LUND INSTITUTE
OF TECHNOLOGY
Lund University

Department of
AUTOMATIC CONTROL

FRTN15 Predictive Control

Final Exam October 23, 2009, 08-13

General Instructions

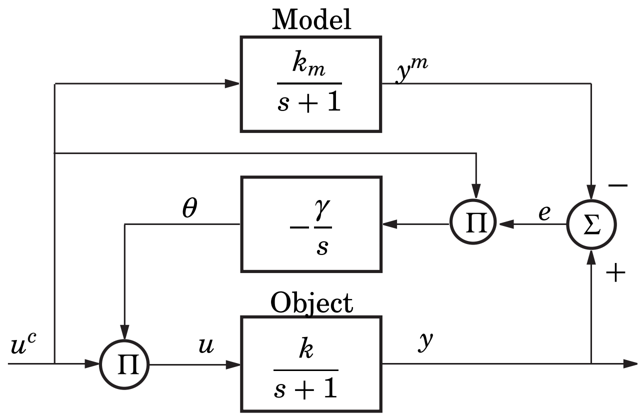
This is an open book exam. You may use any book you want. However, no previous exam sheets or solution manuals are allowed. The exam consists of 6 problems to be solved. Your solutions and answers to the problems should be well motivated. The credit for each problem is indicated in the problem. The total number of credits is 25 points. Preliminary grade limits are:

Grade 3:	12 points
Grade 4:	17 points
Grade 5:	22 points

Results

The results of the exam will be posted at the latest October 26 on the notice board on the first floor of the M-building and they will also be available on the course home page.

Do you accept publication of your grading result on our local web page? (Godkänner du publicering av resultatet på vår lokala hemsida?)



1.

Consider the gain adaptation problem of Fig. 1 for $k > 0$

$$u = \theta u^c$$

Introduce the gain parameter

$$\theta = \frac{k_m}{k}$$

and the output error

$$e = y - y^m = G(s)k\theta u^c - k_m G(s)u^c, \qquad G(s) = \frac{1}{s+1}$$

with u^c as command signal, y^m the reference model output, y system output, θ the gain parameter.

$$\frac{d\theta}{dt} = -\gamma u^c e$$

Show that the gain adaptation is stable in the sense of Lyapunov for $\gamma > 0$. (2 p)

2. A process is modeled by

$$y(k) = b_0 u(k) + b_1 u(k-1) + e(k),$$

where $e(k)$ is a normally distributed white noise process.

a. Derive a least-squares estimator for the process. (2 p)

b. Derive expressions for the estimation error and estimation error covariance. (2 p)

c. Present an input sequence $u(k)$ resulting in a consistent estimator. Prove your claim. (1 p)

3. This problem deals with MRAC design of an STR, similar to that of homework assignment 2. The sampled process and reference models are given by

$$G = \frac{B}{A}, \; G_m = \frac{B_m}{A_m},$$

where $\deg A = \deg A_m = 2$ and $\deg B = \deg B_m = 1$. Also, A, A_m are chosen monic. The controller structure is given by the ARMAX controller

$$Ru = -Sy + Tu^c,$$

where u^c, u and y are reference, control signal and system output, respectively.

a. Assume that the zero of B is poorly damped. Mention a negative consequency of canceling it by a controller pole. Show why it is not possible to avoid this cancellation for an arbitrary choice of B_m. (2 p)

b. Let $B = B_m$ and show that it is generally impossible to find a controller without zero cancellation where $\deg R = 0$. (2 p)

c. Describe how the controller structure can be modified in order to introduce integral action and how this affects the minimal degree solution. (2 p)

d. What is the difference between direct and indirect MRAC? (1 p)

4. The dynamics of a plant are described by

$$\begin{cases} x_{k+1} &= \Phi x_k + \Gamma u_k + d_k \\ y_k &= C x_k, \end{cases}$$

with $\Phi = \frac{1}{2}$, $\Gamma = 1$ and $C = 2$. The disturbance d_k is constant $d_k = 3$.

a. Extend the state to include the disturbance state $d_k = d$ and give the extended dynamics. (2 p)

b. Explain (briefly) why a state observer is needed to use this model for control synthesis, assuming d is unknown and not directly measurable. (1 p)

c. Give the questions for a one-step-ahead linear state estimator, which placed all poles of the error dynamics at $-\frac{1}{2}$. (2 p)

5. Model Predictive Control (MPC) is based on the *receding horizon* principle, illustrated in Fig. 1. The aim is to decide a number of future input values given a prediction of a finite number of future outputs. The first input value is implemented, and the procedure is repeated at the next sampling instance.

The controller is obtained by minimizing a cost function:

$$V(U_t, Y_t) = Y_t^T Q_y Y_t + U_t^T Q_u U_t \tag{1}$$

where U_t and Y_t are sequences of future control signals and outputs up to horizons N and M respectively:

$$U_t = \begin{bmatrix} u(t+N-1) \\ \vdots \\ u(t) \end{bmatrix}, \qquad Y_t = \begin{bmatrix} \hat{y}(t+M|t) \\ \vdots \\ \hat{y}(t+1|t) \end{bmatrix}$$

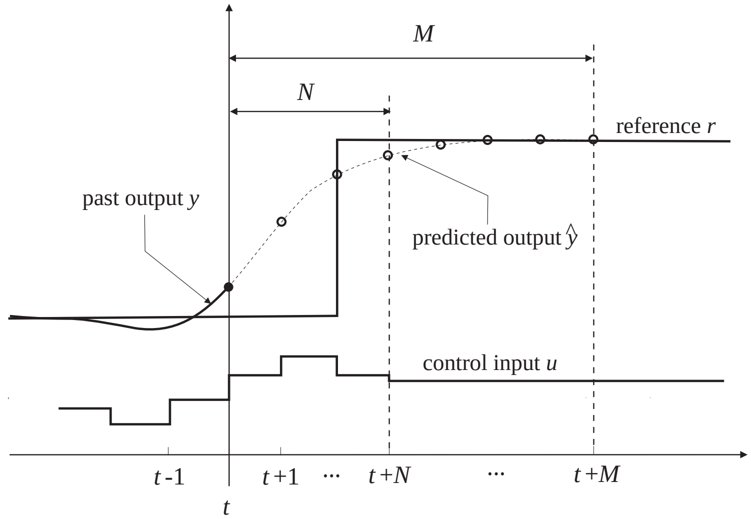


Figure 1 Illustration of the receding horizon principle used in Model Predictive Control

When the system is known, the predicted future outputs are given by the predictor:

$$\begin{bmatrix} \widehat{y}(t+M|t) \\ \vdots \\ \widehat{y}(t+1|t) \end{bmatrix} = \begin{bmatrix} CA^M \\ \vdots \\ CA \end{bmatrix} \widehat{x}(t|t) + \begin{bmatrix} CB & CAB & CA^2B & \dots \\ 0 & CB & CAB & \dots \\ \vdots & \ddots & \ddots & \vdots \end{bmatrix} \begin{bmatrix} u(t+M-1) \\ \vdots \\ u(t+N-1) \\ \vdots \\ u(t) \end{bmatrix}$$

$$Y_t = D_x \widehat{x}(t|t) + D_u U_t$$

a. Show that the cost function (1) can be written as:

$$V(U_t) = \widehat{x}(t|t)^T Q \widehat{x}(t|t) + U_t^T R U_t + 2\widehat{x}(t|t)^T S U_t$$

and that the minimum is attained for:

$$U_t = -R^{-1} S \widehat{x}(t|t)$$

(2 p)

b. The MPC formulation described here assumes that a process model is available. Can you suggest a way of modifying the algorithm to create an adaptive MPC controller? (Hint: Consider the sequence of predicted outputs Y_t, as well as the way in which process parameters are identified in the least-squares algorithm) (1 p)

6. One possible strategy for Iterative Learning Control (ILC) is given by the equations:

$$\begin{aligned} y_k(t) &= G_c(q) u_k(t) \\ e_k(t) &= r(t) - y_k(t) \\ u_k(t) &= Q(q)[u_{k-1}(t) + L(q) e_{k-1}(t)] \end{aligned}$$

where $G_c(q)$ is the closed-loop transfer function of the system and q is the forward time shift operator.

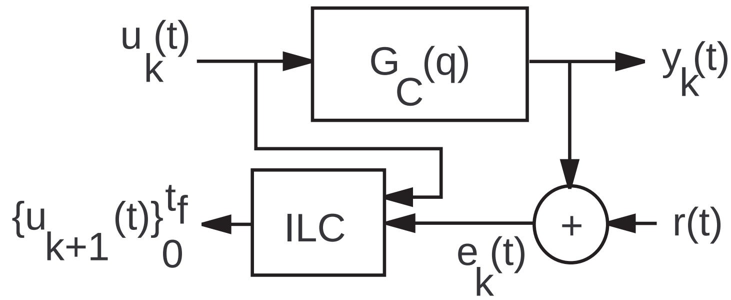


Figure 2 ILC set-up in Problem 6

a. Explain the principle of operation of Iterative Learning Control. (1 p)

b. Assume that $Q(q) = 1$ and that

$$G_C(q) = \frac{1}{(q-0.7)(q-0.9)}, \qquad L(q) = k(q-0.5)(q-0.7)(q-0.9)$$

where k is a positive constant. Does there exist $k > 0$ for which the ILC scheme converges? Motivate your answer. (2 p)