LUND INSTITUTE
OF TECHNOLOGY
Lund University

Department of
AUTOMATIC CONTROL

Multivariable Control Exam

Exam 2014-01-08

Grading

All answers must include a clear motivation and a well-formulated answer. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each subproblem.

Accepted aid

The textbook *Glad & Ljung*, standard mathematical tables like TEFYMA, an authorized "Formelsamling i Reglerteknik"/"Collection of Formulas" and a pocket calculator. Handouts of lecture notes and lecture slides are also allowed.

Results

The results will be reported via LADOK.

1. Consider the feedback system in Figure 1.

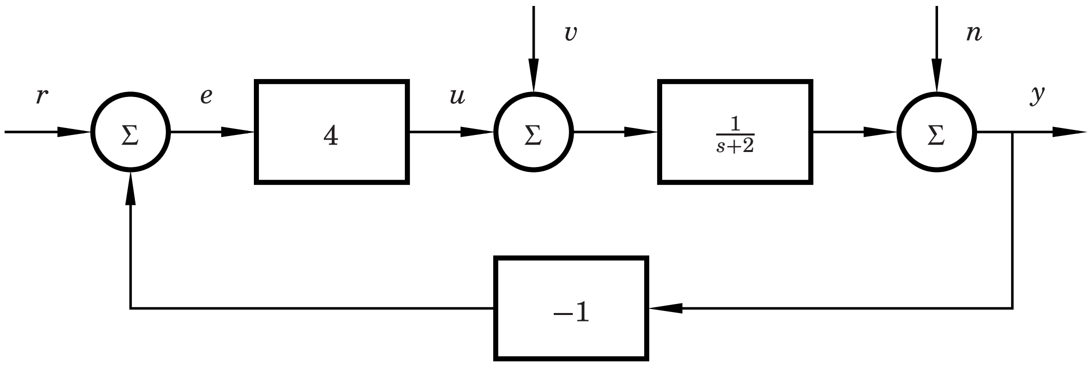

Figure 1 The system in Problem 1.

a. Give a state space realization of the system with inputs r, v and n and outputs e, u and y. (1 p)

b. Determine the entries of the transfer matrix

$$\begin{pmatrix} P_{er}(s) & P_{ev}(s) & P_{en}(s) \\ P_{ur}(s) & P_{uv}(s) & P_{un}(s) \\ P_{yr}(s) & P_{yv}(s) & P_{yn}(s) \end{pmatrix}$$

mapping the inputs r, v and n to the outputs e, u and y. (1 p)

c. After permutation (re-ordering) of the inputs and outputs, the step responses of the system have been plotted in Figure 2. Determine the new order of inputs and outputs. (1 p)

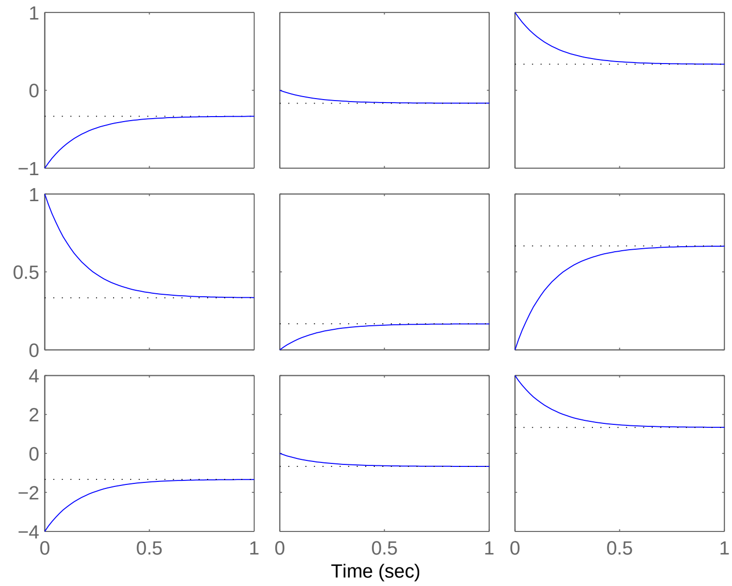

Figure 2 Step response plots for Problem 1. The inputs and outputs have been re-ordered.

d. Suppose that the load disturbance v and the measurement disturbance n are both constant and zero, while the reference signal r is a zero-mean stochastic process with spectral density $\Phi_r(\omega) = \frac{2}{\omega^2+100}$. Determine the spectral density of e. (2 p)

2. Your boss has heard that you are great at automatic control and wants some help finding a good controller for the process $P(s) = \frac{1}{1+s}\mathrm{e}^{-0.5s}$.

a. He wants a fast system; the bandwidth ω_b should be at least 10 rad/s. If it is possible, find a stabilizing controller that meets the specification. If it is not possible, explain why. (1 p)

b. Your boss gets a little impatient and tries to find a controller himself. He claims that he will get a fast enough system with the controller

$$C(s) = \frac{50 + 10s}{s} e^{0.5s}.$$

He shows you the margin plot of the open loop system PC where everything looks nice, see Figure 3. You do however see a very big problem with this design. What is the problem that you need to explain to your boss? (1 p)

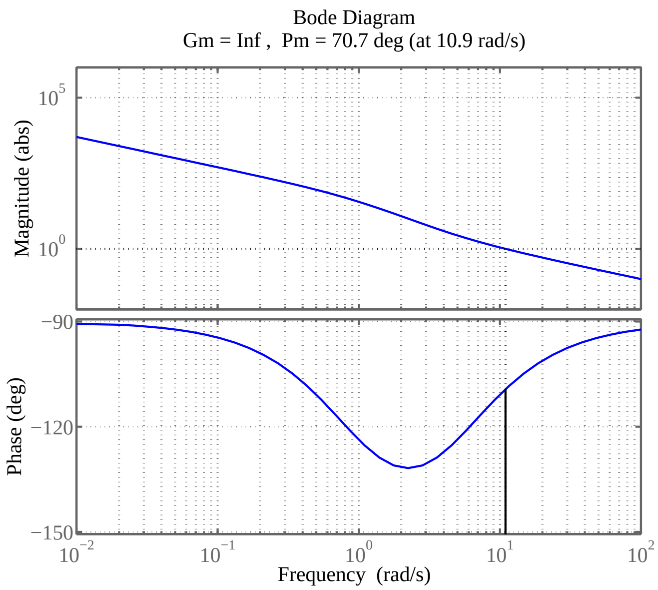

Figure 3 Margin plot for the open loop system in problem 2b.

3. Consider the double integrator

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,$$
$$z = \begin{bmatrix} 1 & 2 \end{bmatrix} x.$$

a. Find the LQR controller that minimizes

$$\int_0^\infty \left(z^T(t)z(t) + u^T(t)u(t) \right) \mathrm{d}t.$$

(2.5 p)

b. Determine the stationary value $u^*(r)$ of the optimal input u when the reference value for z is r. (0.5 p)

c. Find the LQR controller that minimizes

$$\int_0^\infty \left((z(t) - r)^2 + (u(t) - u^*(r))^2\right) \mathrm{d}t.$$

(1 p)

4.

a. Consider control of the process

$$P(s) = \frac{1}{s+1}.$$

If a proportional controller

$$C(s) = K$$

is used, for what values of K will the closed-loop system in Figure 4 be stable? (1 p)

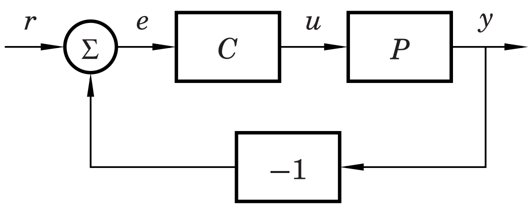

Figure 4 Block diagram of the closed-loop system in Problem 4.

b. Consider the situation where C instead is an unknown time-varying function $K(t)$, with an upper and lower limit of $\pm\alpha$. That is, for all times

$$-\alpha \leq K(t) \leq \alpha.$$

For what positive values of α can stability of the closed-loop system be guaranteed? *Hint: Use the small-gain theorem!* (3 p)

5. Consider a system $\mathcal{S}_1$ described by

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u,$$
$$y = \begin{bmatrix} 0 & 2 \end{bmatrix} x.$$

a. Compute the Hankel singular values. (2 p)

b. Balanced truncation is performed to yield a first-order approximation $\mathcal{S}_2$ of the original system $\mathcal{S}_1$. Provide an upper bound on $||\mathcal{S}_1 - \mathcal{S}_2||$. Note that you do not need to calculate $\mathcal{S}_1$. (1 p)

6. Consider a system described by the following transfer function matrix

$$G(s) = \begin{bmatrix} \frac{1}{1+s} & \frac{-1}{s+1} \\ \frac{2s}{s+2} & \frac{2s}{s+2} \end{bmatrix}$$

Suggest a suitable transformation for doing decoupled control and provide the overall structure of the controller. (You do not have to design any specific controller for the decoupled systems) (2 p)

7.

a. Design an internal model controller for the system

$$P(s) = \begin{pmatrix} \dfrac{s-1}{s+2} & 0 \\ 0 & \dfrac{1}{s+4} \end{pmatrix}.$$

If you have the possibility to decide the crossover frequency ω_c for the open loop of any of the subsystems, choose your design parameters such that $\omega_c = 2$ rad/s. (3 p)

b. Internal model controllers can also be designed by solving a convex optimization problem of the form

$$\min_Q ||T_1 + T_2 Q||.$$

Here T_1 and T_2 can be chosen so that

$$T_1 + T_2 Q = \begin{pmatrix} W_S S \\ W_T T \end{pmatrix},$$

where $S = (I + PC)^{-1}$ is the sensitivity function and $T = PC(I + PC)^{-1}$ is the complementary sensitivity function and W_S as well as W_T are weighting functions. After finding the optimal Q, the controller can be obtained as $C = (I - QP)^{-1}Q$. Find expressions for T_1 and T_2 in terms of a general process P as well as general weighting functions W_S and W_T.

Hint: Use $S + T = I$! (2 p)