



LUNDS  
UNIVERSITET

## Lecture 5

FRTN10 Multivariable Control

Automatic Control LTH, 2018





# Course Outline

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L1–L5 Specifications, models and loop-shaping by hand

- 1 Introduction
- 2 Stability and robustness
- 3 Specifications and disturbance models
- 4 Control synthesis in frequency domain
- 5 **Case study: DVD player**

L6–L8 Limitations on achievable performance

L9–L11 Controller optimization: analytic approach

L12–L14 Controller optimization: numerical approach

L15 Course review



# Loop shaping

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Controller synthesis via loop shaping: Shape the **open loop gain**  $L = PC$  so that

- $|L| > |W_S|$  for low frequencies (disturbance rejection)
- $|L| < |W_T^{-1}|$  for high frequencies (robustness, att. of meas. noise)
- good stability margins ( $\varphi_m$ ,  $A_m$ ,  $M_S$ ) are achieved

The controller  $C$  is typically composed of several factors:

- gain
- lag filters
- lead filters
- other filters (e.g., notch filter)



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# Lecture 5 – Outline

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- 1 Case study: Control of a DVD player
- 2 Review of cascade and midranging control



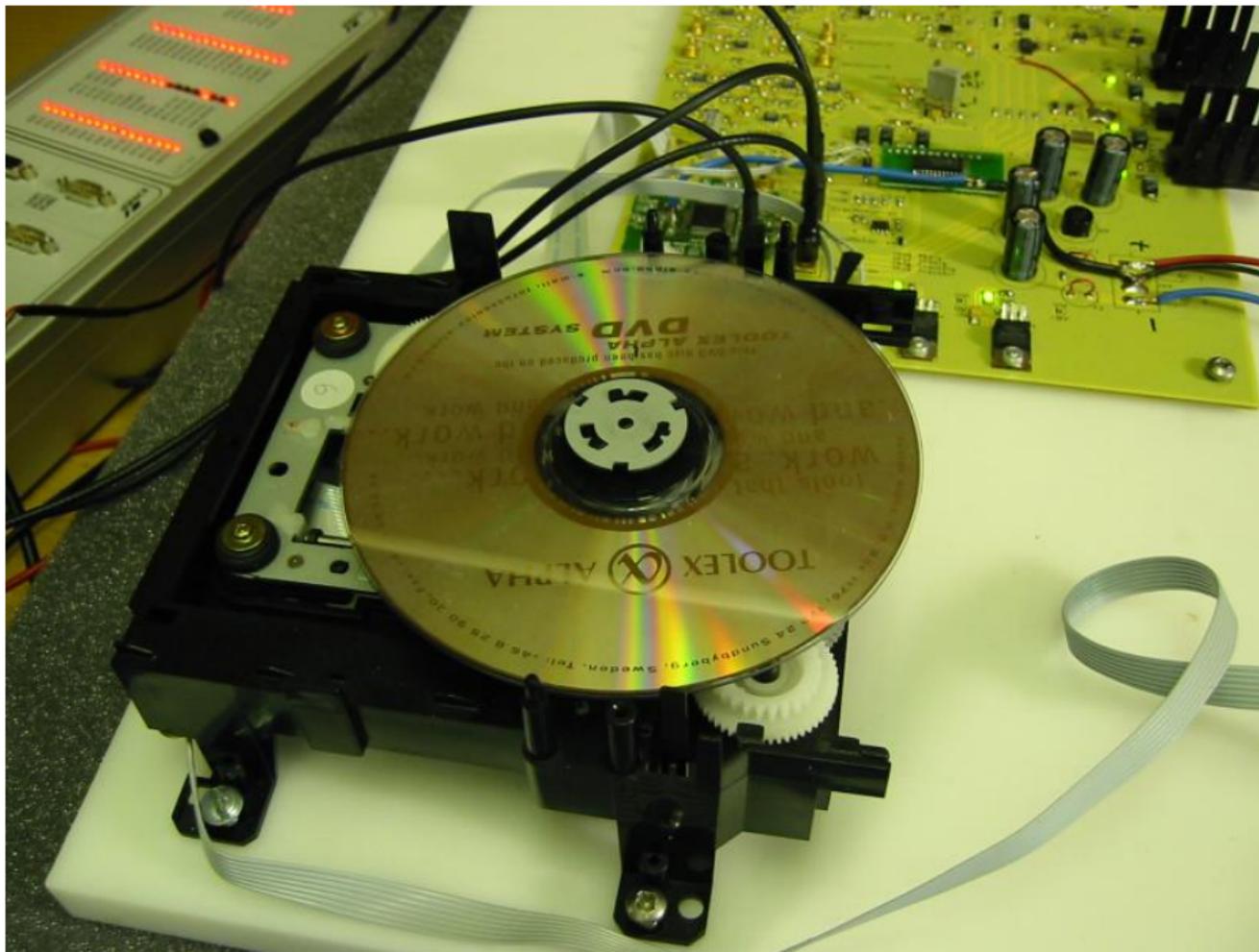
# Case Study: Control of a DVD player

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- The DVD player process
- Problem formulation
- Modeling
- Specifications
- Focus control loop shaping
- Radial control (track following)

Based on work by Bo Lincoln





# The DVD player tracking problem

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Scaled version of the control task in a DVD player:

- Imagine that you are traveling at half the speed of light, along a line from which you may only deviate 1 m
- The line is not straight but oscillates up to 4.5 km sideways up to 25 times per second

Good luck!



# The DVD player tracking problem

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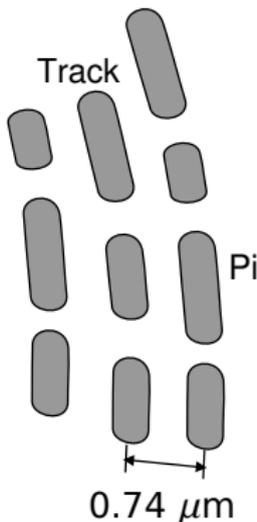
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Good luck!



# The DVD player tracking problem

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- 3.5 m/s speed along track
- 0.022  $\mu\text{m}$  tracking tolerance
- 100  $\mu\text{m}$  deviations at 10–25 Hz due to asymmetric discs

**DVD** Digital Versatile Disc, 4.7–8.5 GB

**CD** Compact Disc, 650–800 MB

**Blu-ray** 25–400 GB



## Can you see the laser spot?

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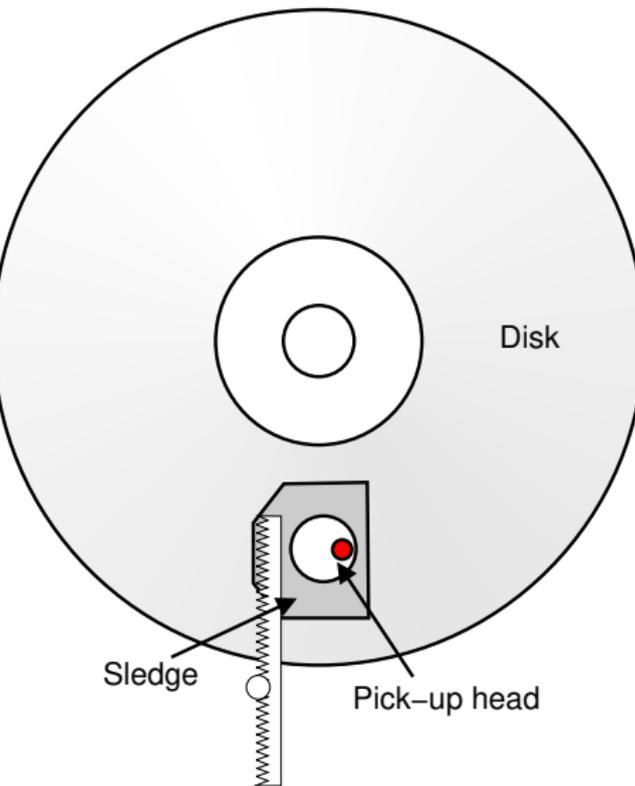


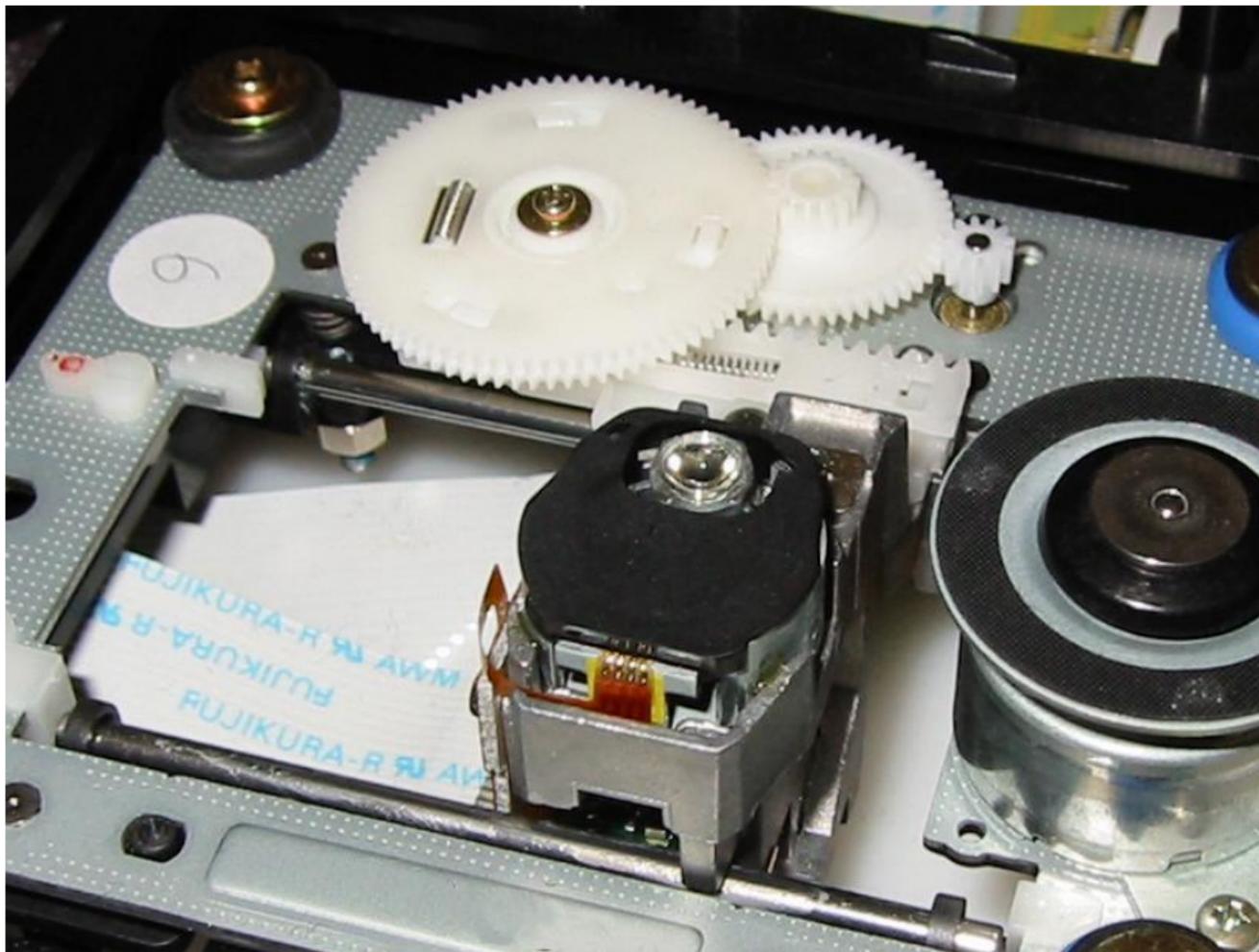


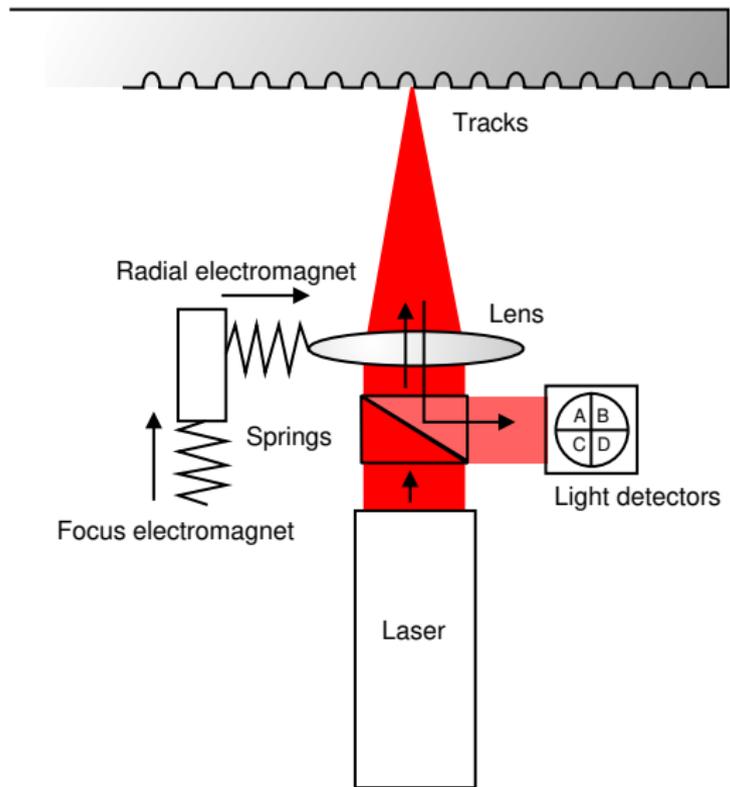


# The DVD Pick-Up Head

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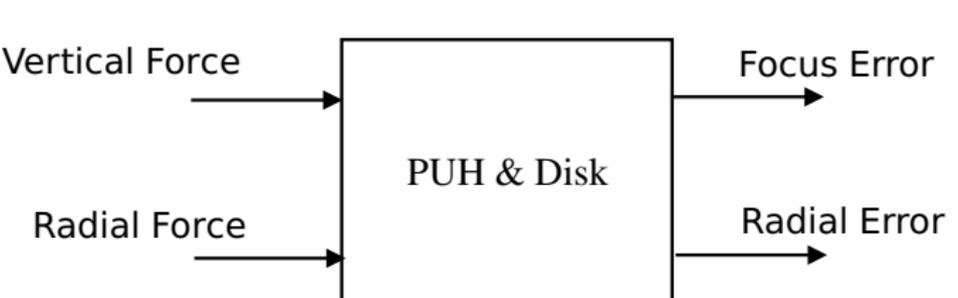






# Input-output diagram for DVD control

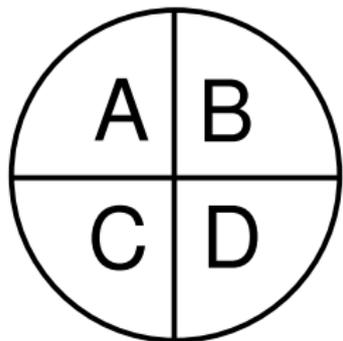
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## The four photo detectors

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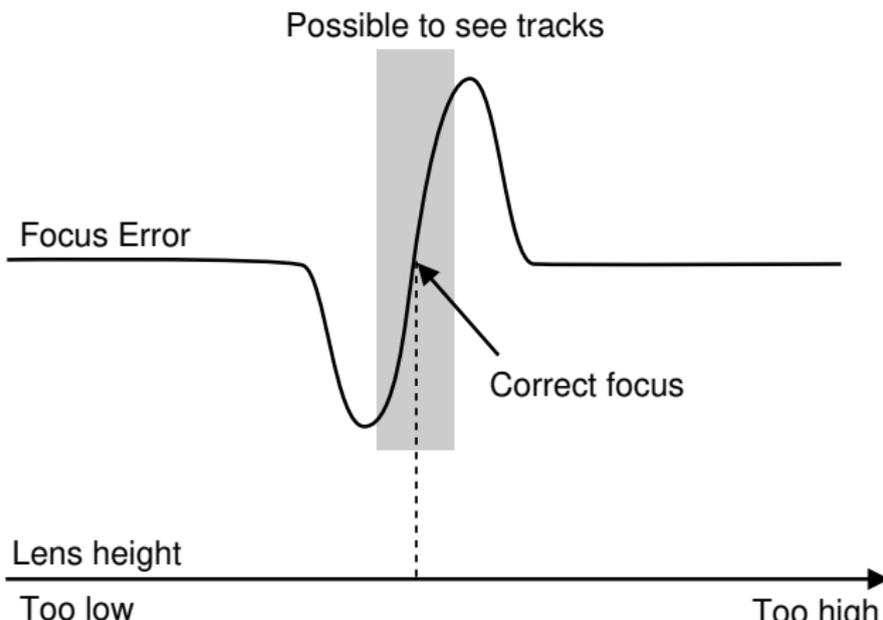


$$\text{focus error} = (A+D) - (B+C)$$

*Note: There are no other sensors in the pick-up head to help keep the laser in the track.*



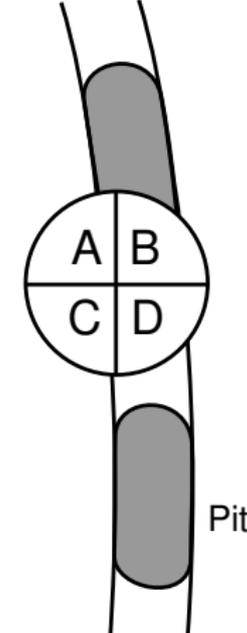
# Focus error signal





# Radial error by push-pull

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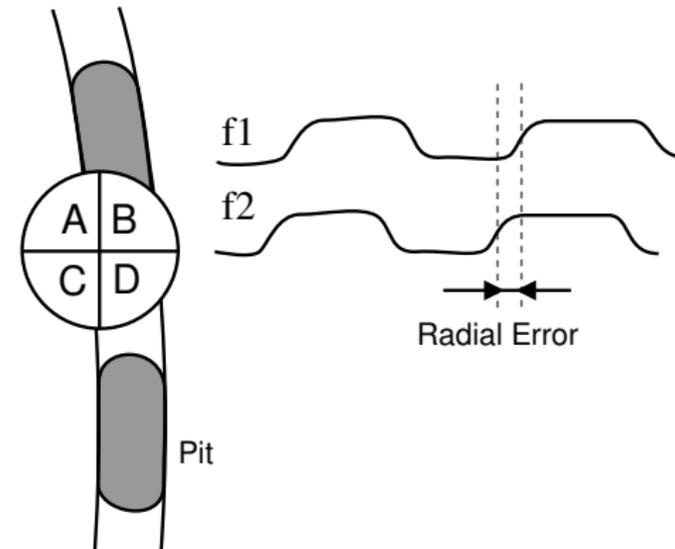


Look at

$$(A + C) - (B + D)$$



## Radial error by phase difference

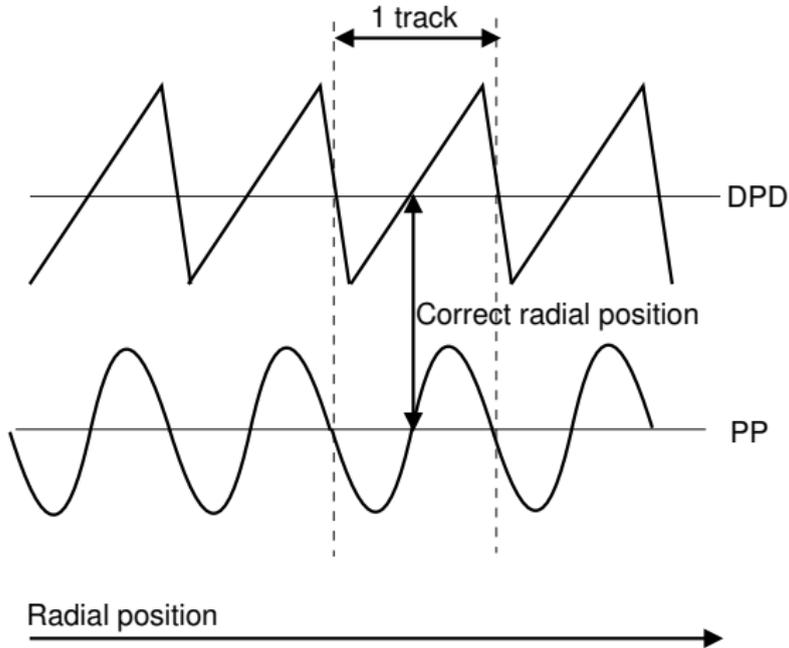


$$f_1 = A + D, \quad f_2 = B + C$$

Error signal  $RE$  created by time difference



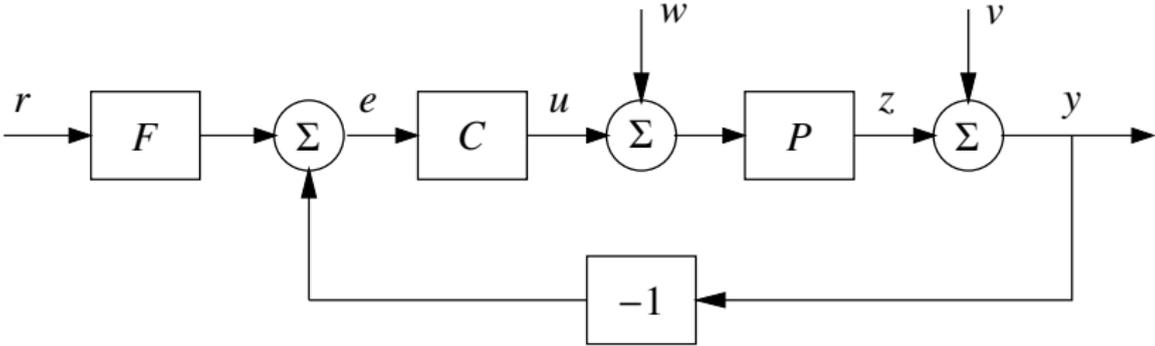
## Radial error signals



Note: Larger linear error region if using phase difference.



# Focus control design

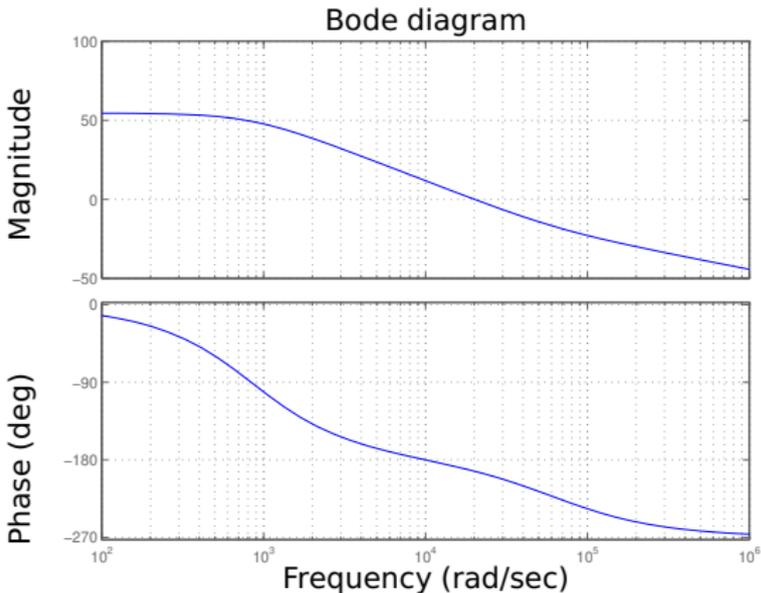


- What blocks and signals are relevant for focus control?
- What disturbances are there?



# Focus process model

Model obtained using system identification:



$$P(s) = 6092 \frac{63168 - s}{s^2 + 1553s + 718214}$$



# From DVD standard ECMA-267

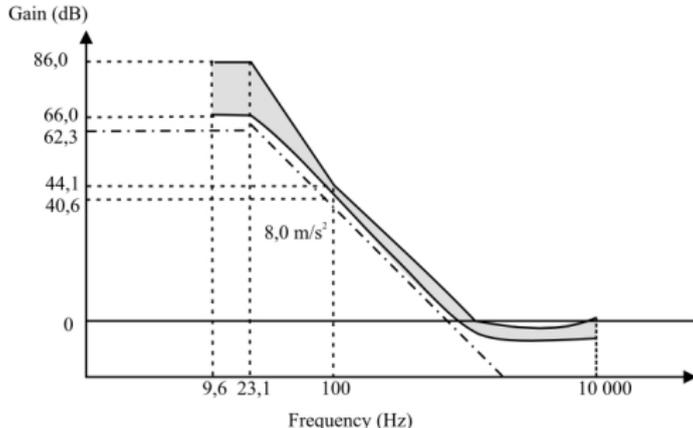


Figure 3 - Reference Servo for axial tracking

## Bandwidth 100 Hz to 10 kHz

$|1 + H|$  shall be within 20 % of  $|1 + H_S|$ .

The crossover frequency  $f_0 = \omega_0 / 2\pi$  shall be specified by equation (II), where  $\alpha_{\max}$  shall be 1,5 times larger than the expected maximum axial acceleration of  $8 \text{ m/s}^2$ . The tracking error  $e_{\max}$  shall not exceed  $0,23 \text{ }\mu\text{m}$ . Thus the crossover frequency  $f_0$  shall be

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{3 \alpha_{\max}}{e_{\max}}} = \frac{1}{2\pi} \sqrt{\frac{8 \times 1,5 \times 3}{0,23 \times 10^{-6}}} = 2,0 \text{ kHz} \quad (\text{II})$$

<http://www.ecma-international.org/publications/standards/Ecma-267.htm>



# Specifications

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- Cancel disturbances due to disc asymmetry

$$|P(i\omega)C(i\omega)| \geq 2000 \quad \text{for } f \leq 23 \text{ Hz}$$

- Robustness towards model errors, rejection of meas. noise

$$|P(i\omega)C(i\omega)| \leq 1 \quad \text{for } f > 2 \text{ kHz}$$

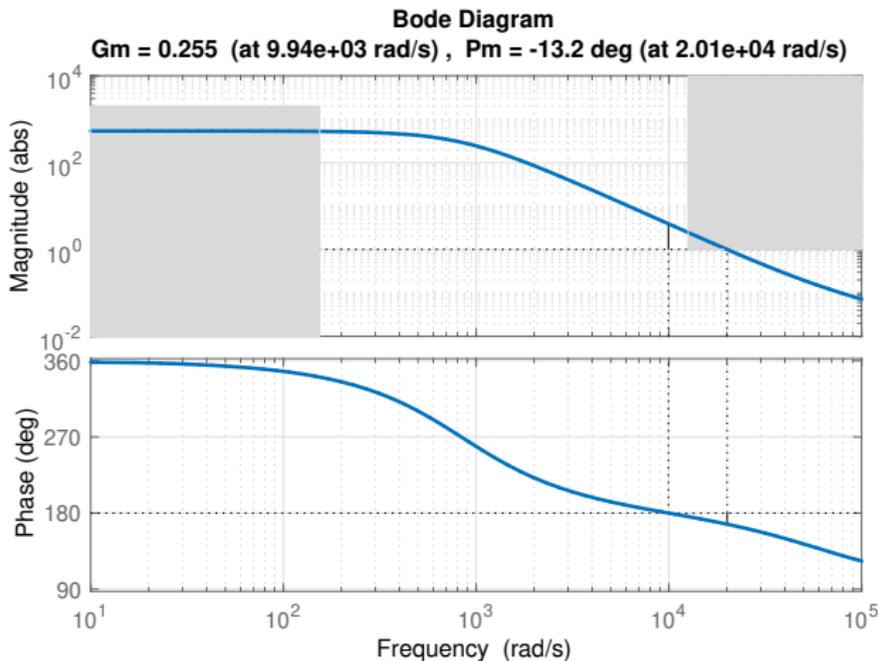
(Compare to the bit rate, which is in the order of 1 MHz)

- Good stability margins



# Open-Loop System

Bode plot of  $P(s)$  with stability margins and specifications:

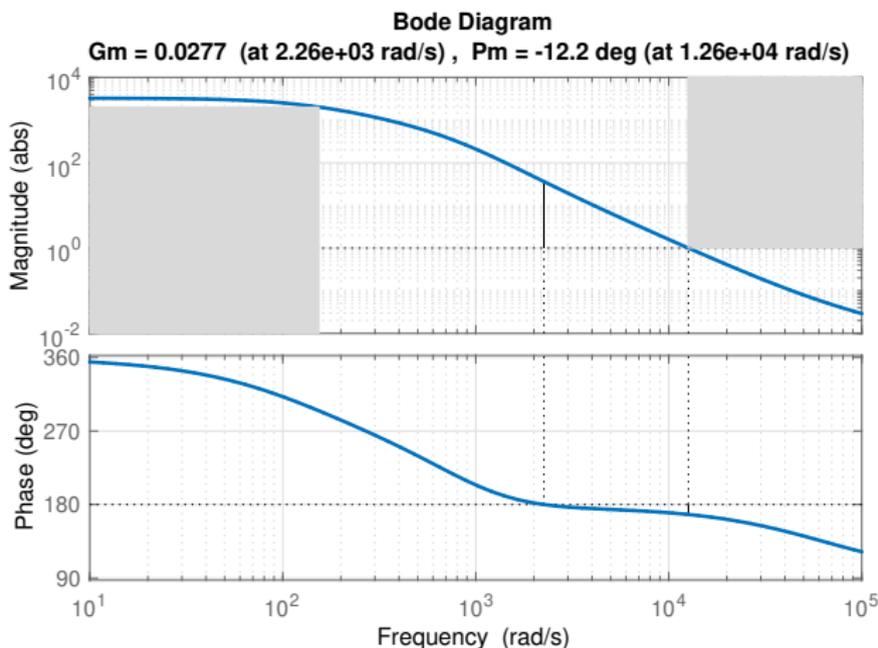


Can a P-controller solve the problem?



# Add Lag Compensator

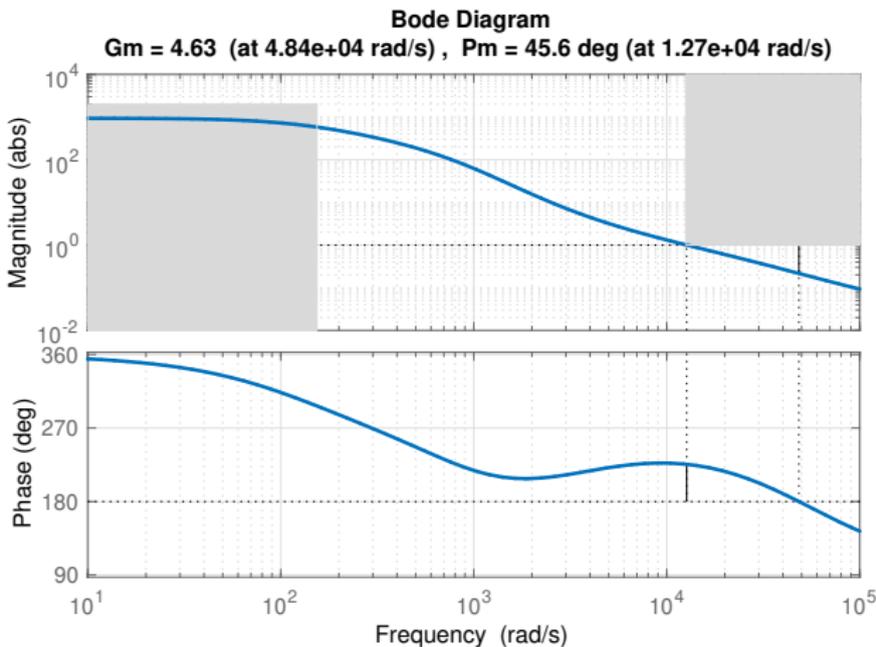
Use lag filter with  $M = 15$  to increase gain below 23 Hz. The break point needs to be well below 2 kHz in order to avoid excessive phase lag at the cross-over frequency:  $C = KC_{lag} = \frac{0.4037(s+1885)}{s+125.7}$





# Add Lead Compensator

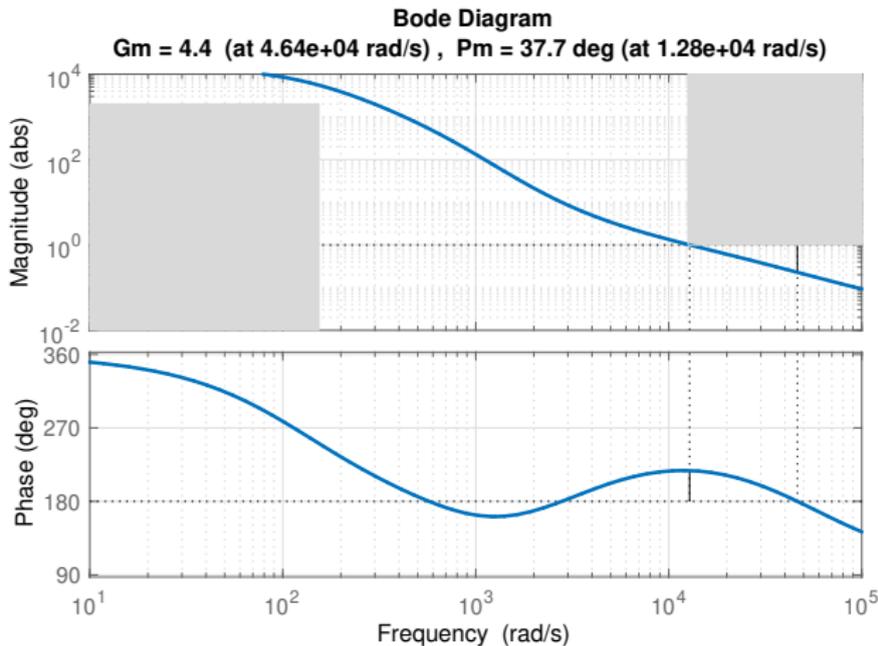
Use lead filter with  $N = 12$  to increase phase by  $57^\circ$  at cross-over frequency.  $C = KC_{lag}C_{lead} = \frac{1.398(s+1885)(s+3228)}{(s+125.7)(s+43530)}$





# Add Another Lag Compensator

Low-frequency gain too low. Add another lag compensator with same parameters:  $C = KC_{lag}^2 C_{lead} = \frac{1.398(s+1885)^2(s+3628)}{(s+125.7)^2(s+43530)}$

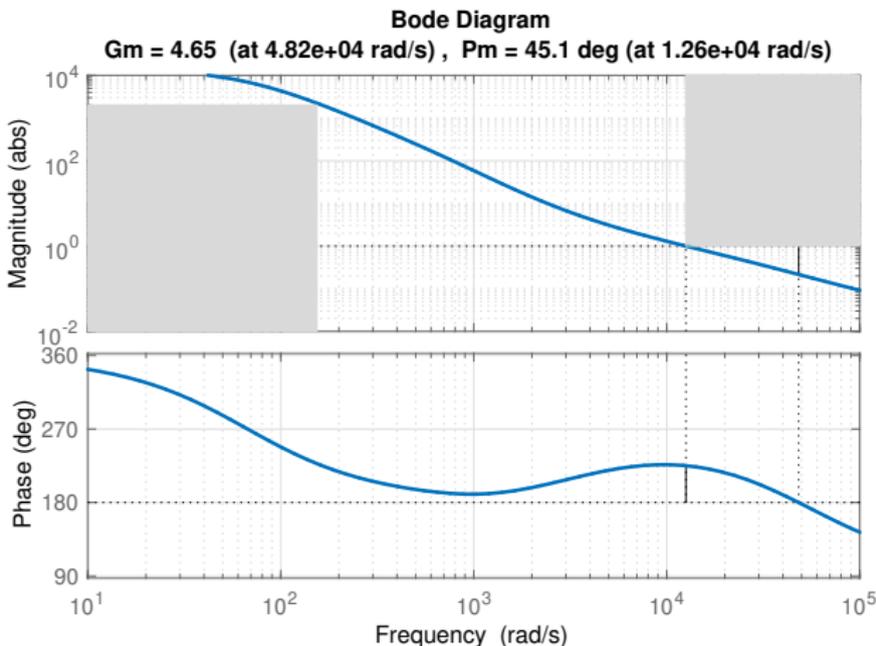




# Final Adjustments

Phase margin too small again. Lower the break frequency of the lag filters to recover some phase:

$$C = KC_{lag}^2 C_{lead} = \frac{1.397(s+1005)^2(s+3628)}{(s+67.02)^2(s+43530)}$$

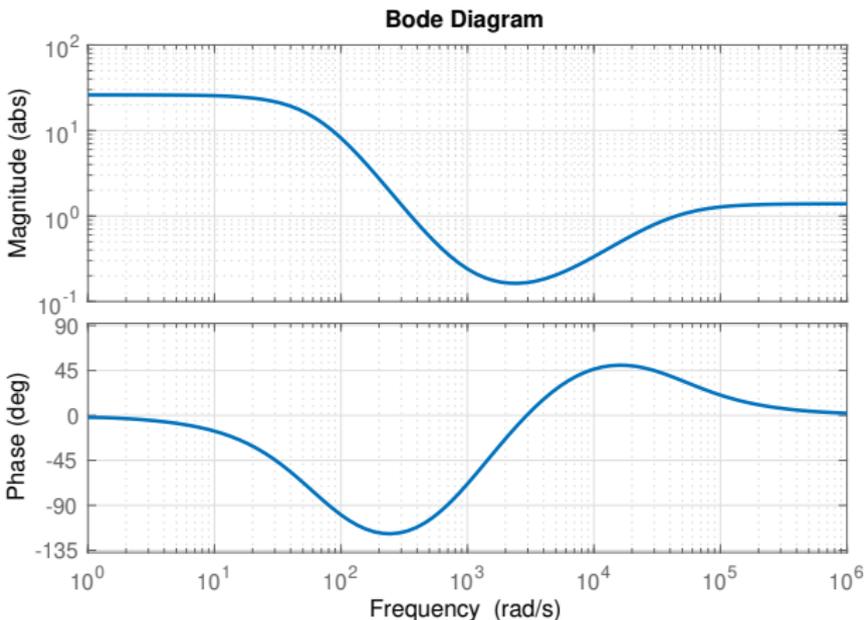




# Final Controller

Bode diagram of final controller

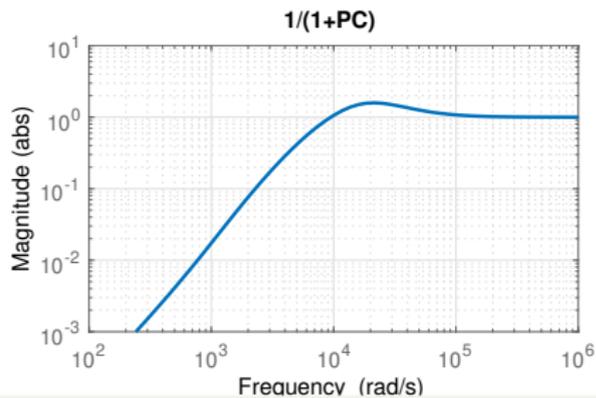
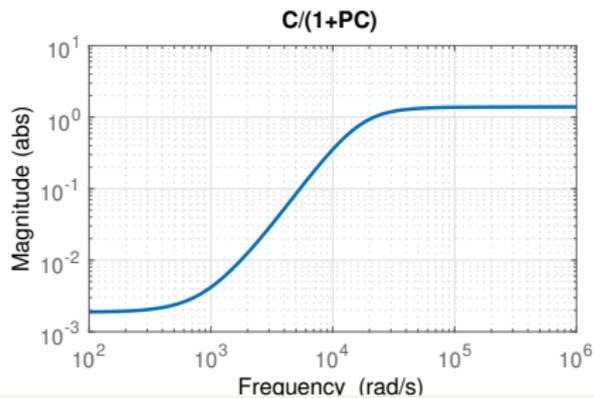
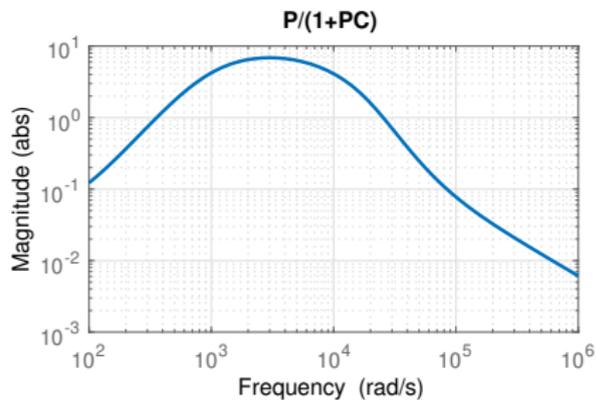
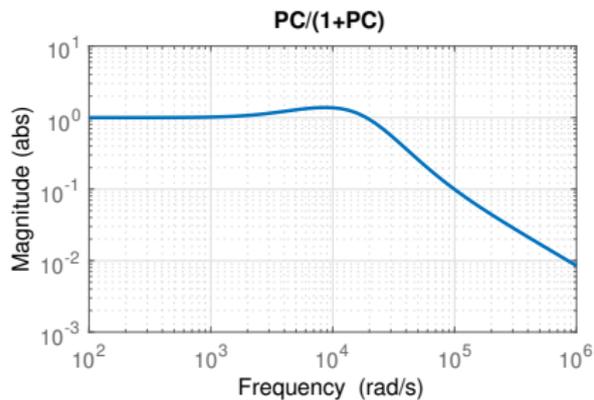
$$C = KC_{lag}^2 C_{lead} = \frac{1.397(s+1005)^2(s+3628)}{(s+67.02)^2(s+43530)}$$



(Could add another pole to have high-frequency roll-off)



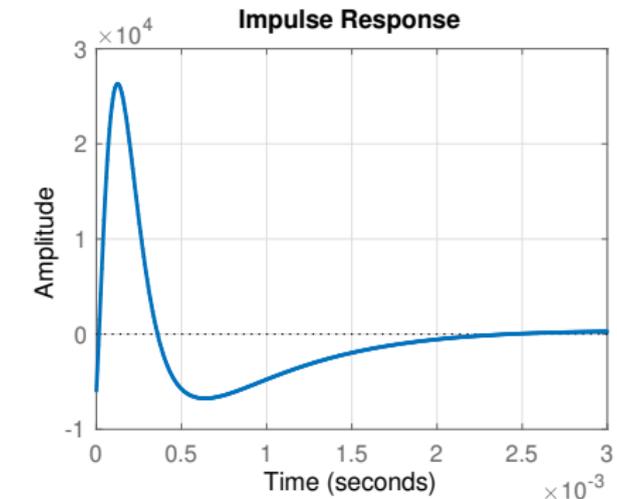
# Gang of Four





# Response to impulse load disturbance

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# Radial control

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Make the laser follow the track by moving “sideways”/radially

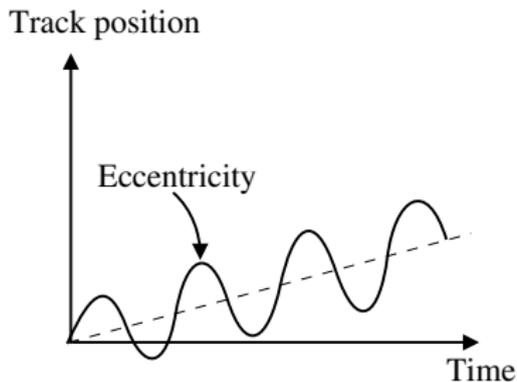
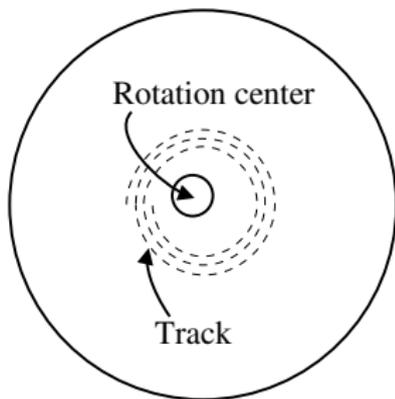
It is essential to solve the Focus control problem first

Tracking via two parallel actuators (midranging):

- Move lens (electromagnet/fast motion)
- Move sledge (slow/large range)

Disturbances:

- eccentricity (up to 100 tracks in one rotation)
- physical vibrations of DVD player
- noise, dirt, etc.

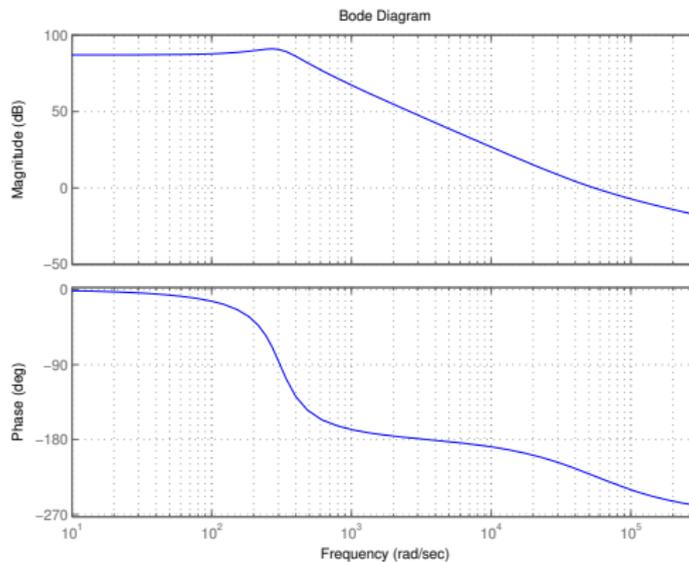


The disc is often a bit eccentric (i.e. not rotating around the track center). The resulting track position, which the Pick-Up-Head has to follow, is sinus-like.



# Experimental radial dynamics model

An estimated transfer function for the radial servo (from the control signal  $u$  to the radial error  $RE$ )



System identification made by sinusoidal excitation.



## From DVD standard ECMA-267

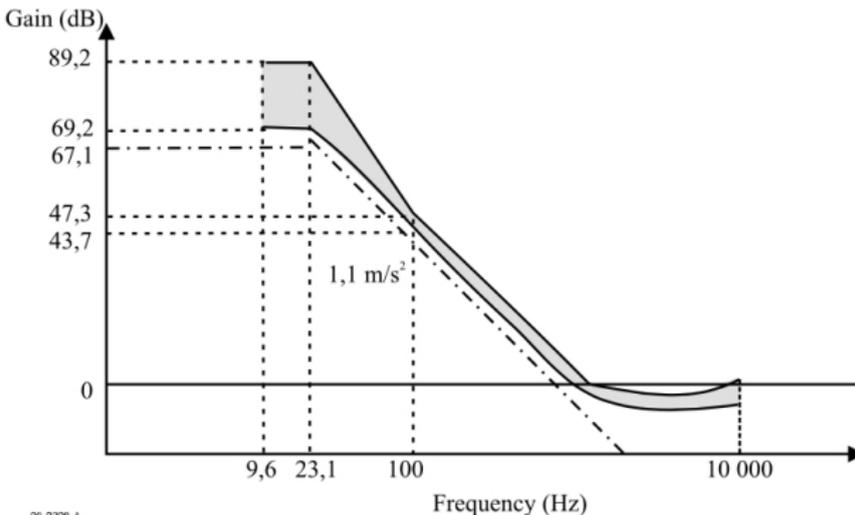


Figure 4 - Reference Servo for Radial Tracking

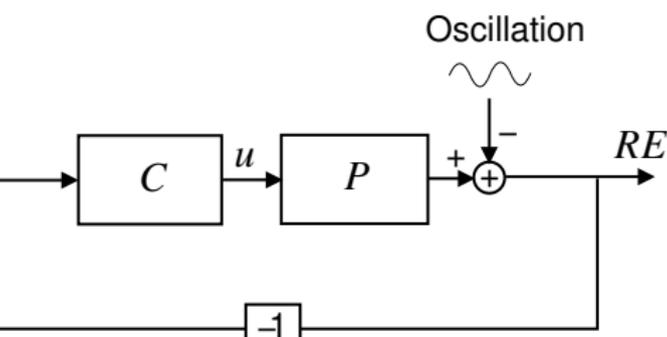
Similar requirements as for the axial (focus) tracking

Many possible design methods (loop shaping, pole placement, LQG)



# Problem with sinusoidal output disturbance

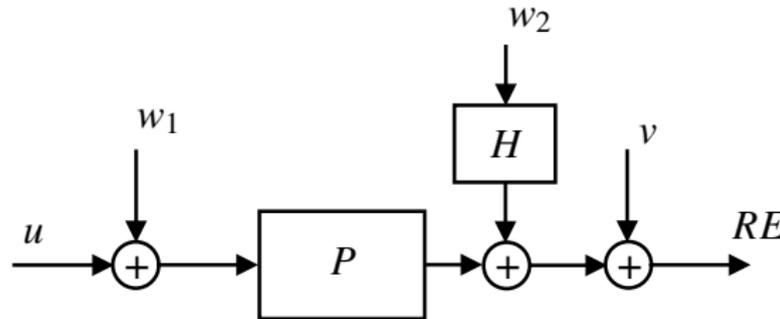
The eccentricity causes problems (at about 10–25 Hz and magnitude up to 100 tracks). Cannot be exactly modeled due to uncertainty.





# Stochastic disturbance modeling

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Noise model: There is both white process noise  $w_1$ , and a track offset, which is modeled as the white noise  $w_2$  through a filter  $H$ .

The filter  $H$  should have a high gain in the frequency range where the oscillation acts (bandpass filter)

Kalman filter + state feedback then solves the problem elegantly



## Further reading

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- Lecture notes on course web page
- "Sensing and Control in Optical Drives – How to Read Data from a Clear Disc" by Amir H. Chaghajerdi, June 2008, *IEEE Control Systems Magazine*, pp. 23–29,

<http://www.ieeecss.org/CSM/library/2008/june08/11-June08ApplicationsOfControl.pdf>



# Lecture 5 – Outline

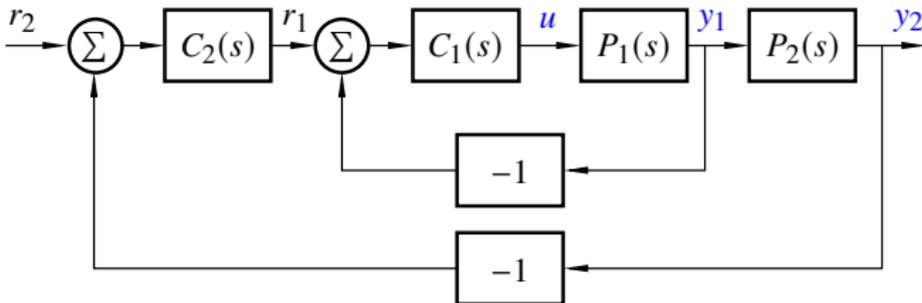
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- 1 Case study: Control of a DVD player
- 2 Review of cascade and midranging control



# Cascade control

For systems with one control signal and two measurement signals:

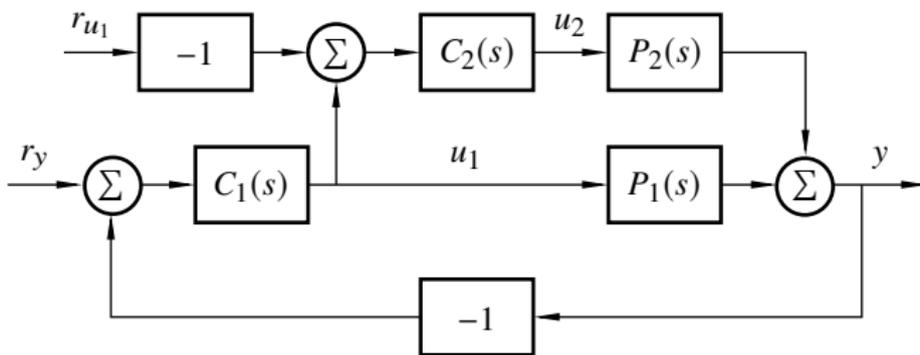


- $C_1(s)$  controls the subsystem  $P_1(s)$ 
  - Fast inner loop,  $G_{y_1 r_1}(s) \approx 1$
- $C_2(s)$  controls the subsystem  $P_2(s)$ 
  - Slow outer loop



## Midranging Control

For systems with one measurement signal and two control signals (e.g. one large-range/slow and one small-range/fast actuator)

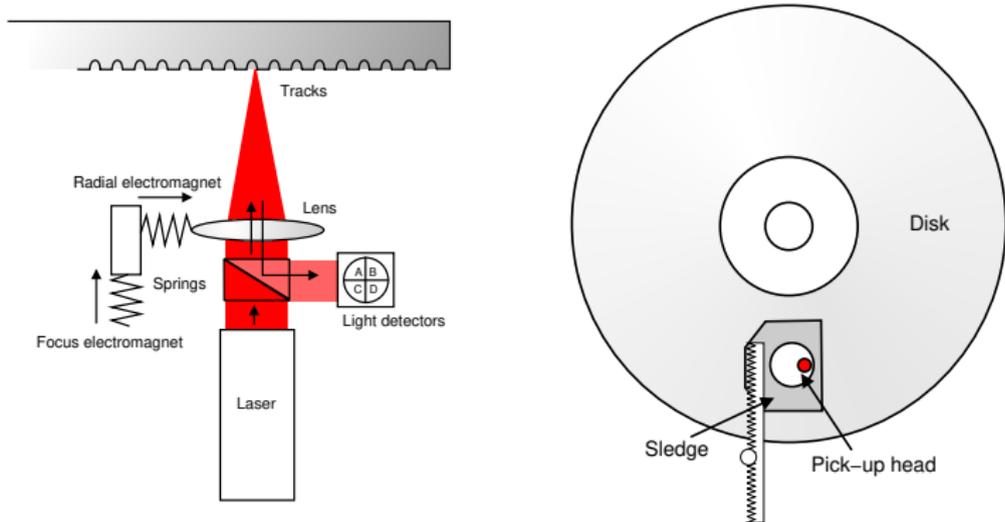


- $C_1(s)$  controls the process output  $y$  with fast actuator  $u_1$
- $C_2(s)$  controls  $u_1$  to the middle of its operating range using slow actuator  $u_2$  (note reverse gain)



# Midranging Control – Example

Radial control of pick-up-head of DVD player



The pick-up-head has two electromagnets for fast positioning of the lens (left). Larger radial movements are taken care of by the sledge (right).