LUND UNIVERSITY

Department of
AUTOMATIC CONTROL

FRTN10 Multivariable Control

Exam 2015-10-29, 14.00–19.00

Points and grades

All answers must include a clear motivation and a well-formulated answer. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each subproblem.

Accepted aid

The textbook Glad & Ljung: "Reglerteori – Flervariabla och olinjära metoder"/"Control Theory—Multivariable and Nonlinear Methods", standard mathematical tables like TEFYMA, an authorized "Reglerteknik Formelsamling"/"Automatic Control Collection of Formulae" and a pocket calculator. Handouts of lecture notes and lecture slides from the course are also allowed.

Results

The results will be reported via LADOK.

1 a. Consider the setup in Figure 1 where the system $P(s) = 1/(s+1)$ is controlled with a P-controller with gain K, $K > 0$. Given the uncertainty Δ, $||\Delta|| \leq 2$, for what values of K is the closed loop guaranteed to be stable? (2 p)

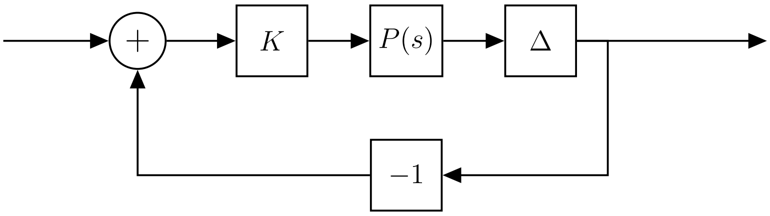

Figure 1: Block diagram for problem 1.

b. Assume that the uncertainty is actually a constant gain $\Delta = 2$. For what values of K will the closed loop system be stable? Explain why the answer is different from subproblem a. (1 p)

2. A zero-mean Gaussian process u with spectrum

$$\Phi_u(\omega) = \frac{1}{\omega^2 + 1}$$

is filtered through a stable linear system $G(s)$, yielding an output signal y with spectrum

$$\Phi_y(\omega) = \frac{2(\omega^2 + 100)}{(\omega^2 + 25)(\omega^2 + 1)}$$

What is the linear system $G(s)$? (2 p)

3. Consider the model of a double integrator

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x$$

which is to be controlled using the control law $u = -Lx + l_r r$. L is found using LQ design with the cost function

$$J = \int_0^\infty \left(x^T Q_1 x + u^T Q_2 u \right) \, dt$$

while l_r is calculated so that the closed loop system has unit static gain. Shown in Figure 2 are the step responses and closed loop poles of three different design cases:

Case 1	**Case 2**	**Case 3**
$Q_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$Q_1 = \begin{pmatrix} 10 & 0 \\ 0 & 0.1 \end{pmatrix}$	$Q_1 = \begin{pmatrix} 0.1 & 0 \\ 0 & 1 \end{pmatrix}$
$Q_2 = 1$	$Q_2 = 1$	$Q_2 = 0.1$

Find the correct pairing of design cases 1–3, step response A–C and closed loop poles I–III. Do not forget to give a clear motivation.

(4 p)

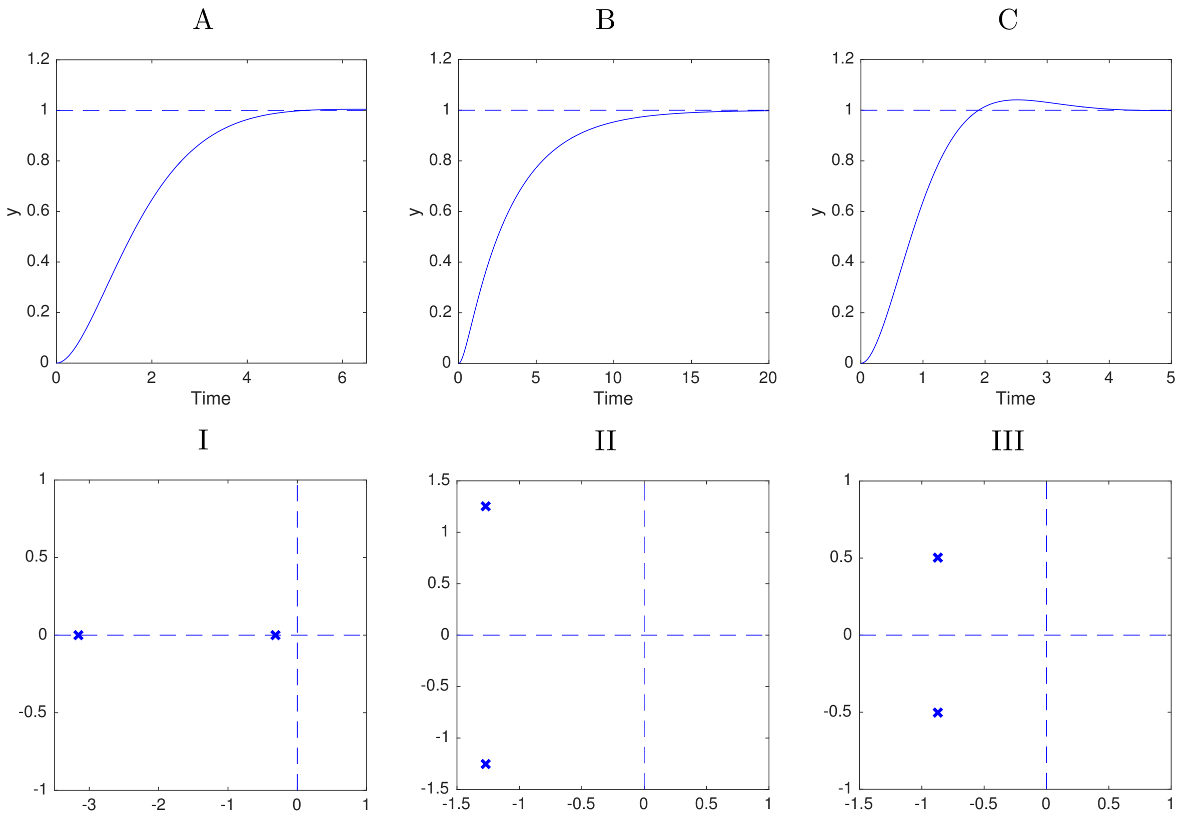

Figure 2: Plots of step responses and pole locations in problem 3

4 a. A two-input, one-output system is to be controlled using a decentralized scheme where one input is to be paired with the output. There is no available system model so an experimental procedure has been used to obtain the RGA in stationarity. The following static gains k_i were computed from experiment data:

- k_1 – The gain from u_1 to y in open loop.
- k_2 – The gain from u_2 to y in open loop.
- k_3 – The gain from u_1 to y when y is controlled with u_2 in closed loop.
- k_4 – The gain from u_2 to y when y is controlled with u_1 in closed loop.

Give an expression for the experimental RGA(0) using the gains k_i. (1.5 p)

b. Assume that the found RGA(0) is given by

$$\begin{pmatrix} 0.8 & 0.2 \end{pmatrix}$$

Which control signal should be used to control y for good control performance at low frequencies? (0.5 p)

5. The system P

$$P(s) = \begin{pmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{pmatrix}$$

in Figure 3 is to be decoupled by the use of a static decoupler D,

$$D = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$$

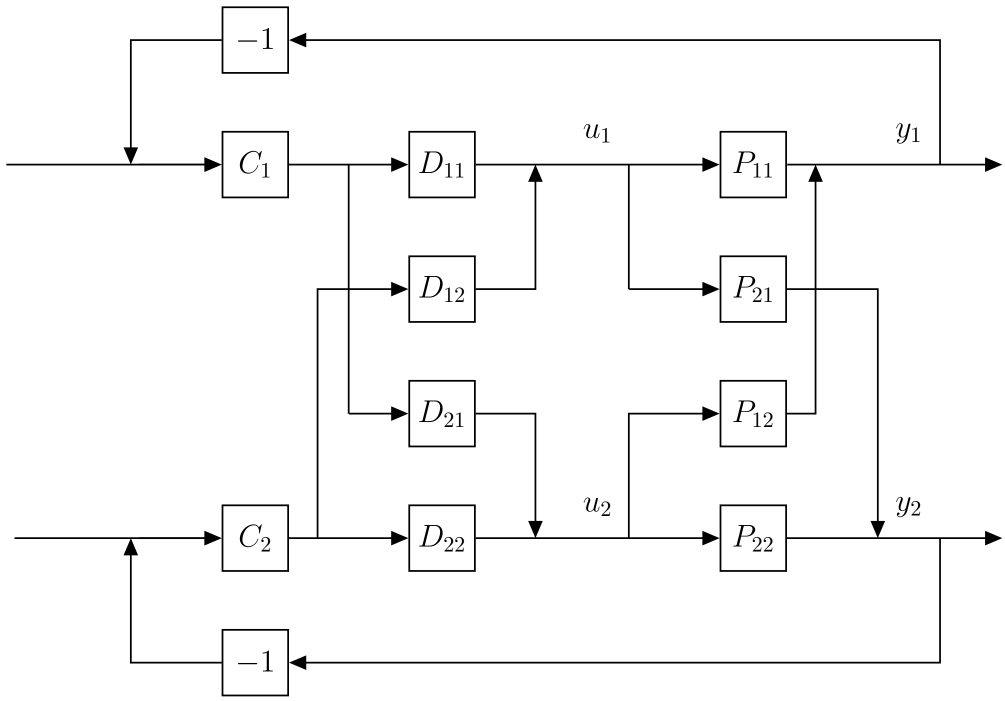

Figure 3: Block diagram of problem 5.

and the diagonal controller C,

$$C = \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix}.$$

Find an expression for D as a function of the elements in $P(0)$ which decouples the system in stationarity. (2 p)

6. Which of the following pairs of systems and closed-loop specifications are possible to achieve? Do not forget to give a clear motivation.

a. The system

$$G_1 = \frac{s-8}{(s-2)(s+3)}$$

with a closed-loop speed of 10 rad/s and reasonable robustness. (1 p)

b. The system

$$G_2 = \frac{s+8}{(s-2)(s+3)}e^{-0.1s}$$

with a closed-loop speed of 5 rad/s and reasonable robustness. (1 p)

c. The system

$$G_3 = \frac{s-50}{(s-3)(s+3)}$$

with the specification that $\|T(i\omega)W_T(i\omega)\|_\infty < 1$, for $W_T(s) = (s+2)/2$. (1 p)

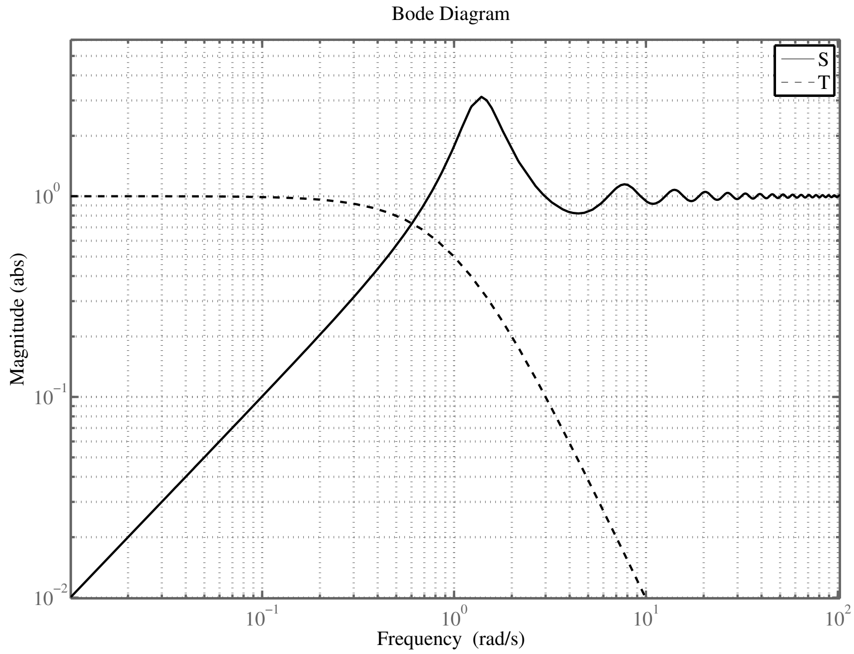

Figure 4: S and T in problem 6 d.

d. The system

$$G_4 = \frac{1}{(s+2)(s+5)}$$

with the specified S and T presented in Figure 4. (1 p)

7. Consider the following system

$$G(s) = \begin{pmatrix} \frac{1}{s+2} & \frac{2}{(s+2)(s+3)} \end{pmatrix}$$

a. What are the (multivariable) poles and zeros of the system? (1 p)

b. Write the system in state-space form with as few states as possible. (1 p)

c. A student suggested the following state-space realization. Does it correspond to the same transfer function? (1 p)

$$\dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -2 & 2 \\ 0 & 0 & -3 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} x$$

8. Consider the system

$$\dot{x} = \begin{pmatrix} -1 & -4 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 2 \\ 0 & 6 \end{pmatrix} x$$

a. Verify that the controllability and observability gramians are given by

$$S = \begin{pmatrix} 2.5 & 0 \\ 0 & 0.5 \end{pmatrix} \quad O = \begin{pmatrix} 0.5 & 0 \\ 0 & 20 \end{pmatrix}$$

(2 p)

b. Determine the Hankel singular values. (1 p)

c. Find a balanced realization of the system. (2 p)