LUND INSTITUTE OF TECHNOLOGY
Lund University

Department of
AUTOMATIC CONTROL

Multivariable Control Exam

Exam 2013-10-23

Points and grades

All answers must include a clear motivation and a well-formulated answer. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each subproblem.

Accepted aid

The textbook *Glad & Ljung*, standard mathematical tables like TEFYMA, an authorized “Formelsamling i Reglerteknik”/”Collection of Formulas” and a pocket calculator. Handouts of lecture notes and lecture slides are also allowed.

Results

The result of the exam will be posted on the notice-board at the Department. The result as well as solutions will be available on the course home page:
`http://www.control.lth.se/Education/EngineeringProgram/FRTN10.html`

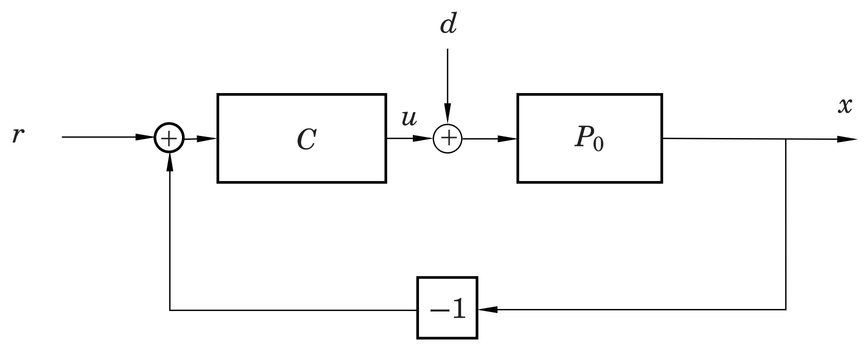

Figure 1 The system in problem 3.

1. Consider control of a MIMO system described by the transfer matrix

$$G(s) = \begin{bmatrix} \dfrac{s-1}{s+1} & \dfrac{9}{s+1} \\ \dfrac{1}{s+2} & \dfrac{1}{s+2} \end{bmatrix}.$$

a. Are there any fundamental limitations due to right half plane poles or zeros in the system? If so, which? (2 p)

b. Find a state-space realization of the system. (2 p)

2. Consider a system described by

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{1+s} & \frac{10s}{s+1} \\ \frac{10s}{s+1} & \frac{1}{1+s} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

We want to use decentralized control to keep y_1, y_2 at their reference values in spite of disturbances close to sinusoidal with frequency 0.1 Hz.

Suggest (if possible) a suitable input-output pairing for controlling y_1, y_2 with two single-input/single-output controllers. (2 p)

3.

a. Consider the set-up in Figure 1. A controller will be designed to make x follow the reference signal r while limiting the control signal u. Introduce appropriate variables and rewrite the system on the form given in Figure 2. (2 p)

b. For the plant $P_0(s) = \frac{1}{s+1}$, two controllers $C_1 = \frac{1}{s(s+1)}$ and $C_2 = 2$ have been designed. Their performance have the following characteristics:

	C_1	C_2
Minimum value of $x(t)$ for $t \geq 5$ after a unit step in r	0.94	0.68
Maximum value of $u(t)$ for $t \geq 0$ after a unit step in r	1.3	2
H_∞ norm of the closed loop transfer function from r to x	1.35	0.67

Design a controller with $\min_{t\in[5,\infty)} x(t) \geq 0.81$ and $\max_{t\in[0,\infty)} u(t) \leq 1.65$ when r is a unit step, while the H_∞ norm of the closed loop transfer function from r to x is at most 1.05. (2 p)

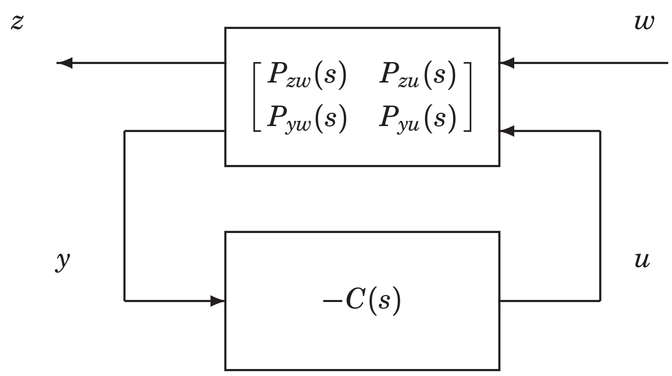

Figure 2 General form of a closed loop system.

4. Consider the system in Figure 3.

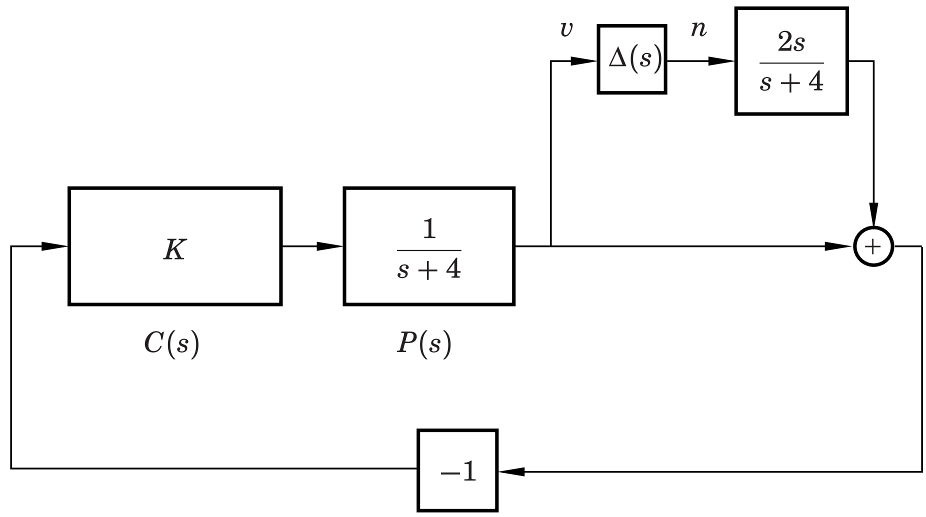

Figure 3 System for Problem 4.

Let the controller be a given by a static feedback gain K.

a. For what values of K is the closed loop system stable when $\Delta = 0$. (1 p)

b. What values of K keeps the closed loop system stable for every (stable) Δ with gain less than one? (2 p)

5. Consider the system

$$\dot{x} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + v,$$
$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x + e,$$
$$\Phi_e \equiv 1, \quad \Phi_v \equiv \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

We want to control the system using LQG, where the cost function to minimize is

$$\mathbb{E}\left[y^T Q y + u^T R u\right], \quad Q = 10, \quad R = 1$$

a. Determine the vector L in the feedback law $u = -L\hat{x}$. (3 p)

b. Determine the vector K in the Kalman filter equation

$$\dot{\hat{x}} = Ax + Bu + K(y - Cx)$$

Assume that e, v are uncorrelated white noise processes. (2 p)

6. Consider the system

$$\dot{x} = \begin{bmatrix} -1 & 0.1 \\ 0 & 0.1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + v$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + e$$

Assume that v, e are uncorrelated and white.

I. Intensity of $e = 1$. Intensity of $v = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

II. Intensity of $e = 1$. Intensity of $v = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$.

III. Intensity of $e = 100$. Intensity of $v = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

IV. Intensity of $e = 0.1$. Intensity of $v = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

a. Pair the noise models I-IV with the corresponding (noise free) initial state convergence for the Kalman filters in plots A-D in Figure 4 (1 p)

b. In which case is the Kalman-filter most sensitive to modelling errors in the system dynamics? (1 p)

c. In which case is the Kalman-filter estimate most sensitive to measurement outliers (measurements were our noise-model isn't correct) (1 p)

7. Consider the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix} u,$$
$$y = \begin{bmatrix} 0.1 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a. Is the system controllable and/or observable? (1 p)

b. Determine the Hankel singular values. (1 p)

c. Find a balanced realization. (1 p)

d. Find a first order approximative model using balanced truncation. (1 p)

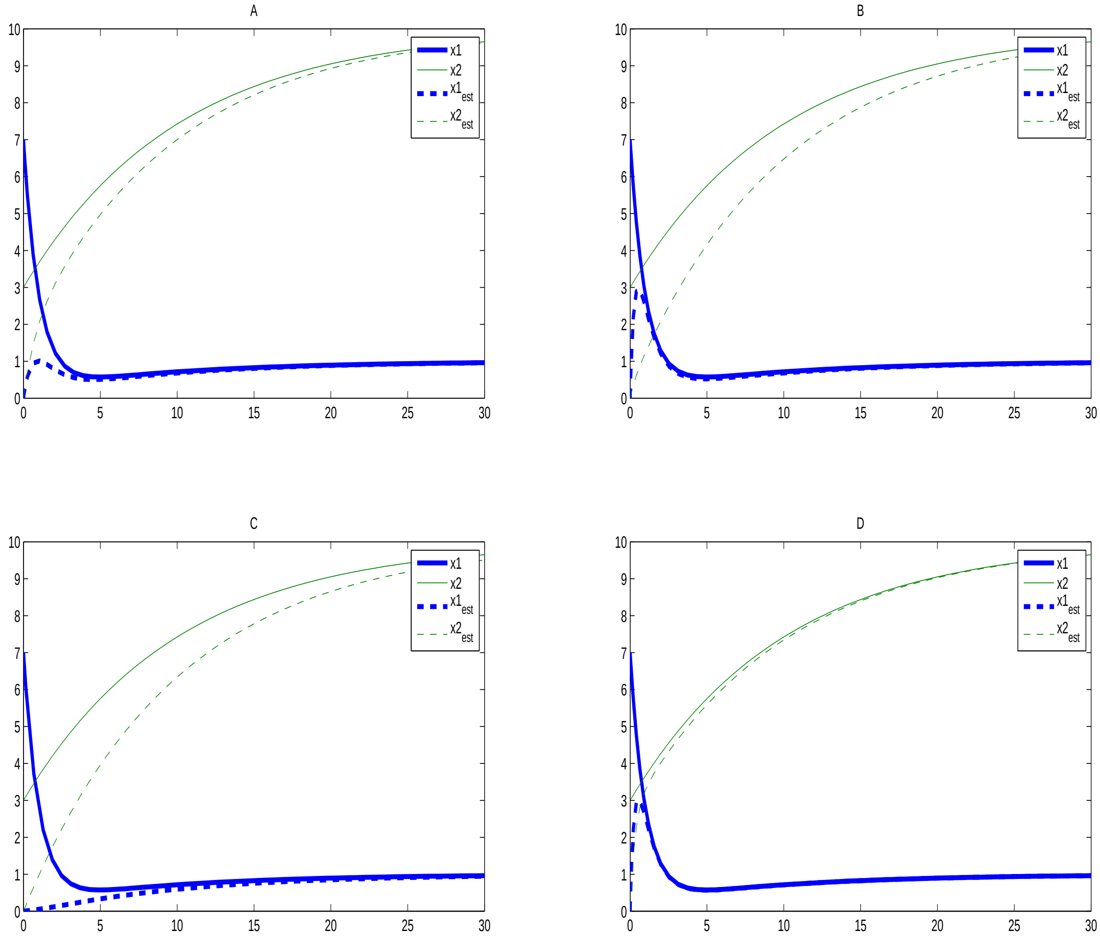

Figure 4 Initial state convergence of different Kalman filters for problem 6