LUND INSTITUTE
OF TECHNOLOGY
Lund University

Department of
AUTOMATIC CONTROL

Multivariable Control (FRTN10)

Exam October 24, 2012, hours: 8.00-13.00

Points and grades

All answers must include a clear motivation and a well-formulated answer. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each subproblem.

Accepted aid

The textbook *Glad & Ljung*, standard mathematical tables like TEFYMA, an authorized "Formelsamling i Reglerteknik"/"Collection of Formulas" and a pocket calculator. Handouts of lecture notes and lecture slides are also allowed.

Results

The result of the exam will be posted on the notice-board at the Department. The result as well as solutions will be available on the course home page:
`http://www.control.lth.se/Education/EngineeringProgram/FRTN10.html`

1. Consider a system with inputs (u_1, u_2) and outputs (y_1, y_2), given by the differential equations

$$\ddot{y}_1 + 2\dot{y}_1 - y_2 + y_1 = u_1 + \dot{u}_2 + 2u_2$$
$$y_2 + \dot{y}_1 + y_1 = u_1 - 2u_2$$

Calculate the transfer matrix of the system. (2 p)

2. Consider the system

$$G(s) = \begin{pmatrix} \frac{\alpha}{s+1} & -\frac{s+2}{s+1} \\ \frac{1}{s+1} & \frac{1}{s+2} \end{pmatrix}$$

a. What are the poles of the system? For which values of α is the system non-minimum phase? (2 p)

b. Derive a state space realization of the system. (2 p)

3. Consider the control system in the block diagram in Figure 1. The process $P(s)$ is modeled to have an unstructured uncertainty Δ.

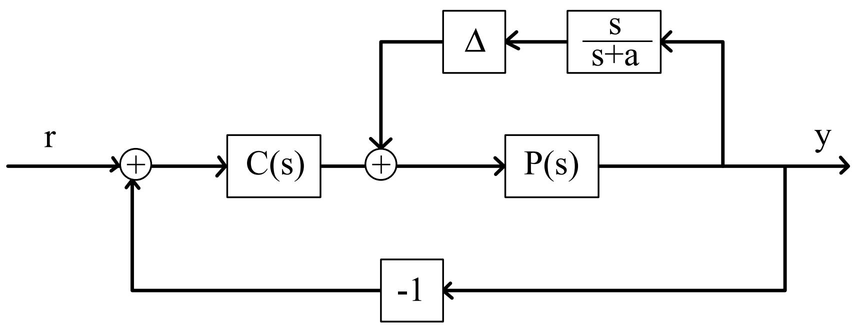

Figure 1 Block diagram for the system in problem 3.

a. Explain the role of the transfer function $\frac{s}{s+a}$ $(a > 0)$ in the block diagram. (1 p)

b. Give an expression for γ^* such that stability of the closed loop system is guaranteed by the small gain theorem if and only if $\|\Delta\|_\infty < \gamma^*$. Assume that the nominal system (i.e. with $\Delta = 0$) is asymptotically stable. (2 p)

4. We will consider optimal state estimation for the setup shown in Figure 2 with $P(s) = \frac{6}{s+2.5}$.

a. The process disturbance v_1 has the power spectrum $\Phi_{v_1}(\omega) = \frac{1}{\omega^2+1}$. The measurement error v_2 can be considered as white noise with intensity 1; $\Phi_{v_2}(\omega) = 1$. Rewrite the system to a form where the only inputs are the control signal and white noise signals with intensity 1. (2 p)

b. Determine the optimal Kalman filter gain for the system derived in the previous subproblem.

Hint: For some choice of state variables, the Riccati equation has the solution $P = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 3/8 \end{pmatrix}$. (2 p)

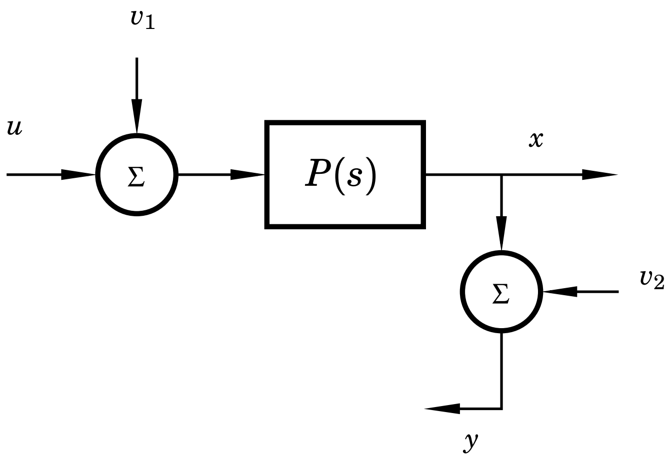

Figure 2 The setup for the system in problem 4.

5. Consider control of the process with Bode diagram shown in Figure 3. For each of the following statements, determine if it is true or false. Short motivations are required! (2 p)

1. Without disturbances, a P-controller will be sufficient to follow reference steps without stationary error.
2. A step-shaped input load disturbance can be removed by a P-controller.
3. It is impossible to design a PI-controller that achieves cut-off frequency $\omega_c = 5$ rad/s and phase-margin $\varphi_m = 35$ degrees.
4. A P-controller will be sufficient to follow a ramp-shaped reference signal without stationary error.

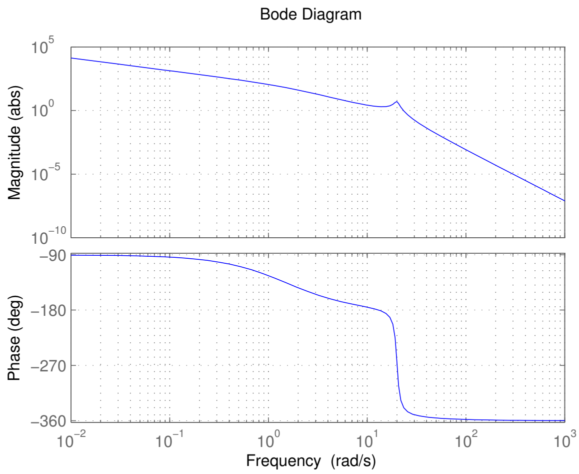

Figure 3 Bode plot for problem 5.

6. Consider the following system

$$\dot{x} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ -2 & -3 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 \end{pmatrix} x + \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} x$$

Is it controllable? Is it observable? Determine the controllable and observable subspaces. Motivate! (3 p)

7. In this problem we will study model reduction of the MIMO controller

$$\dot{\xi} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \xi + \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & -1 - \frac{\sqrt{2}}{2} \end{pmatrix} y$$

$$u = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & -1 - \frac{\sqrt{2}}{2} \end{pmatrix} \xi$$

a. Verify that the state space realization is balanced. Calculate the Hankel singular values. (2 p)

b. Based on your results from **a** perform a model reduction by eliminating the state corresponding to the smallest Hankel singular value. (2 p)

c. What is the transfer matrix of the reduced controller? (1 p)

8. You are called in as a consultant at the company SuperControl to analyze the latest developments of one of their main rivals, UltraControl. For various reasons both companies are working hard to find good controllers for the following systems

A $P(s) = \frac{s-3}{(s+4)(s-2)}$

B $P(s) = \frac{1}{s-10}$

C $P(s) = \frac{s-1}{(s+1)^2}$

D $P(s) = \frac{1}{s^2+0.4s+4}$

UltraControl claims that they have designed stabilizing controllers that result in the sensitivity functions (S) and complementary sensitivity functions (T) in Figure 4.

It is known within SuperControl that UltraControl sometimes makes serious mistakes (their entire control group has been recruited from KTH), and your task is therefore to analyze if their control designs are reasonable. For each of the systems A–D, analyze if the given plots of S and T is even possible to achieve. Motivate your answers. (2 p)

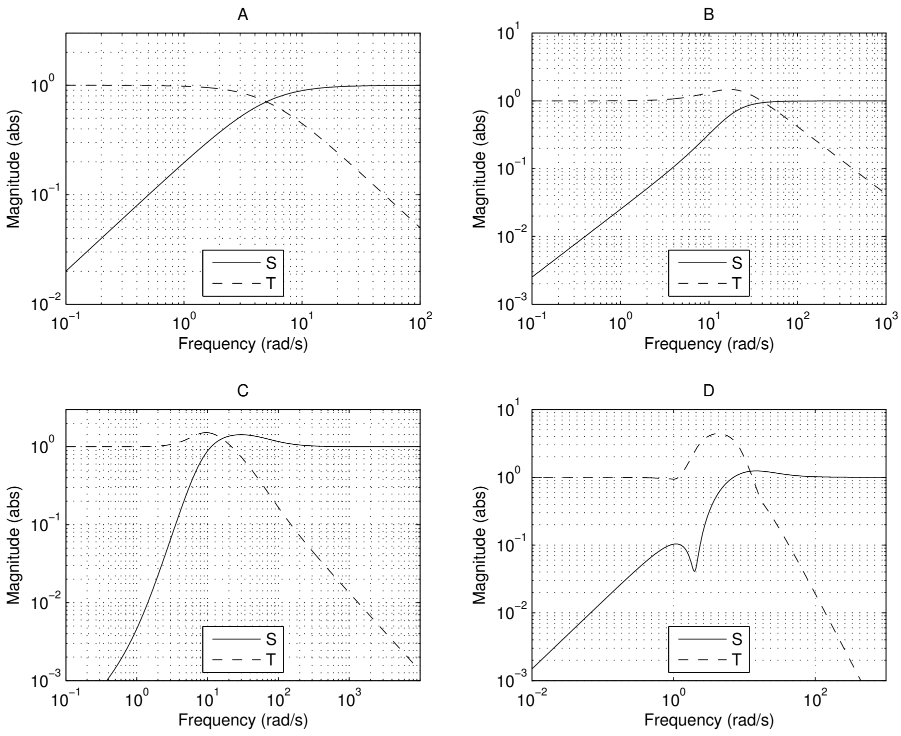

Figure 4 Plots of S and T for the different systems in problem 8.