LUNDS TEKNISKA HÖGSKOLA
Lunds universitet

Institutionen för
REGLERTEKNIK

FRTN15 Predictive Control

Final Exam October 25, 2008, 8-13

General Instructions

This is an open book exam. You may use any book you want. However, no previous exam sheets or solution manuals are allowed. The exam consists of 6 problems to be solved. Your solutions and answers to the problems should be well motivated. The credit for each problem is indicated in the problem. The total number of credits is 25 points. Preliminary grade limits are:

Grade 3:	12 points
Grade 4:	17 points
Grade 5:	22 points

Results

The results of the exam will be posted at the latest November 1 on the notice board on the first floor of the M-building and they will also be available on the course home page.

Do you accept publication of your grading result on our local web page? (Godkänner du publicering av resultatet på vår lokala hemsida?)

1. The following system is to be controlled using Model Predictive Control.

$$
\begin{aligned}
x(k+1) &= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(k) \\
y(k) &= \begin{pmatrix} 1 & 0 \end{pmatrix} x(k)
\end{aligned}
$$

where $y(k)$ is available for measurement. The controller should fulfill $y(k) = r(k)$, where $r(k)$ is a reference signal, and respect the constraints

$$
\begin{aligned}
x_{min} \leq \; & x_2(k) \; \leq x_{max} \\
u_{min} \leq \; & u(k) \; \leq u_{max}
\end{aligned}
$$

a. Determine the additional output signals z and z_c which correspond to controlled and constrained outputs respectively, i.e. determine C_z and C_c so that the system can be written on the following form

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) \\
y(k) &= Cx(k) \\
z(k) &= C_z x(k) \\
z_c(k) &= C_c x(k)
\end{aligned}
$$

(1 p)

b. For the numerical values $u_{min} = x_{min} = -2$ and $u_{max} = x_{max} = 2$, and given that $x(k) = (0 \quad 1)^T$ and $u(k-1) = 0.5$, what restrictions do we have on $\Delta u(k) = u(k) - u(k-1)$? (2 p)

c. What computational problems can arise if the plant is operated near the constraint on x_2 and noise disturbances are affecting the states? How can the MPC problem be modified to limit these problems? (1 p)

d. Assume that a constant disturbance is acting on the process input. How can the state space model be extended to include this? (1 p)

2. We want to estimate the parameters a and b using input-output data for a system with transfer function $H(z)$ given by

$$
H(z) = \frac{b}{z+a}
$$

The parameters are likely to be time-varying, so Recursive Least Squares (RLS) estimation with forgetting is employed.

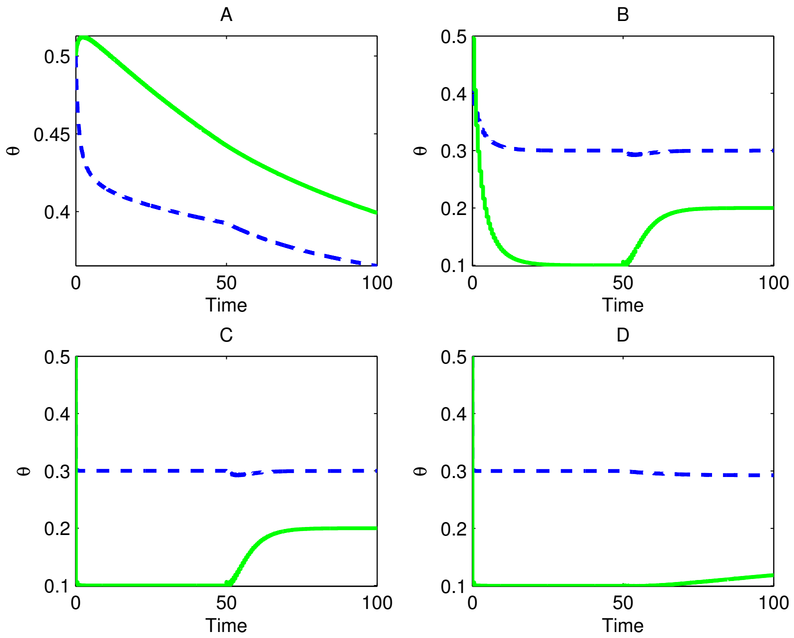
a. State the equations for the RLS-algorithm with forgetting and explain its operation. Also, write the model on regressor form. (2 p)

b. What problems might arise when a small value of λ is used? (1 p)

c. Figure 1 shows the results when using the four configurations i-iv of λ and κ, where $P_0 = \kappa \cdot I$.

$$
\begin{aligned}
\text{i}: \quad & \lambda = 0.998, \quad && \kappa = 10^3 \\
\text{ii}: \quad & \lambda = 1, && \kappa = 10^3 \\
\text{iii}: \quad & \lambda = 0.998, && \kappa = 1 \\
\text{iv}: \quad & \lambda = 1, && \kappa = 10^{-2}
\end{aligned}
$$

The initial estimates were 0.5 for both parameters. The correct values are $a = 0.1$ and $b = 0.3$. At $t = 50$, a changes to 0.2. Determine which result A-D that corresponds to which configuration i-iv. Motivate your answer. (2 p)



Figur 1 Responses to the parameter choices i-iv in Problem 2.

d. Assume that some input-output data are available off-line. Explain how this can be used to improve the initial behaviour of the on-line algorithm. (1 p)

3.

a. Explain the basic principles in Model Reference Adaptive Control (MRAC). What is the difference between *direct* and *indirect* MRAC? What design choices are needed? What information about the plant is needed? (2 p)

b. Consider the system $H(q)$ given by

$$H(q) = \frac{b_0 q + b_1}{q^2 + a_1 q + a_2}$$

where b_0 and b_1 are such that the process zero is stable. Show how a direct MRAC on the form

$$R(q)u(t) = T(q)u_c(t) - S(q)y(t)$$

is obtained so that the closed-loop transfer function from u_c to y is given by

$$H_m(q) = \frac{b_{m1} q + b_{m2}}{q^2 + a_{m1} q + a_{m2}}$$

What are the orders of the polynomials $R(q)$, $S(q)$, and $T(q)$? Assume that the process zero is cancelled. (2 p)

4. Consider the following system

$$
\begin{aligned}
x_{k+1} &= ax_k + v_k \\
y_k &= x_k + e_k
\end{aligned}
$$

where v and e are white-noise processes with zero mean and the covariances

$$
\begin{aligned}
\mathbf{E}\{v_j v_k\} &= r_1 \delta_{jk} \\
\mathbf{E}\{v_j e_k\} &= 0 \\
\mathbf{E}\{e_j e_k\} &= r_2 \delta_{jk}
\end{aligned}
$$

a. Determine the Kalman filter for the system. (1 p)

b. What are the steady-state filter gain and the estimation covariance? (1 p)

c. Determine the filter gain and estimation covariance in steady-state when $r_1 >> r_2$. Comment on the result. (1 p)

5.

a. Calculate a one-step-ahead optimal predictor for the system

$$y_{k+1} = 0.8y_k - 0.5y_{k-1} + w_{k+1} + 0.4w_k$$

where w is a stochastic process with zero mean and variance $\mathbf{E}\{w_k w_j^T\} = \sigma_w^2 \delta_{kj}$. Also determine the prediction covariance. (2 p)

b. Determine a one-step-ahead minimum-variance controller for the system

$$y_{k+1} = 0.8y_k - 0.5y_{k-1} + u_k + 0.6u_{k-1} + w_{k+1} + 0.4w_k$$

Also determine the output variance under closed-loop minimum variance control. (2 p)

6. An Iterative Learning Control (ILC) strategy for the system

$$y_k(t) = G(q)u_k(t)$$

is given by

$$
\begin{aligned}
e_k(t) &= r(t) - y_k(t) \\
u_{k+1}(t) &= u_k(t) + L(q)e_k(t)
\end{aligned}
$$

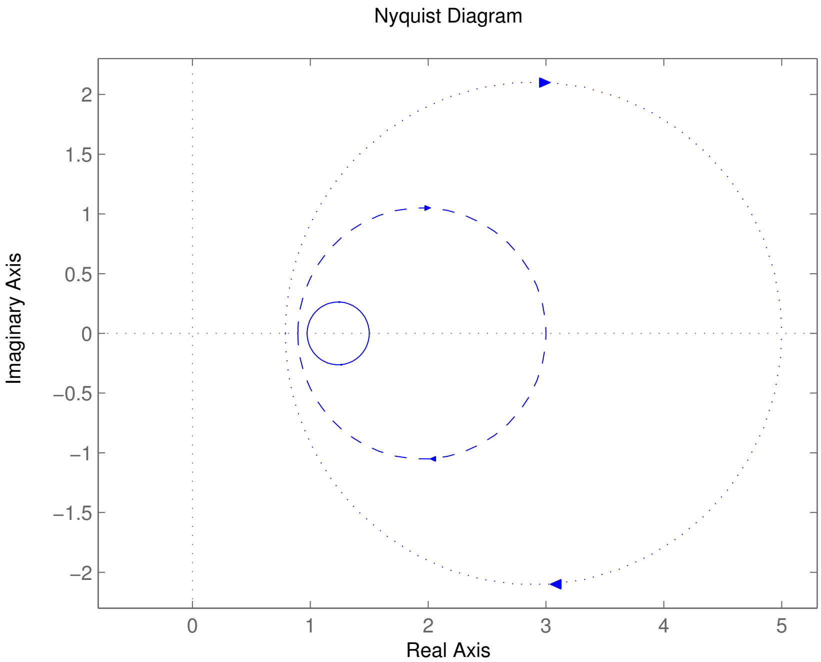
a. Explain the principle of ILC and a draw a block diagram of the system. (1 p)

b. Give two examples of practical situations where ILC would be a suitable control strategy. (1 p)

c. Figure 2 shows the nyquist plot for the three choices of $L(q)$:

$$
\begin{aligned}
L(q) &= q + 0.5 \quad \textit{(dotted)} \\
L(q) &= q + 0.7 \quad \textit{(dashed)} \\
L(q) &= q + 0.85 \quad \textit{(solid)}
\end{aligned}
$$

For which choice of $L(q)$ will the control error converge? (1 p)



Figur 2 Nyquist plots for G(q)L(q) in Problem 6.