LUND UNIVERSITY

Department of
AUTOMATIC CONTROL

FRTN10 Multivariable Control

Exam 2018-10-27, 08:00–13:00

Points and grades

All answers must include a clear motivation and a well-formulated answer. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each subproblem. *Preliminary* grade limits:

- Grade 3: 12 points
- Grade 4: 17 points
- Grade 5: 22 points

Accepted aid

The textbook *Glad & Ljung*, standard mathematical tables like TEFYMA, an authorized "Formelsamling i Reglerteknik"/"Collection of Formulas" and a pocket calculator. Handouts of lecture notes and lecture slides (including markings/notes) are also allowed.

Results

The result of the exam will be entered into LADOK. The solutions will be available on the course home page: `http://www.control.lth.se/course/FRTN10`

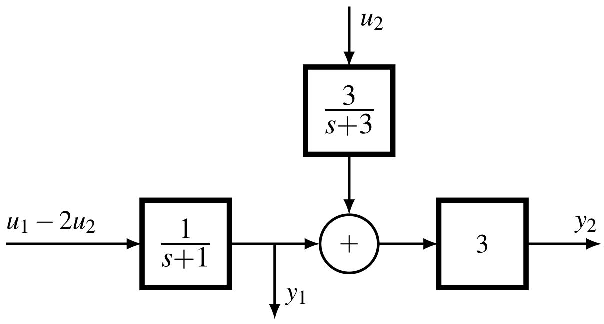

Figure 1 Block diagram of Problem 1

1. Consider the multivariable process in Figure 1.

a. Find the transfer matrix of the process from $u = [u_1 \ \ u_2]^T$ to $y = [y_1 \ \ y_2]^T$. (1 p)

b. Find the poles and zeros of the system. (1.5 p)

c. If u_1 is unit intensity white noise and $u_2 = 0$, what is the stationary variance of y_2? (1.5 p)

2. Give an example of a continuous-time linear time-invariant system in state-space form that fulfills all of the following properties:

- The system should have three stable poles.
- The system should have one input and two outputs.
- The system should be observable but not controllable.
- The system should be proper. (3 p)

3. A control system is shown in the block diagram in Figure 2(a). Assuming $r_1 = r_2 = 0$, we want to isolate the uncertainty block Δ as shown in Figure 2(b).

a. Find the transfer function H from w to v expressed in terms of P, Q, R, F and G. Assume that all blocks are MIMO. (1.5 p)

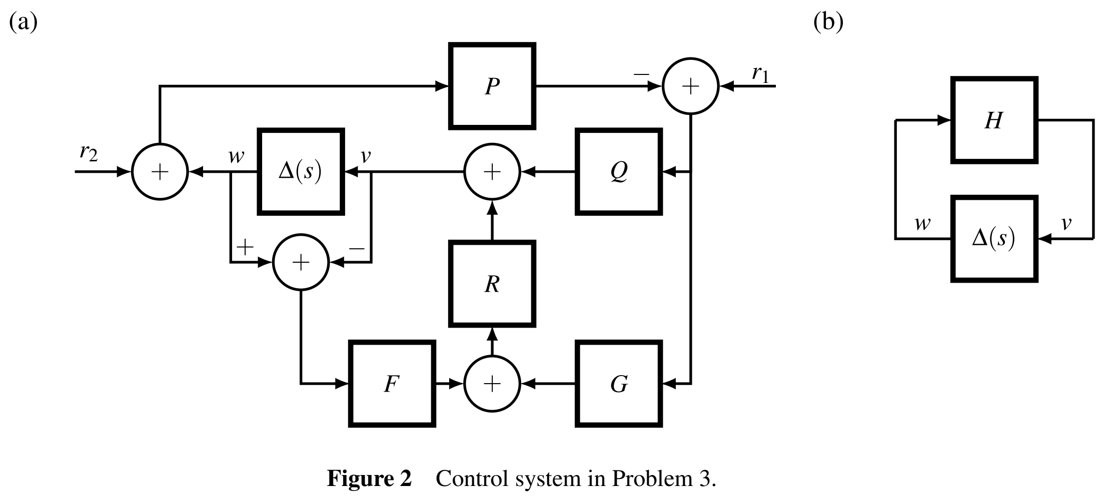

Figure 2 Control system in Problem 3.

b. Now assume that all blocks are SISO and that H is stable with $\|H\|_\infty = 4$. For which of the following Δ_i's can you guarantee stability of the closed-loop system using the Small Gain Theorem?

$$\Delta_1(s) = -\frac{1}{s+8} \qquad \Delta_2(s) = \frac{0.5s+2}{s-0.5}$$

$$\Delta_3(s) = \frac{0.5s-2}{s+5} \qquad \Delta_4(s) = \frac{2}{s+10}e^{-4s}$$

(2 p)

4. You have designed a two-degree-of-freedom controller for the non-minimum-phase process

$$P(s) = \frac{e^{-0.1s}}{s^2+13s+100}$$

according to the standard configuration shown in Figure 3. The "Gang of Six" closed-loop Bode magnitude diagrams are shown in Figure 4.

a. Does the controller have integral action? (0.5 p)

b. Is the control system robust? (0.5 p)

c. What is the maximum gain from measurement noise to the control signal? (0.5 p)

d. Is is possible to make the closed loop ten times faster while retaining the same level of robustness? (0.5 p)

e. Sketch the response of the plant output to a unit reference step change. (0.5 p)

Do not forget to motivate your answers above!

5. Consider the MIMO transfer function

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+1} & \frac{1}{s+0.5} \\ 0 & 0 & \frac{1}{s-1} \end{bmatrix}$$

a. We wish to control the stationary output of the system with a decentralized controller. Find suitable input–output pairings for the controller with the help of the Relative Gain Array. You may use the fact that

$$\begin{bmatrix} a & b & 0 \\ 0 & c & d \\ 0 & 0 & e \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{a} & -\frac{b}{ac} & \frac{bd}{ace} \\ 0 & \frac{1}{c} & -\frac{d}{ce} \\ 0 & 0 & \frac{1}{e} \end{bmatrix}$$

Looking beyond the RGA, is this the only reasonable pairing for this process? (2 p)

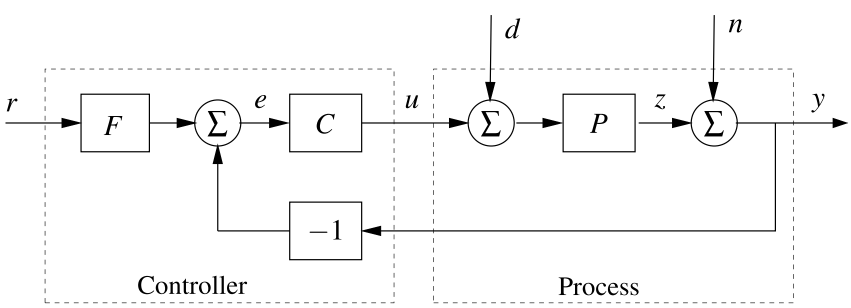

Figure 3 2-DOF controller structure in Problem 4.

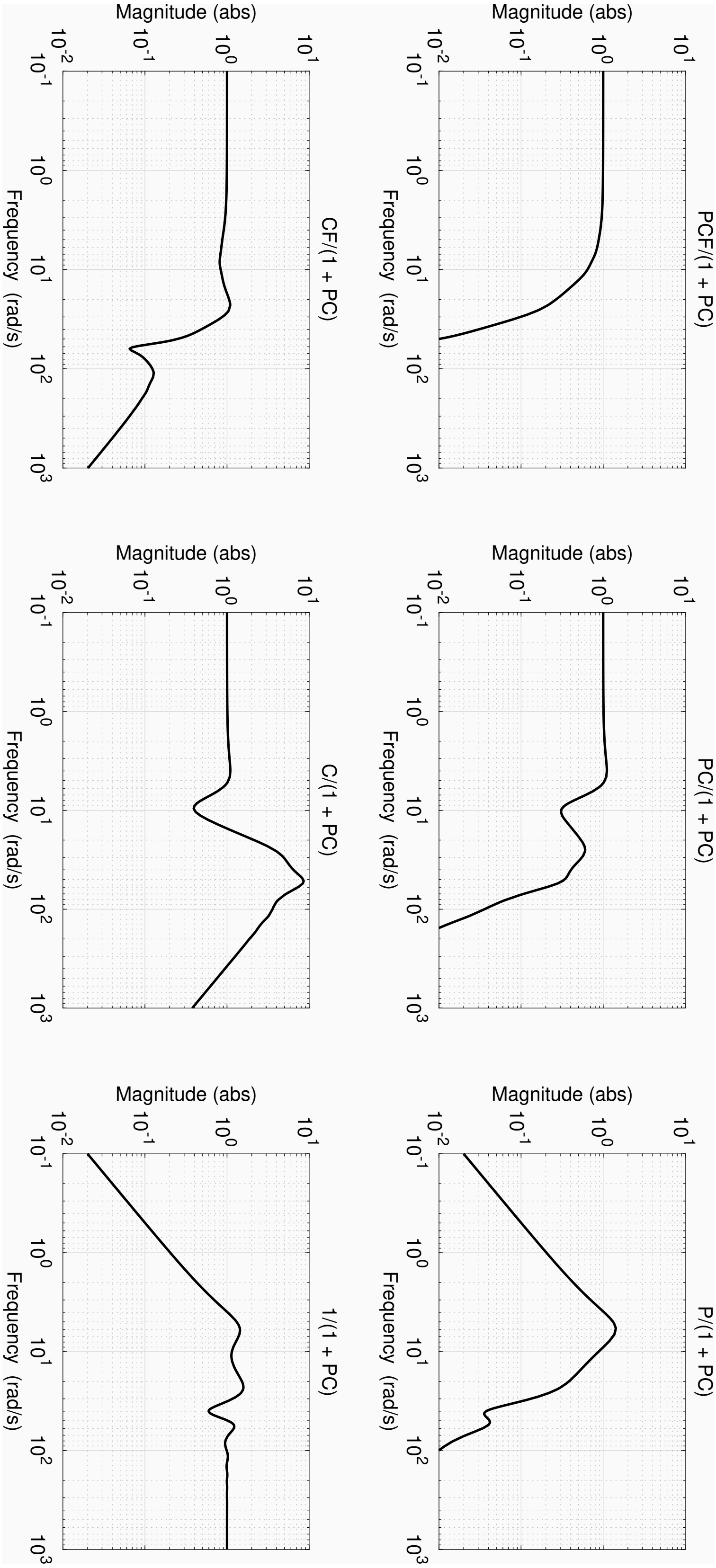

Figure 4 Bode magnitude diagrams of the "Gang of Six" closed-loop transfer functions in Problem 4.

b. Assume that we have a pairing for the last output. We worry about having unmodeled high-frequency dynamics in our real process and we therefore want this specific closed loop to have first-order roll-off. This means that we require $|T(i\omega)| \leq |\frac{k}{i\omega}|$ for some k where $T(s)$ is the complementary sensitivity function of the control loop. What is the largest attenuation (smallest k) that can be achieved with a stable controller? (1 p)

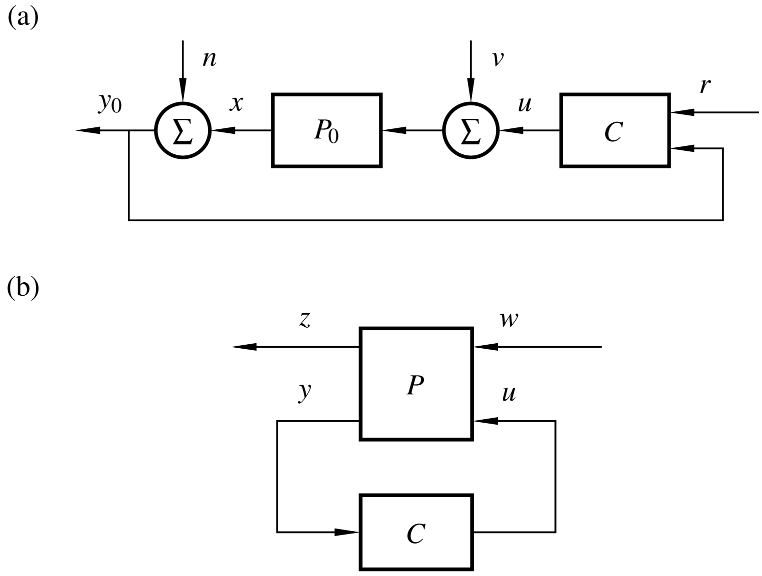

Figure 5 Control loop in Problem 6: (a) original control loop, (b) desired general form.

6. We wish to design a controller for the system in Figure 5(a), where P_0 is a SISO process.

a. The goal is to control the process output x. Transform the system in Figure 5(a) to the general closed-loop form in Figure 5(b) by writing down the transfer function matrix

$$P = \begin{bmatrix} P_{zw} & P_{zu} \\ P_{yw} & P_{yu} \end{bmatrix}$$

where

$$z = x, \quad y = \begin{bmatrix} y_0 \\ r \end{bmatrix}, \quad w = \begin{bmatrix} n \\ v \\ r \end{bmatrix}$$

(1 p)

b. We try to design a controller for the process in the previous question with the help of convex optimization but notice that we have no way of specifying that the process output should follow a step change in the reference r. Add an extra output to z that allows us to add a constraint to capture this need and find the new P. (1 p)

c. Write down the closed-loop transfer function for the general form in Figure 5(b) expressed in terms of the Youla parameterization $Q = C(I - P_{yu}C)^{-1}$. When designing controllers with the help of optimization, why is it preferable to use Q as a design variable over simply expressing the closed-loop transfer function in terms of C?

(1 p)

7. Consider the following system:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x$$

a. Assume full state information and design a state feedback controller $u = -Lx$ that minimizes the cost function

$$\int_0^{\infty} \left(x^T Q_1 x + u^T Q_2 u\right) dt$$

where

$$Q_1 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \quad Q_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(2 p)

b. Assume that there are uncorrelated unit white noise disturbances on the two measurements and the two inputs. Write down the state-space model that includes the disturbances and design a Kalman filter to estimate the states. Answer with the Kalman gain and a state-space representation of the Kalman filter. (2 p)

c. Given the unsure estimates $\hat{x}$ of our true states given by the Kalman filter, design a state feedback law $u = -L\hat{x}$ that minimizes the average cost per time unit, i.e.,

$$\mathrm{E}\left[x^T Q_1 x + u^T Q_2 u\right]$$

(1 p)

d. When you test your controller you notice that it performs well but on further inspection you find that the control signal has a lot of high-frequency noise in it. Suggest a change of our design variables that can help remove this high-frequency content.

(1 p)