LUNDS TEKNISKA HÖGSKOLA
Lunds universitet

Institutionen för
REGLERTEKNIK

FRTN10 Multivariable Control

Exam 2013-01-09

Points and grades

All answers must include a clear motivation. The total number of points is 25. The maximum number of points is specified for each subproblem.

Preliminary grades

3 12-16 points

4 17-21 points

5 22-25 points

Accepted aid

The textbook *Glad & Ljung*, standard mathematical tables like TEFYMA, an authorized "Formelsamling i reglerteknik" and a pocket calculator. Copies of lecture notes will be distributed at the exam.

Results

The exam results will be posted on the course web page.

1. A process with two inputs and two outputs is described by the following state-space equation,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

with

$$A = \begin{pmatrix} -10 & 0 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 10 & c \\ 10 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

a. For what values of c does the system have right half plane zeros? (1 p)

b. Determine observability for all real values of the parameter c. (1 p)

c. Find the controllablity gramian for the system. (1 p)

2. A two-input-one-output system has the transfer matrix

$$G(s) = \begin{pmatrix} G_{yu}(s) & G_{yd}(s) \end{pmatrix} = \begin{pmatrix} \frac{2}{s^2+3s+2} & \frac{1}{s+1} \end{pmatrix}.$$

from control signal u and disturbance d to output y.

a. Find a state-space realization of this system. The realization should be of order two. (2 p)

b. The disturbance d has spectrum

$$\Phi_d(\omega) = \frac{1}{\omega^4 + 5\omega^2 + 4}$$

Find a linear system $H(s)$ such that d is the output when the input to $H(s)$ is white noise with intensity 1. (1 p)

c. Expand the state-space model from **a** so that the disturbance input d is replaced by a white-noise input w. (1 p)

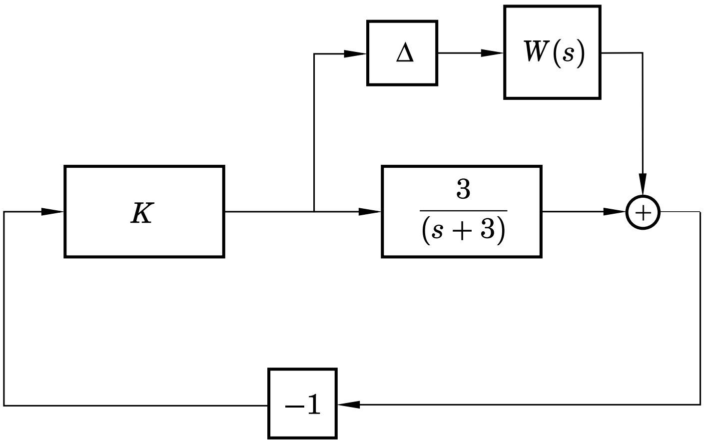
3. Consider the system in Figure 1. The process is modelled as a first order system, but there is some doubt on how good this model is. Therefore, an uncertain Δ-block has been added in the block diagram. It is assumed that $||\Delta||_\infty < 1$, but nothing else is known about Δ.

a. The controller is a simple proportional controller with gain K and the weight is

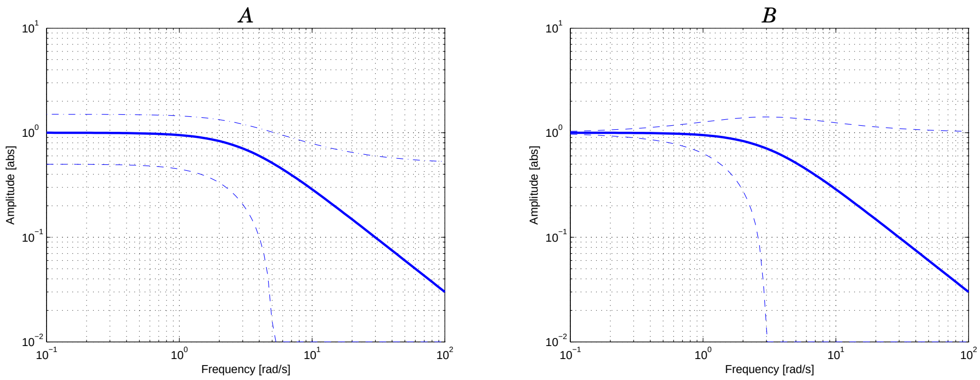
$$W(s) = \frac{s}{s+3}.$$

For what values, positive or negative, of K can you guarantee that the system is stable? (2 p)

b. In which of the two diagrams in Figure 2 do the dotted lines illustrate the uncertainty in the Bode amplitude plot of the process? Motivate your choice. (1 p)



Figur 1 System in problem 3.



Figur 2 Amplitude uncertainties in system in problem 3.

4. A control engineer is asked to find a controller for a double tank system consisting of a large and a small tank in series. The following transfer function has been chosen to model this process

$$G(s) = \frac{1}{(s+1)(100s+1)}.$$

The engineer decides to use an IMC controller with

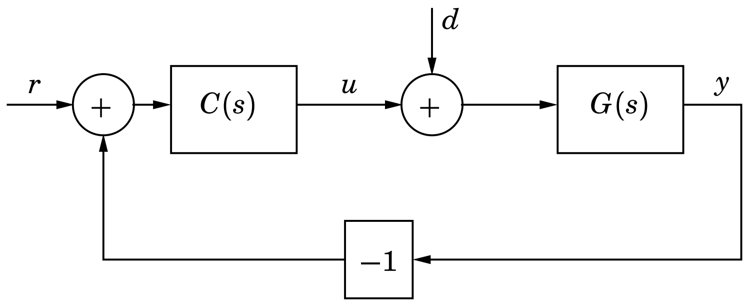
$$Q(s) = \frac{G^{-1}(s)}{(\lambda s + 1)^n}$$

where n is chosen such that $Q(s)$ has relative degree 0.

To the engineer's surprise, the closed loop system from the reference r to the process output y, see Figure 3, can be made arbitrarily fast.

a. Find parameter values that makes the slowest poles in the transfer function from r to y at least 100 times faster than the slowest pole of $G(s)$. Determine the corresponding controller $C(s)$. (2 p)

b. The control engineer soon discovers that no matter how fast the response from r to y is made, the closed loop response due to a load disturbance (d to y) will still be very slow. Explain why. (1 p)



Figur 3 The closed loop control system in Problem 4.

5. Consider the first order system

$$\dot{x} = -x + u + v_1$$
$$y = x + v_2$$

where $u(t)$ is the control signal, while $v_1(t)$ and $v_2(t)$ are independent zero mean white noise processes with intensity 1.

a. Construct a state feedback controller that minimizes the stationary cost

$$\mathbf{E}\left(2x(t)^2 + u(t)^2\right)$$

(2 p)

b. Construct an output feedback LQG controller that minimizes

$$\mathbf{E}\left(2y(t)^2 + u(t)^2\right)$$

(2 p)

c. Where are the poles of the closed loop system located in the previous two subproblems? (2 p)

d. Consider instead the stationary cost

$$\mathbf{E}\left(y(t)^2 + \psi u(t)^2\right)$$

where $\psi > 0$. What value of ψ gives the same control law as in **a**? (1 p)

6. Consider the closed loop SISO system presented in Figure 4 where y is the output, u control signal, and d, v are disturbances. The same system can be represented by the diagram Figure 5 where

$$w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \qquad z = \begin{pmatrix} y \\ \bar{u} \end{pmatrix}$$

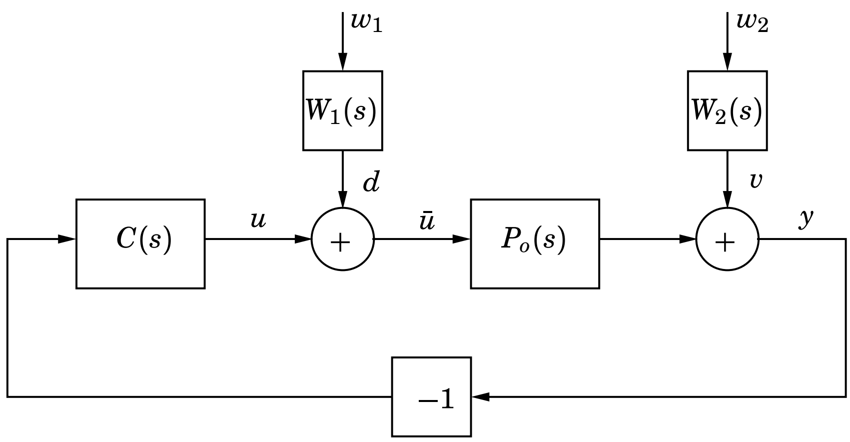
a. Given the process transfer function $P_o(s)$ and the weighting functions $W_1(s)$, $W_2(s)$, determine the transfer matrix

$$\begin{pmatrix} P_{zw}(s) & P_{zu}(s) \\ P_{yw}(s) & P_{yu}(s) \end{pmatrix}$$

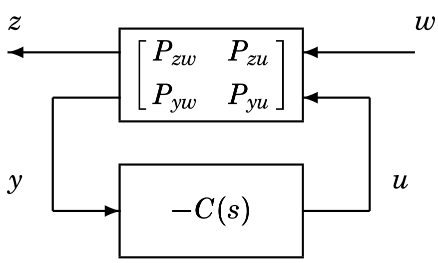
such that the two diagrams become equivalent. (2 p)

b. Determine the closed loop transfer function matrix from w_2 to z. (2 p)

c. Assume that v is a low frequency output disturbance. Choose a $W_2(s)$ such that the constraint $\|W_2(i\omega)S(i\omega)\|_\infty \le 1$, where $S(s)$ is the sensitivity function, will give closed-loop attenuation of v at low frequencies. (1 p)



Figur 4 The closed loop system in Problem 6.



Figur 5 General form of a closed loop system in Problem 6.