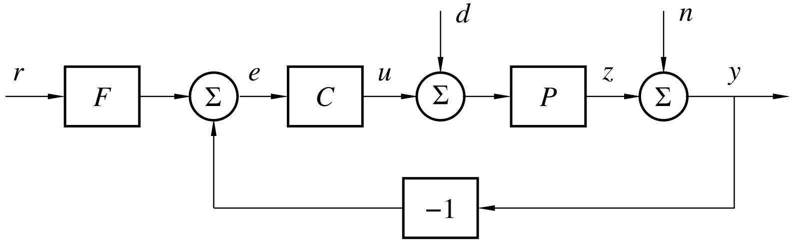
Lecture 4 – Outline

1. Frequency domain specifications
2. Loop shaping
3. Feedforward design

Relations between signals



$$Z = \frac{P}{1+PC}D - \frac{PC}{1+PC}N + \frac{PCF}{1+PC}R$$
$$Y = \frac{P}{1+PC}D + \frac{1}{1+PC}N + \frac{PCF}{1+PC}R$$
$$U = -\frac{PC}{1+PC}D - \frac{C}{1+PC}N + \frac{CF}{1+PC}R$$

Design specifications

Find a controller that

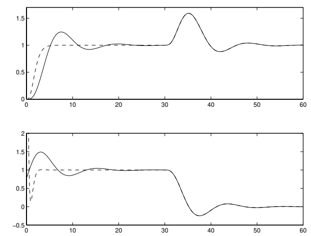
A: reduces the effect of load disturbances
B: does not inject too much measurement noise into the system
C: makes the closed loop insensitive to process variations
D: makes the output follow the setpoint

Common to have a controller with **two degrees of freedom** (2 DOF), i.e. separate signal transmission from y to u and from r to u. This gives a nice separation of the design problem:

1. Design feedback to deal with A, B, and C
2. Design feedforward to deal with D

Time-domain specifications

- Specifications for deterministic signals, e.g., step response w.r.t. reference change, load disturbance
  - Rise-time T_r
  - Overshoot M
  - Settling time T_s
  - Static error e_0
  - ...



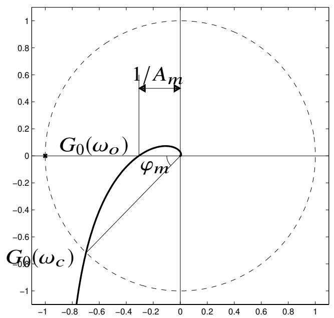
reference step disturbance step

- Stochastic specifications, e.g.,
  - Process output variance
  - Control signal variance

Frequency-domain specifications

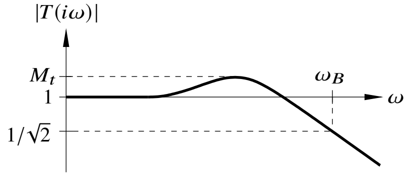
Open-loop specifications (for loop gain $G_0 = L = PC$)

- cross-over frequency ω_c
- phase margin φ_m
- amplitude margin A_m
- ...



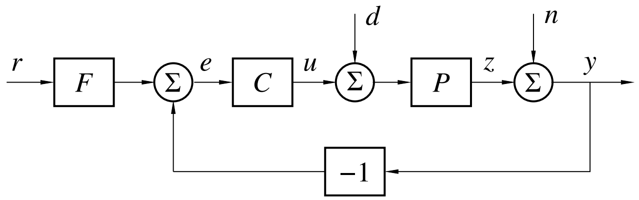
Closed-loop specifications, e.g.

- maximum sensitivity M_s
- resonance peak M_t
- closed-loop bandwidth ω_B
- ...



Frequency domain specifications

Closed-loop specifications, cont'd:



Desired properties:

- Small influence of load disturbance d on z $\Leftrightarrow$ $PS \approx 0$
- Limited amplification of noise n in control u $\Leftrightarrow$ $CS \approx 0$
- Small influence of model errors on z $\Leftrightarrow$ $S \approx 0$
- Robust stability despite model errors $\Leftrightarrow$ $T \approx 0$
- Accurate tracking of setpoint r $\Leftrightarrow$ $TF \approx 1$

Frequency domain specifications

$S + T = 1$ and other constraints makes the above impossible to achieve at all frequencies.

Typical design compromise:

- $T \to 0$ for high frequencies ($\omega > \omega_B$)
- $S \to 0$ for low frequencies (+ possibly other disturbance dominated frequencies)

Expressing specifications on S and T

Maximum sensitivity specifications:

- $\|S\|_\infty \le M_s$
- $\|T\|_\infty \le M_t$

Frequency-weighted specifications:

- $\|W_S S\|_\infty \le 1 \quad \Leftrightarrow \quad |S(i\omega)| \le |W_S^{-1}(i\omega)|,\ \forall\omega$
- $\|W_T T\|_\infty \le 1 \quad \Leftrightarrow \quad |T(i\omega)| \le |W_T^{-1}(i\omega)|,\ \forall\omega$

where $W_S(s)$ and $W_T(s)$ are some weighting functions

Loop shaping

Idea: Look at the **loop gain** $L = PC$ for design and translate specifications on S and T into specifications on L

$$S = \frac{1}{1+L} \approx \frac{1}{L} \qquad \text{if } L \text{ is large}$$

$$T = \frac{L}{1+L} \approx L \qquad \text{if } L \text{ is small}$$

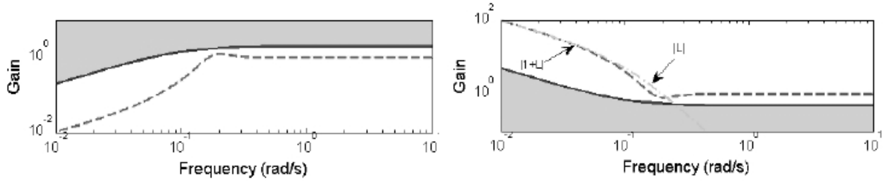
Classical loop shaping: Design C so that $L = PC$ satisfies specifications on S and T

- how are the specifications related?
- what to do with the region around cross-over frequency ω_c (where $|L| \approx 1$)?

Sensitivity vs loop gain

$$S = \frac{1}{1+L}$$

$$|S(i\omega)| \le |W_S^{-1}(i\omega)| \quad \Leftrightarrow \quad |1 + L(i\omega)| > |W_S(i\omega)|$$



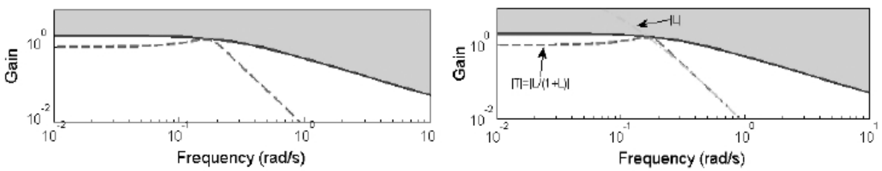
For small frequencies, W_S large $\Rightarrow$ $1 + L$ large and $|L| \approx |1 + L|$.

$$|L(i\omega)| \ge |W_S(i\omega)| \quad \text{(approx.)}$$

Complementary sensitivity vs loop gain

$$T = \frac{L}{1+L}$$

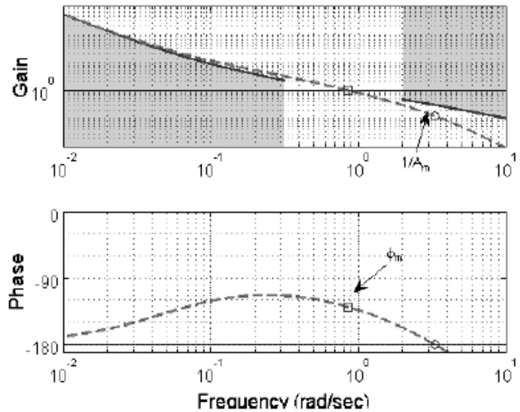
$$|T(i\omega)| \le |W_T^{-1}(i\omega)| \quad \Leftrightarrow \quad \frac{|L(i\omega)|}{|1 + L(i\omega)|} \le |W_T^{-1}(i\omega)|$$



For large frequencies, $W_T^{-1} \approx 0 \quad \Rightarrow \quad |T| \approx |L|$

$$|L(i\omega)| \le |W_T^{-1}(i\omega)| \quad \text{(approx.)}$$

Resulting constraints on loop gain L:



Approximations are inexact around cross-over frequency ω_c. In this region, focus is on stability margins (A_m, φ_m)

Lead–lag compensation

Shape the loop gain $L = PC$ using a compensator $C = C_1C_2C_3\ldots$ composed of various elements, such as

- gain

$$K$$

- lag (phase retarding) elements

$$C_{lag}(s) = \frac{s+a}{s+a/M}, \quad M > 1$$

- lead (phase advancing) elements

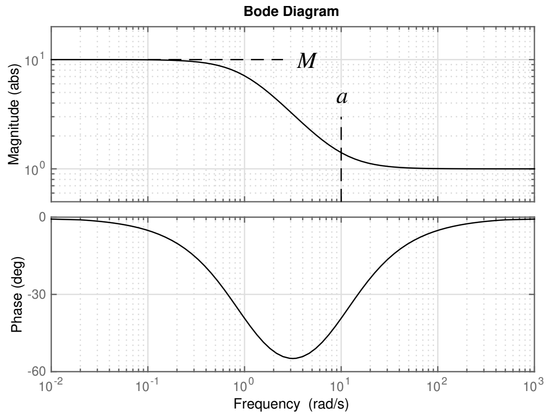
$$C_{lead}(s) = N\frac{s+b}{s+bN}, \quad N > 1$$

Example:

$$C(s) = K\frac{s+a}{s+a/M} \cdot N\frac{s+b}{s+bN}$$

Lag filter

$$C_{lag}(s) = \frac{s+a}{s+a/M}, \quad M > 1$$

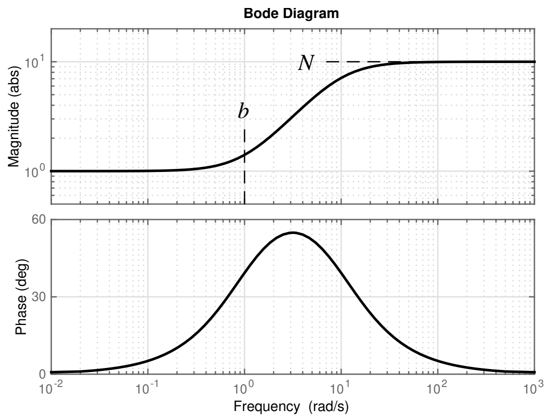


Lag filter

- Increases low-frequency loop gain by factor M
  - $M = \infty \implies$ PI controller
  - Reduces static error by factor M if $L(s)$ contains an integrator
- Break frequency a should be as high as possible for fast disturbance rejection, but too high a reduces stability margins
  - Rule of thumb $a = 0.1\omega_c$ guarantees that φ_m is reduced less than 6°

Lead filter

$$G_{lead}(s) = N\frac{s+b}{s+bN}, \quad N > 1$$



Lead filter

- Increases phase by amount that depends on N (see Collection of Formulae), maximum phase lead at $\omega = b\sqrt{N}$
  - Typically placed at desired cross-over frequency ω_c
- Gain at $b\sqrt{N}$ increases by $\sqrt{N}$. To retain the same cross-over frequency, the overall controller gain must be decreased

Iterative lead–lag design

Typical workflow:

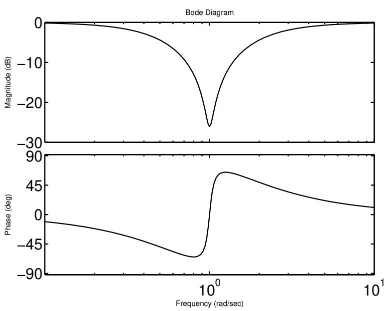
- Adjust gain to obtain the desired cross-over frequency
- Add lag element to improve the low-frequency gain
- Add lead element to improve the phase margin

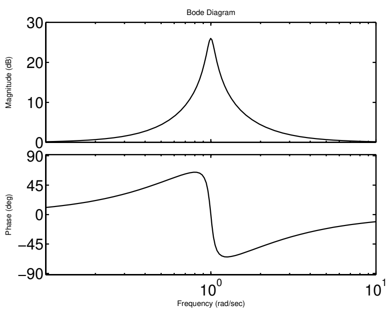
Adding a lead element while retaining the cross-over frequency affects the low-frequency gain

Need to iterate!

Links with complex poles/zeros

Example (notch/resonance filters): $\dfrac{s^2 + 2\zeta_a \omega_0 s + \omega_0^2}{s^2 + 2\zeta_b \omega_0 s + \omega_0^2}$



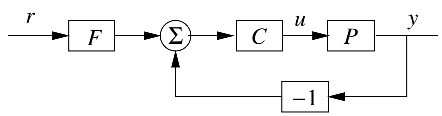


$\omega_0 = 1,\ \zeta_a = 0.05,\ \zeta_b = 1$ $\omega_0 = 1,\ \zeta_a = 1,\ \zeta_b = 0.05$

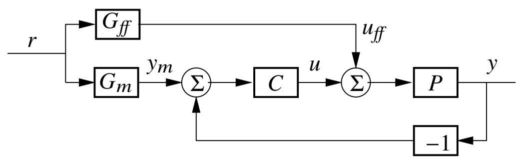
Feedforward design

Two common 2-DOF configurations:

(1)

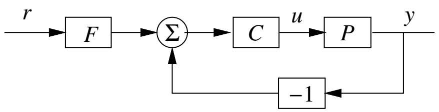


(2)



Ideally, we would like the output to follow the setpoint perfectly, i.e. $y = r$

Feedforward design (1)



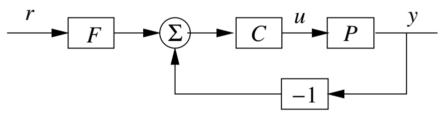
Perfect following requires

$$F = \frac{1 + PC}{PC} = T^{-1}$$

In general impossible because of pole excess in T. Also

- T might contain non-minimum-phase factors that can/should not be inverted
- u must typically satisfy some upper and lower limits

Feedforward design (1)

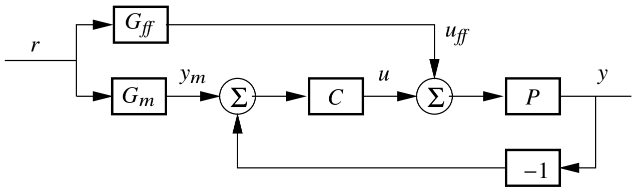


Assume T minimum phase. An implementable choice of F is then

$$F(s) = \frac{1 + P(s)C(s)}{P(s)C(s)(sT_f + 1)^d}$$

where d is large enough to make F proper

Feedforward design (2)

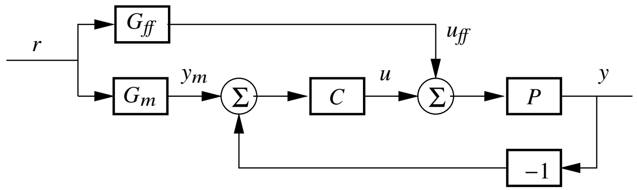


G_m and G_{ff} can be viewed as generators of the desired output y_m and the feedforward u_{ff} that corresponds to y_m

For y to follow y_m, select

$$G_{ff} = G_m / P$$

Feedforward design (2)



Since $G_{ff} = G_m / P$ should be stable, causal and proper we find that

- Unstable zeros of P must be included in G_m
- Time delays of P must be included in G_m
- The pole excess of G_m must not be smaller than the pole excess of P

Take process limitations into account!

Feedforward design – example

Process:

$$P(s) = \frac{1}{(s+1)^4}$$

Selected reference model:

$$G_m(s) = \frac{1}{(sT_m+1)^4}$$

Then

$$G_{ff}(s) = \frac{G_m(s)}{P(s)} = \frac{(s+1)^4}{(sT_m+1)^4} \qquad G_\infty(\infty) = \frac{1}{T_m^4}$$

Fast response (small T_m) requires high gain in G_{ff}.

Bounds on the control signal limit how fast response we can obtain in practice

Lecture 4 – summary

Frequency domain design:

- Good mapping between S, T and $L = PC$ at low and high frequencies (mapping around cross-over frequency less clear)
- Simple relation between C and L $\Rightarrow$ “easy” to shape L
- Lead–lag design: iterative procedure

Feedforward design

- Must respect unstable zeros, time delays and pole excess of plant