LUND INSTITUTE
OF TECHNOLOGY
Lund University

Department of
AUTOMATIC CONTROL

Multivariable Control (FRTN10)

Exam October 19, 2011, hours: 8.00-13.00

Points and grades

All answers must include a clear motivation and a well-formulated answer. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each subproblem.

Accepted aid

The textbook *Glad & Ljung*, standard mathematical tables like TEFYMA, an authorized "Formelsamling i Reglerteknik"/"Collection of Formulas" and a pocket calculator. Handouts of lecture notes and lecture slides are also allowed.

Results

The result of the exam will be posted on the notice-board at the Department. The result as well as solutions will be available on the course home page:
http://www.control.lth.se/Education/EngineeringProgram/FRTN10.html

1. Consider the system

$$y + 2\dot{y} + \ddot{y} = u_1 + 3u_2.$$

a. Find a state-space representation of the system. (2 p)

b. Find the transfer function matrix of the system. (1 p)

c. Is the system controllable? Is it observable? (1 p)

2. The system

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

is chosen to be LQ-controlled using the cost function

$$J = \int_0^\infty (x_2^T Q x_2 + u^T r u)\mathrm{d}t.$$

Four responses to the initial conditions $x_0 = (5 \quad 3)^T$ are shown in Figure 1. Pair these plots to the corresponding weights shown below. Motivate.

$$\mathbf{A}: \quad Q = 1,\ r = 1 \qquad \mathbf{B}: \quad Q = 1,\ r = 0.01$$
$$\mathbf{C}: \quad Q = 0.01,\ r = 1 \qquad \mathbf{D}: \quad Q = 1,\ r = 100$$

(3 p)

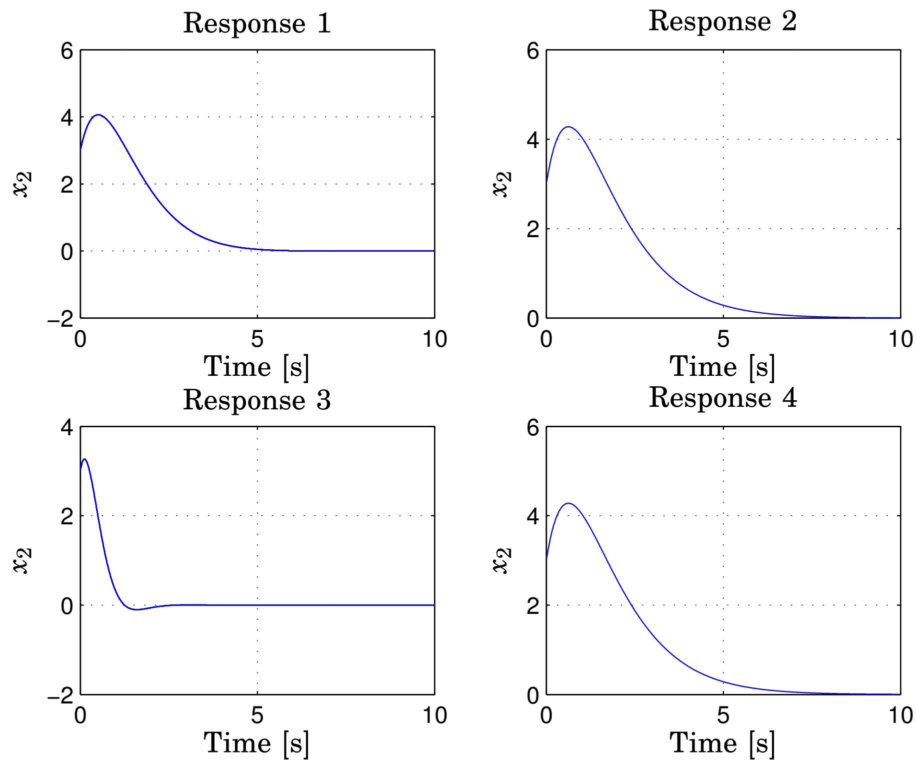

Figure 1 Figure belonging to Problem 2. Note that the diagrams do not have the identical scalings for the amplitude axes.

3. Consider the block diagram for the dynamic system in Figure 2. For perfect reference following one might like to have that the complementary sensitivity function $T(i\omega) = 1$ for all frequencies ω. However, give at least one reason for why this is in general not desirable (also in general not feasible). (1 p)

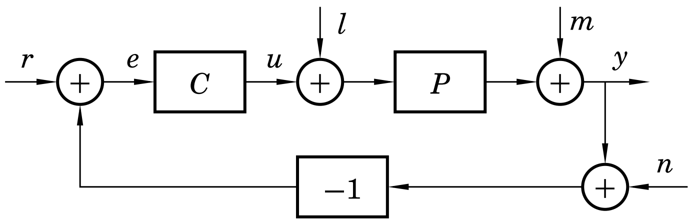

Figure 2 Figure belonging to Problem 3.

4. Consider the block diagram in Figure 3. The process P is given by

$$P(s) = \frac{s-3}{(s+1)(s+2)}$$

You have been assigned the task to design the controller C such that the following specifications are fulfilled:

- The system should be internally stable.
- Good tracking of reference signals for frequencies $\omega \leq 1$ rad/s.

Your colleague claims to have solved the problem and presents you the sensitivity function in Figure 3 Would you implement your colleague's controller? Motivate your answer! (2 p)

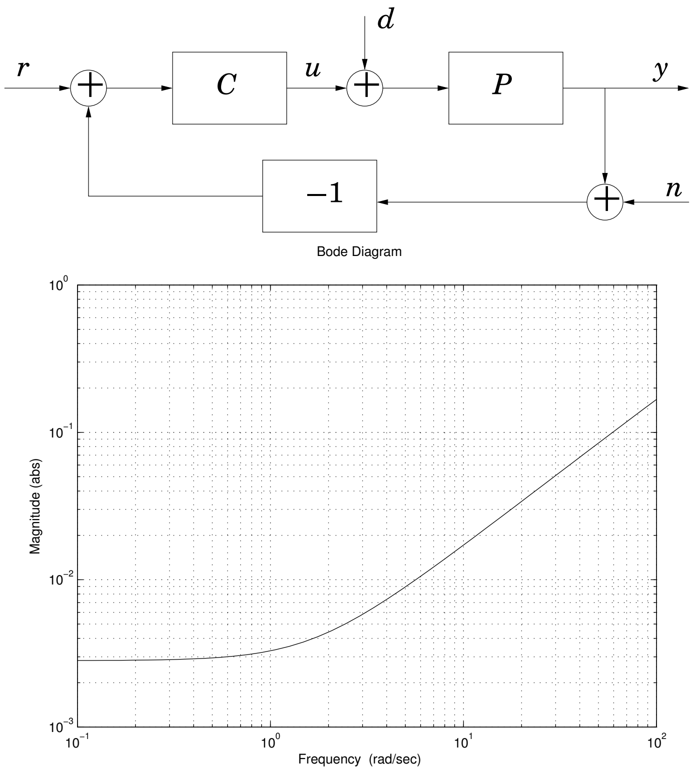

Figure 3 Block diagram and sensitivity function in Problem 4

5. Consider the following transfer matrix:

$$G(s) = \begin{pmatrix} \frac{s+2}{(s+1)^2} & 0 \\ 0 & \frac{s+1}{(s+2)^2} \end{pmatrix}.$$

a. Find the poles and transmission zeros of the transfer matrix. (2 p)

b. The system is observable and controllable, which means that there are no pole-zero cancellations. Explain how this relates to your answer in **a**) and how it differs from a single input single output case. (1 p)

6. Find a minimal state space realization of

$$G(s) = \begin{pmatrix} \frac{s+2}{s+1} & \frac{1}{s+1} \\ \frac{2s+3}{s^2+3s+2} & \frac{1}{s+1} \end{pmatrix}$$

(3 p)

7. Consider the 2-DOF control setup in Figure 4. P is the plant and C_1 and C_2 are the control blocks to be designed. The objective is to make the control error $e = r - y$ small without making the control effort u too large.

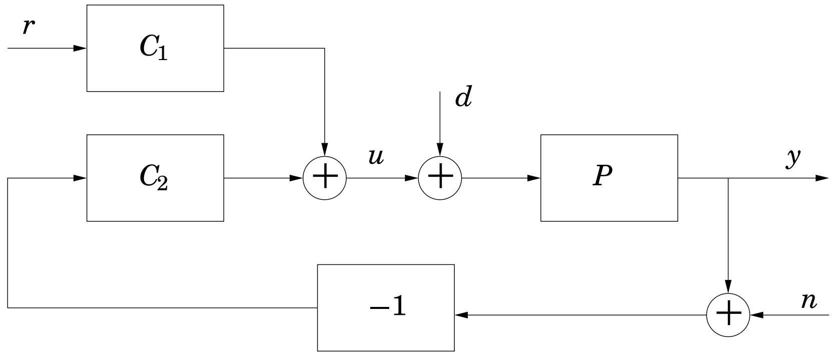

Figure 4 2-DOF control setup in problem 7

a. What is the relation between the signals in the generalized plant depicted in Figure 5 and the signals in Figure 4? (1 p)

b. Derive the transfer function G in Figure 5. (2 p)

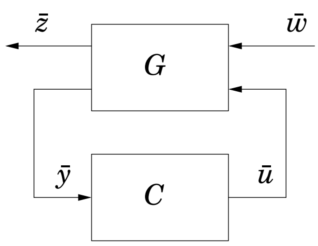

Figure 5 Generalized plant in problem 7

8. This problem is about choosing noise models in LQG design. Let P be a single input single output transfer function and set

$$H(s) = \frac{\omega_0^2}{s^2 + 0.1\omega_0 s + \omega_0^2}$$

The controller C is to be determined by LQG-design. The objective is to minimize the following cost function:

$$J(u) = \int_0^\infty z^2(t) + u^2(t)dt$$

The model used in the design will be chosen as one of the models depicted in the block diagrams in Figure 6–8. The signals w_1 and w_2 are independent Gaussian white noise processes with variance 1.

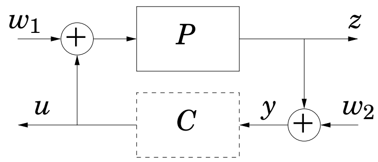

Figure 6 Model I

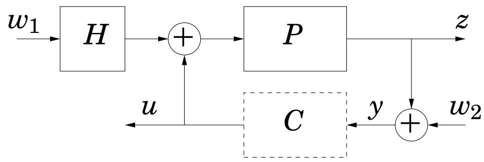

Figure 7 Model II

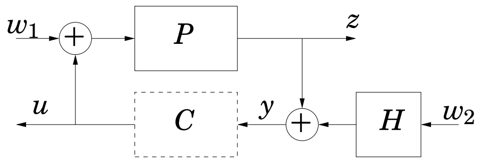

Figure 8 Model III

a. Which of the block diagrams in Figure 6–8 results in the controller with the best robustness to *measurement noise* at frequencies around ω_0. Motivate your answer! (1 p)

b. Which of the block diagrams in Figure 6–8 results in the controller with the best robustness to *load disturbance noise* at frequencies around ω_0. Motivate your answer! (1 p)

c. Which of the models in Figure 6–8 will result in the controller that gives the **largest** value of $|S(i\omega_0)P(i\omega_0)|$, where $S = \dfrac{1}{1+PC}$ is the sensitivity function.
Hint: Remember that $S + T = 1$. (2 p)

9. Consider the system

$$
\begin{aligned}
\dot{x} = A_x x + B_x u \quad &= \begin{bmatrix} -1 & -1 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u \\
y = C_x x \quad &= \begin{bmatrix} 1 & 0 \end{bmatrix} x
\end{aligned}
$$

Show that the state transformation

$$
\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = Tx = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
$$

will transform the system into a new state space form which is a *balanced realization*

$$
\begin{aligned}
\dot{z} &= A_z z + B_z u \\
y &= C_z z
\end{aligned}
$$

by showing that the controllability Gramian S_z and the observability Gramian O_z will be the same.

Hint: To show this you don't need to calculate the explicit values of S_z and O_z, even though that is also a possibility. (2 p)

and O_z will corrispondingly get the same value (omitted here) which shows that the state space form with the z-states is a balanced realization.

Good luck!