LUNDS TEKNISKA HÖGSKOLA
Lunds universitet

Institutionen för
REGLERTEKNIK

FRTN15 Predictive Control

Final Exam October 16, 2007, 14-19

General Instructions

This is an open book exam. You may use any book you want. However, no previous exam sheets or solution manuals are allowed. The exam consists of 6 problems to be solved. Your solutions and answers to the problems should be well motivated. The credit for each problem is indicated in the problem. The total number of credits is 25 points. Preliminary grade limits are:

Grade 3:	12 points
Grade 4:	17 points
Grade 5:	22 points

Results

The results of the exam will be posted at the latest October 26 on the notice board on the first floor of the M-building and they will also be available on the course home page.

Do you accept publication of your grading result on our local web page? (Godkänner du publicering av resultatet på vår lokala hemsida?)

1. Consider the system:

$$y_k = -a_1 y_{k-1} - a_2 y_{k-2} + b_1 u_{k-1} + b_2 u_{k-1} + e_k \qquad (1)$$

where e_k is white noise with unit variance, and a_i, b_i are unkown parameters.

a. Assume that a sequence of N input-output data points has been collected for the system (1). Describe a method for producing an estimate $\hat{\theta}$ of the unkown parameters. (1 p)

b. It is desired to obtain real time estimates of the parameters, $\hat{\theta}_k$, while the system is running. Outline an algorithm capable of providing such estimates. What information is needed to initialize this algorithm? (2 p)

c. It has been discovered that the unkown parameters are not constant but instead are slowly time-varying. Describe how to modify the estimation algorithm to take into account this parameter variation, and provide better estimates. (1 p)

2. Model Predictive Control (MPC) is based on the *receding horizon* principle.

a. Explain, with the help of a diagram, the operation of the receding horizon principal. Clearly explain the terms *prediction horizon*, *control horizon* and *predicted output*. (1 p)

b. An important feature of MPC is its ability to handle constraints on inputs, states and outputs. In order to do this, however, these constraints must be expressed in terms of the optimization variables, which are the future control moves Δu_k.

Consider the state-space system:

$$\begin{aligned} x_{k+1} &= -0.5x_k + 2u_k \\ y_k &= 5x_k \end{aligned}$$

with the following constraint on the output:

$$-10 \leq y_k \leq 10, \quad \forall\ k$$

If the current state $x_k = 1$ and the previous control input $u_{k-1} = -1$, show that the corresponding constraint on the control signal change Δu_k is given by:

$$-0.25 \leq \Delta u_k \leq 1.75$$

(2 p)

c. The state space system above is disturbed by a constant but unknown disturbance d_k entering the plant at the input. Explain how the state space model can be augmented to include a representation of this disturbance. Given that an MPC controller has been designed for the augmented model, what additional system component will be required for the controller to work? (2 p)

3. A DC servo motor can be described by the discrete-time transfer function:

$$G_p(q) = \frac{b_o q + b_1}{q^2 + a_1 q + a_2}$$

To control the motor, a two degree of freedom controller will be used:

$$R(q)u(t) = -S(q)y(t) + T(q)u_c(t)$$

a. Determine the orders of the polynomials $R(q)$, $S(q)$ and $T(q)$, and the degee γ of the observer polynomial $(q + a_o)^\gamma$ such that the controller has integral action and the transfer function from u_c to y is:

$$G_m(q) = \frac{b_{m_1} q + b_{m_2}}{q^2 + a_{m_1} q + a_{m_2}}$$

Assume that the process zero is cancelled. (2 p)

b. Briefly describe how to modify the design to obtain a Direct Self Tuning Regulator according the the specifications in Part a. (2 p)

4. Consider an autoregressive moving-average (ARMA) model:

$$Y(z) = \frac{C(z)}{A(z)} W(z) = \frac{C^*(z^{-1})}{A^*(z^{-1})} W(z)$$

where the input $W(z)$ is white noise with zero mean and variance σ^2, and with:

$$\begin{cases} A(z) = z - 0.9 \\ C(z) = z + 0.7 \end{cases} \Leftrightarrow \begin{cases} A^*(z^{-1}) = 1 - 0.9z^{-1} \\ C^*(z^{-1}) = 1 + 0.7z^{-1} \end{cases}$$

a. Calculate the minimum variance two-step-ahead predictor. What is its prediction variance? (2 p)

b. Now assume that the model above describes 'coloured noise' corrupting another system:

$$Y(z) = \frac{B(z)}{A(z)} U(z) + \frac{C(z)}{A(z)} W(z)$$

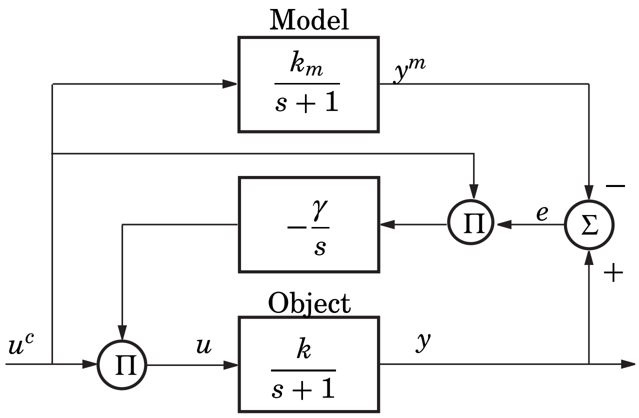
where $A(z)$ and $C(z)$ are the same as before and $B(z) = z + 0.5$. What is the two-step ahead minimum variance controller for this system? (*Hint: Consider the Diophantine equation* $C = AF + z^{-2}G$) (2 p)

5. Consider the gain adaptation problem of Fig. 2 for $k > 0$

$$u = \theta u^c$$

Introduce the gain parameter

$$\theta = \frac{k_m}{k}$$



Figur 1 Gain adaptation

and the output error

$$e = y - y^m = G(s)k\theta u^c - k_m G(s)u^c, \qquad G(s) = \frac{1}{s+1}$$

with u^c as command signal, y^m the reference model output, y system output, θ the gain parameter.

$$\frac{d\theta}{dt} = -\gamma u^c e$$

Show that the gain adaptation is stable in the sense of Lyapunov for $\gamma > 0$. (3 p)

6. One possible strategy for Iterative Learning Control (ILC) is given by the equations:

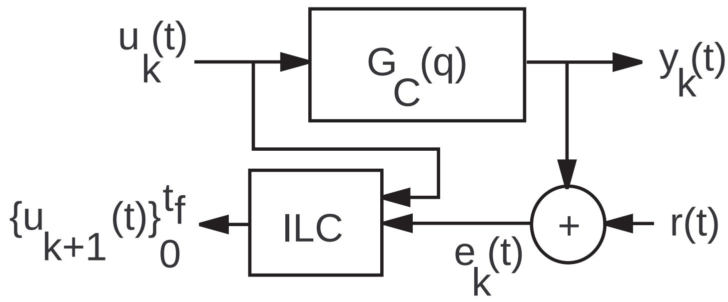
$$\begin{aligned} y_k(t) &= G_c(q)u_k(t) \\ e_k(t) &= r(t) - y_k(t) \\ u_k(t) &= Q(q)[u_{k-1}(t) + L(q)e_{k-1}(t)] \end{aligned}$$

where $G_c(q)$ is the closed-loop transfer function of the system and q is the forward time shift operator.

a. Explain the principle of operation of Iterative Learning Control. (1 p)

b. For each of the following scenarios, state whether or not you think ILC will give an improvement in performance:

1. A robot machining Formula 1 engine components to very high tolerances
2. Trajectory optimization for fluid-filled containers on an assembly line
3. Tracking improvement for an automated machine tool operation with outputs corrupted by Gaussian noise
4. Trajectory optimization for landing an Unmanned Aerial Vehicle (UAV)



Figur 2 ILC set-up in Problem 6

Clearly state your reasoning for each case. (2 p)

c. Assume that $Q(q) = 1$ and that

$$G_C(q) = \frac{1}{(q-0.7)(q-0.9)}, \qquad L(q) = k(q-0.5)(q-0.7)(q-0.9)$$

where k is a positive constant. Determine a condition on k which, if fulfilled, guarantees that the error of the resulting ILC scheme converges. (2 p)