. For what type of control problems is ILC a suitable strategy? What types of disturbances can be handled? (1 p)

c. What condition on $L(q)$ must hold for the control error to converge? Give a graphical interpretation of this in terms of the Nyquist plot of $L(q)G(q)$. (1 p)

6. A stochastic problem is generated as

$$\begin{aligned} x_{k+1} &= 0.9x_k + v_k \\ y_k &= 0.1x_k + e_k \end{aligned}$$

with uncorrelated white-noise processes e_k and v_k. The covariances of these noise processes are $E\{v_k v_j^T\} = \sigma_v^2 \cdot \delta_{kj}$ and $E\{e_k e_j^T\} = \sigma_e^2 \cdot \delta_{kj}$. Furthermore, x_0 is normally distributed with zero mean and variance σ_0^2.

a. Determine the Kalman Filter for the process. (1 p)

b. Determine the estimation covariance P and the filter gain K in case of a stationary Kalman Filter.
Hint: You do not need to calculate the estimation covariance exactly. It is sufficient to give an equation that the estimation covariance can be calculated from including conditions for an admissible solution. (1 p)

c. What are the estimation covariance P and the filter gain K in steady-state when $\sigma_e^2 >> \sigma_v^2$ is true for the noise-processes? Also, interpret the result. (1 p)