LUND UNIVERSITY

Department of
AUTOMATIC CONTROL

Multivariable Control Exam

Exam 2016-01-08

Grading

All answers must include a clear motivation and a well-formulated answer. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each subproblem.

Accepted aid

The textbook "Control Theory" by *Glad & Ljung*, standard mathematical tables like TEFYMA, an authorized “Formelsamling i Reglerteknik”/”Collection of Formulas” and a pocket calculator. Handouts of lecture notes and lecture slides are also allowed.

Results

The results will be reported via LADOK.

Notice that several sub-problems can be solved independently.

1. For perfect attenuation of input disturbances one might like to have that the sensitivity function $S(i\omega) = 0$ for all frequencies ω. However, give at least one reason for why this is in general not desirable (also in general not feasible).

(1 p)

2. Consider the following system

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$
$$y = \begin{pmatrix} 1 & 1 \end{pmatrix} x$$

which is to be controlled so that the following cost function is minimized:

$$J = \int_0^\infty 10y^2 + u^2 \, dt.$$

Calculate the in LQ sense optimal state feedback vector L and l_r in the control law $u = -Lx + l_r r$ that ensures unit static gain for the closed loop system.

(3 p)

3. Due to your proficiency in control engineering, you have been hired to help with the design of a controller for a certain system. You remember that you have seen a similar system, for which the Bode diagram is shown in Figure 1, during your long years in school. Your employer cannot figure out why the P-controller that is currently being used is capable of making the system output follow constant references without any errors, but as soon as there is a load disturbance on the input, errors start showing up. Explain why this is the case. The previous employee tasked with developing the control system had also made plans to change the controller to a PI-controller and to increase the cutoff frequency to 5 rad/s. Explain why this likely is a bad idea, and give a modification suggestion that could improve the situation (no calculations needed, just paint a broad picture). (2 p)

4. The block diagram for an electrical system is shown in Figure 2. A state-space model where the noise has been ignored can be written

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u \\ y &= \begin{pmatrix} 0 & 1 \end{pmatrix} x. \end{aligned}$$

a. The disturbances on the states of the system is due to noise from the power supply. It is therefore expected that the same disturbance v is observed on both states. Extend the state-space model to account for v and w. (1 p)

b. It is known that the power supply is driven by AC power at 50Hz. We therefore assume that the noise v will have the spectral density

$$\Phi_v(\omega) = \frac{(\frac{\omega}{\omega_0})^2}{\left((\frac{\omega}{\omega_0})^2 + 1\right)^2},$$

where $\omega_0 = 2\pi 50$ rad/s. Find a transfer function $H(s)$ so that v can be written $v = H(s)n$, where n is white noise with intensity 1. (1 p)

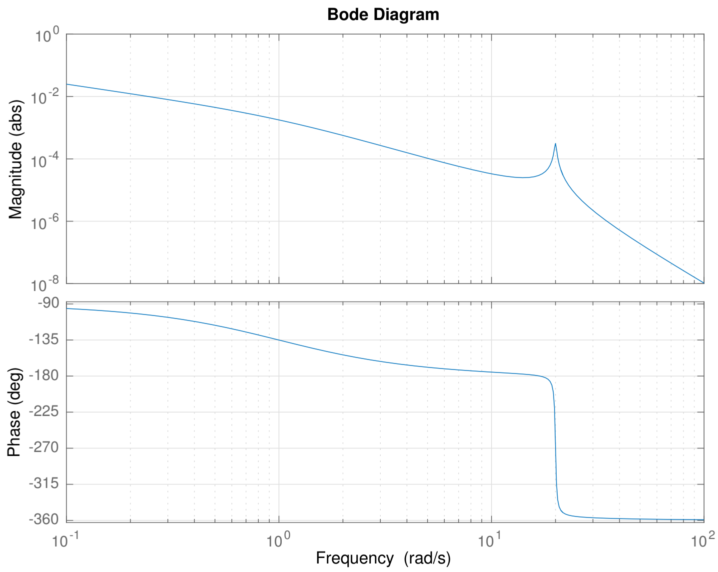

Figure 1 The Bode diagram of the system in problem 3

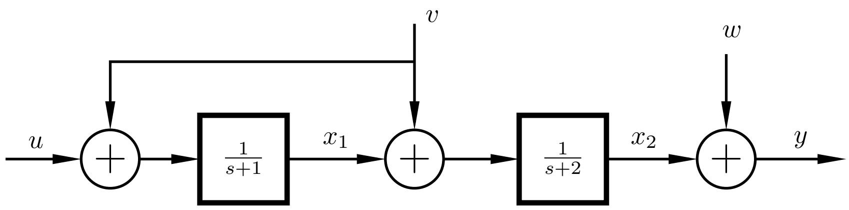

Figure 2 The system in Problem 4.

c. Extend the state-space representation so that the input is n (white noise with intensity 1) instead of v. (1 p)

d. Assume that n and w are uncorrelated and that w is of intensity 2. Write down the equation that you would solve if you wanted to design a Kalman filter. Make sure to define all the matrices. You don't need to solve the equation. (1 p)

5. We want to design a controller C for the SISO control loop in Figure 3 using Youla parametrization and convex optimization. To do this, the control loop must first be transformed into the standard form of Figure 4, where z are the signals that we want to control, y are the signals available to the controller, w are the exogenous inputs and u is the control signal.

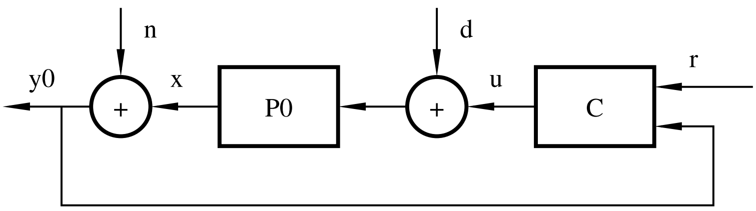

Figure 3 The control loop in Problem 5.

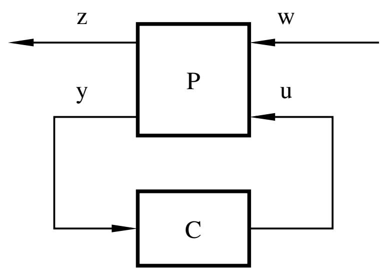

Figure 4 Desired form of the control loop in Problem 5.

The signals z and w are given by

$$z = \begin{pmatrix} e \\ u \end{pmatrix}, \qquad w = \begin{pmatrix} d \\ n \\ r \end{pmatrix},$$

where the control error is $e = r - x$. The controller C, which is a 1×2 transfer function, is the same in both figures, as is the control signal u.

a. What is the controller input y of Figure 4 according to Figure 3? What is the size of the transfer function matrix P? (1 p)

b. Find the transfer function matrix P so that Figure 3 and Figure 4 describe the same control problem. (1 p)

c. The Youla parametrization results in the closed loop system

$$z = Hw,$$

where the transfer function H is given by

$$H = P_{zw} + P_{zu} Q P_{yw}.$$

The control objective is

a) To make the $\mathcal{L}_2$-gain $||H_{ij}||_2 \le 5$ for all elements H_{ij}.

b) During an impulse disturbance experiment in d, the control signal should satisfy $|u(t)| \le 1.6$.

c) During an impulse disturbance experiment in d, the control error should be small from three seconds and onward: $|e(t)| \le 0.3, t \ge 3$ if the impulse occurs at $t = 0$.

Two transfer functions Q_1 and Q_2 have been found that satisfy objective a). Figure 5 shows impulse responses from d to e and u when using the corresponding controllers C_1 and C_2. Find a Q that satisfies all three objectives a), b) and c). (2 p)

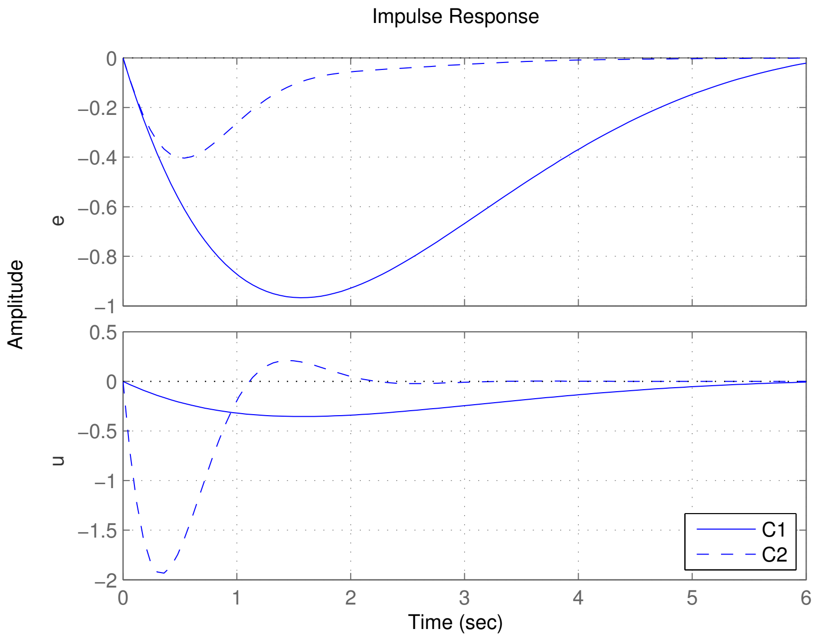

Figure 5 Impulse responses from disturbance d to control error e (top) and control signal u (bottom) for the controllers C_1 and C_2 in Problem 5.

6. Consider the following system

$$G(s) = \begin{pmatrix} \frac{1}{s+1} & \frac{1}{s+2} \\ \frac{2}{s+2} & \frac{1}{s+2} \end{pmatrix}$$

a. What are the (multivariable) poles and zeroes of the system? (1 p)

b. Write the system in state-space form with as few states as possible. (1 p)

c. Something happens to the system at $s = 0$, explain how this affects the ability to control the system. (1 p)

7. Consider the setup presented in Figure 6 where the system $P = 1/(s+1)$ is controlled with the P-controller $C = K$, $K > 0$ given the uncertainty Δ, $||\Delta||_\infty \leq 2$. For what values of K is the closed loop stable? (2 p)

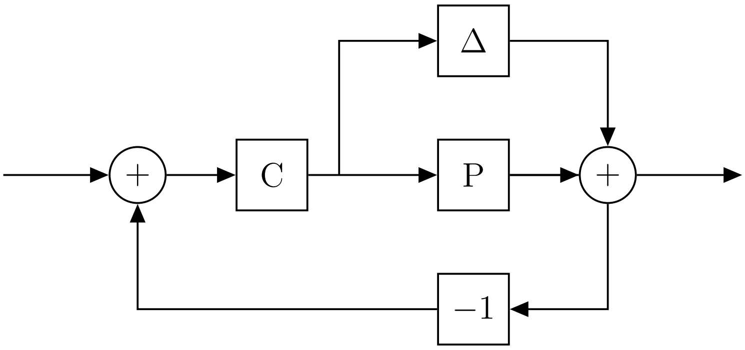

Figure 6 Block diagram for problem 7.

8.

a. The behaviour of the MIMO system G is dependent on the variable γ which takes either the value 0.1 or 1. For which value of γ should u_1 be paired with y_1 and u_2 be paired with y_2 for good decentralized controller performance at low frequencies? Motivate your answer. (2 p)

$$G = \begin{pmatrix} \frac{1}{s+\gamma} & \frac{2\gamma}{10s+1} \\ \frac{\gamma}{10s+1} & \frac{e^{-s\gamma}}{s+5\gamma} \end{pmatrix}$$

b. Find a static decoupler W_1, W_2 for the system G with $\gamma = 1$. Note that there are several possible solutions. (1 p)

9. In this problem we will study model reduction of the MIMO controller

$$\dot{\xi} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \xi + \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & -1 - \frac{\sqrt{2}}{2} \end{pmatrix} y$$
$$u = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & -1 - \frac{\sqrt{2}}{2} \end{pmatrix} \xi$$

a. Verify that the state space realization is balanced. Calculate the Hankel singular values. (2 p)

b. Based on your results from **a**, perform a model reduction by eliminating the state corresponding to the smallest Hankel singular value. (1 p)