LUNDS TEKNISKA HÖGSKOLA
Lunds universitet

Institutionen för
REGLERTEKNIK

FRTN10 Multivariable Control

Exam 2013-04-03, hours: 14.00-19.00

Points and grades

All answers must include a clear motivation and a well-formulated answer. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each subproblem.

Preliminary grades

3 12-16 points

4 17-21 points

5 22-25 points

Accepted aid

The textbook *Glad & Ljung*, standard mathematical tables like TEFYMA, an authorized "Formelsamling i reglerteknik"/"Collection of Formulas" and a pocket calculator. Handouts of lecture notes and lecture slides are also allowed.

Results

The result of the exam will be posted on the notice-board at the Department. The result will also be posted on the course web page.

1. Consider a system with transfer matrix

$$G(s) = \begin{pmatrix} \dfrac{2.4}{s+1} & -\dfrac{16}{s+5} \\ \dfrac{1.8}{s+1} & -\dfrac{12}{2s+5} \end{pmatrix}$$

a. Where in the complex plane are the poles and zeros located? (2 p)

b. Verify that $G(0)$ has a singular value decomposition $U\Sigma V^T$ with

$$U = \begin{pmatrix} -0.8 & -0.6 \\ -0.6 & 0.8 \end{pmatrix} \qquad V = \begin{pmatrix} -0.6 & -0.8 \\ 0.8 & -0.6 \end{pmatrix}$$

What are the singular values of $G(0)$? (1 p)

c. For constant input- and output-vectors $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ and $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ satisfying

$$y = G(0)u$$

Find u with $|u| = \sqrt{u_1^2 + u_2^2} = 1$ and maximal output norm $|y|$. Moreover, find u with $|u| = 1$ and minimal output norm $|y|$. (1 p)

d. Find a state space realization corresponding to $G(s)$. (2 p)

2. Consider the 2-input-2-output system

$$G(s) = \begin{pmatrix} \dfrac{1}{4s+1}e^{-1.1s} & \dfrac{4}{s+2} \\ \dfrac{3}{s^2+10s+1} & \dfrac{1}{s+1} \end{pmatrix}$$

a. Determine, if possible, a dynamic decoupling matrix $W(s)$ that gives a decoupled system $G(s)W(s) = I$ for use in a decentralized control structure. If this is not possible, explain why and choose $W(s)$ to satisfy the condition at least for $s = 0$. (2 p)

b. Compute $S(0)$ and $T(0)$ where

$$S(s) = [I + G(s)W(s)C(s)]^{-1}$$
$$T(s) = G(s)W(s)C(s)[I + G(s)W(s)C(s)]^{-1}$$

and $W(s)$ is used together with the decentralized controller

$$C(s) = \begin{pmatrix} \left(1 + \dfrac{1}{s}\right) & 0 \\ 0 & 2\left(1 + \dfrac{1}{2s}\right) \end{pmatrix}$$

Explain how the result is related to the integrators in $C(s)$. (2 p)

3. Consider the following system

$$\begin{cases} \dot{x} &= \begin{pmatrix} -3 & 1 \\ 1 & -1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y &= \begin{pmatrix} 0 & 1 \end{pmatrix} x \end{cases}$$

a. Calculate the controllability Gramian and its eigenvalues. What conclusion can be drawn from this? (2 p)

b. A balanced realization of the system is given by

$$\begin{cases} \dot{x} &= \begin{pmatrix} \dfrac{-28-2\sqrt{34}}{34+5\sqrt{34}} & \dfrac{10+2\sqrt{34}}{34+5\sqrt{34}} \\ \dfrac{10+2\sqrt{34}}{34+5\sqrt{34}} & \dfrac{-108-18\sqrt{34}}{34+5\sqrt{34}} \end{pmatrix} x + \begin{pmatrix} \dfrac{5+\sqrt{34}}{\sqrt{68+10\sqrt{34}}} \\ \dfrac{-3}{\sqrt{68+10\sqrt{34}}} \end{pmatrix} u \\ y &= \begin{pmatrix} \dfrac{5+\sqrt{34}}{\sqrt{68+10\sqrt{34}}} & \dfrac{-3}{\sqrt{68+10\sqrt{34}}} \end{pmatrix} x \end{cases}$$

Calculate the Hankel singular values. No exact answer is required, it is enough to give rounded answers. (1 p)

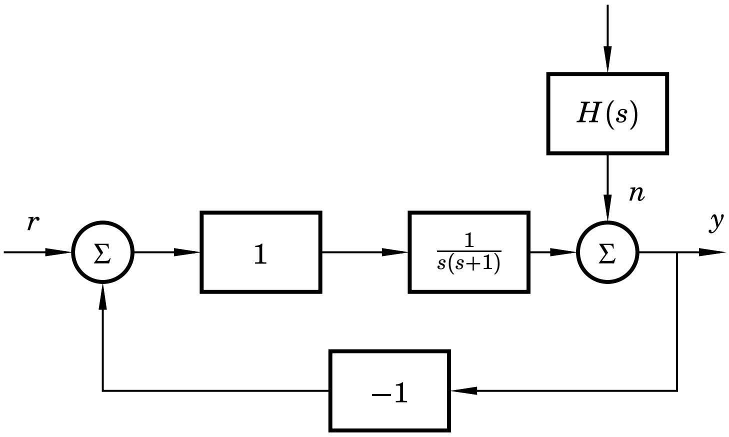
c. Calculate the reduced order system when the state corresponding to the lowest Hankel singular value is eliminated. (1 p)

4. Consider the system in figure 1. The reference signal r is constantly zero, while n is obtained by filtering white noise of intensity 1.

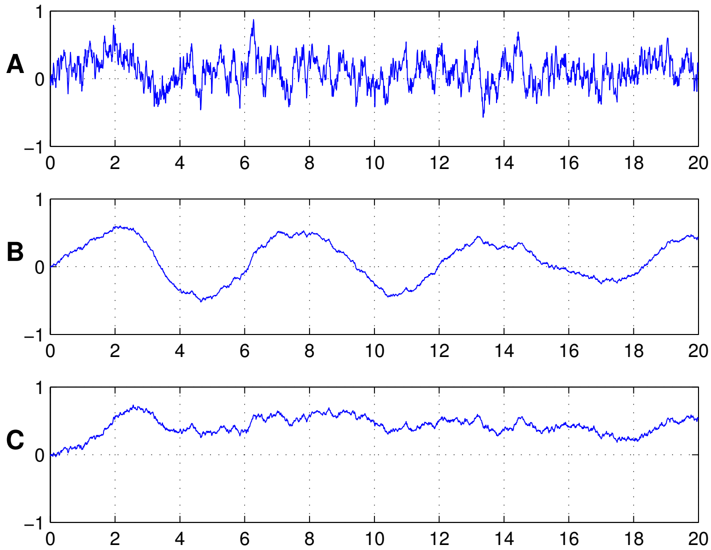
The output signal y is plotted in figure 2 for three different filters $H(s)$. Explain which of the following filters correspond to which realization. (As always, a clear motivation is required.)

$$H_1(s) = \frac{0.15}{10s+1} \qquad H_2(s) = \frac{1}{s+10} \qquad H_3(s) = \frac{0.1s+0.05}{s^2+0.2s+1}$$

(2 p)



Figur 1 Block diagram of system in problem 4.



Figur 2 Three realizations A, B and C, of the output signal y in problem 4.

5. After many years of hard work, you have just graduated from LTH as a full-fledged expert on control engineering. On the first day of your new job at the local factory, you get an assignment from your boss. He says: "*The guy who used to have your job designed a controller using some really old method — I think he called it slope-shaping. Anyway, he only did half the job, and it's up to you to finish it.*"

Your boss hands you a paper (Figure 3) and tells you: "*Here you have some notes that the other guy left us. We have tested the design he made, and it works well for handling disturbances. However, the process can not track step reference changes. I don't really care about how you fix this, the only thing that is important is that the output is exactly equal to the reference at ALL times, no matter what the input is! Get it?*".

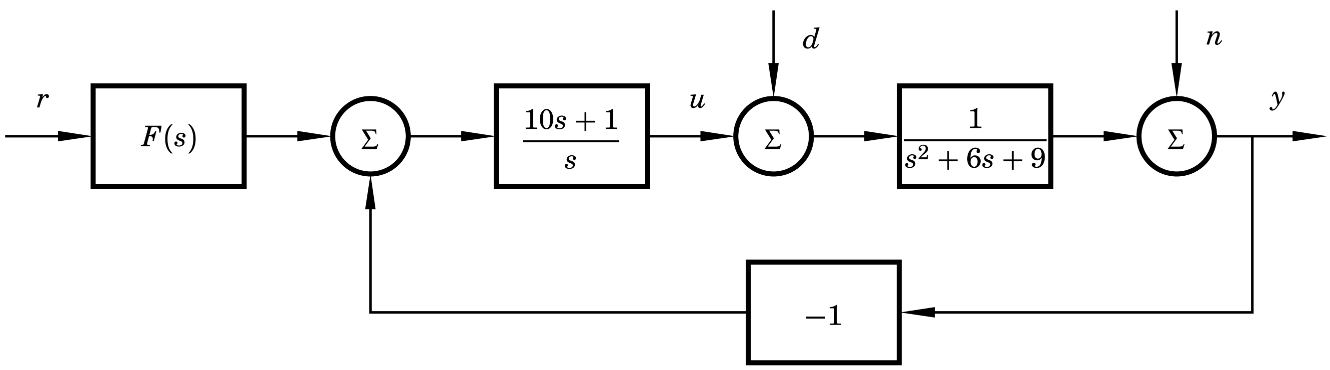
a. You quickly realize that with the model of Figure 3, it is impossible to design $F(s)$ to achieve what your boss wants. Why is this the case? (1 p)

b. Suppose that $F(s)$ is chosen to make $y(t)$ follow exactly a sinusoidal input $r(t) = \sin(\omega t)$ both for $\omega = 0$ and for $\omega = 100$. What are then the values of $|F(0)|$ and $|F(100i)|$? (1 p)

c. Suppose that the filter $F(s)$ must be implemented as a linear combination of existing filters, namely

$$F(s) = f_0 + \frac{f_1}{s+1} + \frac{f_2}{s+10}$$

where f_0, f_1 and f_2 are to be determined by optimization. State an optimization problem in f_0, f_1 and f_2 that minimizes the amplitude A of the the error $y(t) - r(t) = A\sin(t + \phi)$ when $r(t) = \sin t$. Further, add a constraint on f_0, f_1 and f_2 that gives zero stationary error $y - r$ when $r(t)$ is constant. (2 p)



Figur 3 The diagram your boss gives you in problem 5.

6. Consider the first order system

$$\begin{aligned} \dot{x} &= -x + u + v_1 \\ y &= x + v_2 \end{aligned}$$

where $u(t)$ is the control signal, while $v_1(t)$ and $v_2(t)$ are independent zero mean white noise processes with intensity 1.

a. Construct an output feedback LQG controller that minimizes

$$\mathbf{E}\left(2y(t)^2 + u(t)^2\right)$$

(3 p)

b. Where are the poles of the closed loop system located? (1 p)

c. Consider instead the stationary cost

$$\mathbf{E}\left(y(t)^2 + \psi u(t)^2\right)$$

where $\psi > 0$. What value of ψ gives the same control law as in **a**? (1 p)