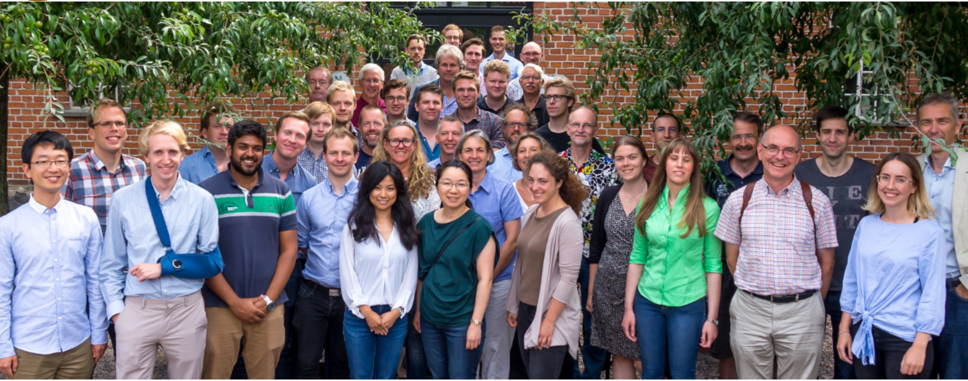
Lunds Universitet

Welcome to

FRTN10 Multivariable Control

Anton Cervin

Department of Automatic Control



- Founded 1965 by Karl Johan Åström (IEEE Medal of Honor)
- Approx. 45 employees
- Education for B, BME, C, D, E, F, I, K, M, N, Pi, W
- Research in autonomous systems, distributed control, robotics, cloud control, automotive systems, . . .

Lecture 1 – Outline

1. Course program
2. Course introduction
3. Signals and systems

Administration

Anton Cervin

Course responsible and lecturer

anton@control.lth.se

046-222 44 75

M:5145

Mika Nishimura

Course administrator

mika@control.lth.se

046-222 87 85

M:5141

Prerequisites

FRT010 Automatic Control, Basie Course or FRTN25 Automatic Process Control is required prior knowledge.

It is assumed that you have taken the basic courses in mathematics, including linear algebra and calculus in several variables, and preferably also a course in systems & transforms.

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NEW

New Exercise 1 that reviews the Control Basics:

- System representations
- Bode diagrams
- Block diagrams
- Stability

Course material

The course material is available on the homepage:

`http://www.control.lth.se/course/FRTN10`

- Lecture slides (handed out, allowed on the exam)
- Lecture notes (being completed this year)
- Exercise problems with solutions
- Laboratory assignments

Course material

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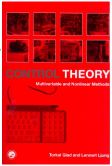
- Exercise problems with solutions
- Laboratory assignments

Optional reading:

- Glad & Ljung: Glad & Ljung: *Control Theory: Multivariable and Nonlinear Methods*, Taylor & Francis
  (Available as e-book through Lund University Libraries)



Lectures

The lectures (30 hours in total) are given by Anton Cervin on Mondays, Tuesdays, and Thursdays.

See the LTH schedule generator for details.

Exercise sessions and TAs

The exercise sessions (28 hours in total) are arranged in two groups (free choice):

Group	*Times*	*Room*
1	Wed 10–12, Fri 10–12	M:M1 (Exercise) or Lab A (Comp. exercise)
2	Wed 13–15, Fri 13–15	M:M1 (Exercise) or Lab A (Comp. exercise)

Hamed Sedaghi

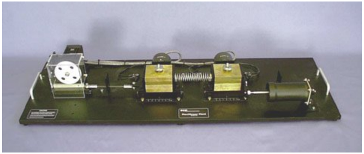
Martin Heyden

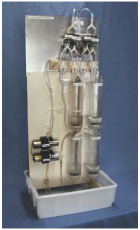
Martin Morin

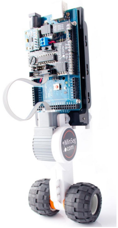
Laboratory experiments

The three laboratory sessions (12 hours in total) are mandatory. Links to the booking system (SAM) will be posted on the course homepage. You must sign up before the first session starts. Before each session there are pre-lab assignments that must be completed. No reports are required afterwards.

Lab	*Weeks*	*Booking*	*Room*	*Responsible*	*Process*
1	38–39	Sep 6	Lab C	Hamed Sedaghi	Flexible linear servo
2	39–40	Sep 17	Lab C	Martin Heyden	Quadruple tank
3	41–42	Sep 27	Lab B	Martin Morin	MinSeg (NEW)







Exam

The exam is given on Saturday October 27 at 8:00–13:00.

Retake exams are offered in April and August, 2019.

Lecture slides (with markings/small notes) are allowed on the exam. You may also bring *Automatic Control—Collection of Formulae*, standard mathematical tables (TEFYMA), and a pocket calculator.

Matlab

Matlab is used in laboratory sessions as well as in the five computer exercise sessions

- Control System Toolbox
- Simulink
- CVX (`http://cvxr.com/cvx`), used in exercise session 12

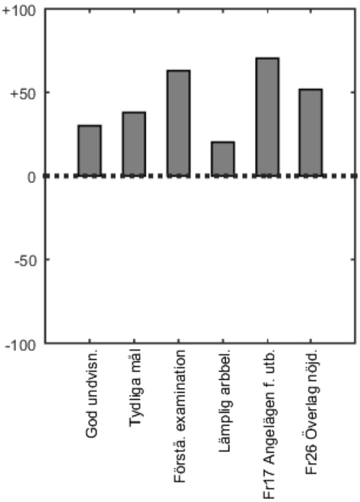
Feedback and Q&A

Feedback mechanisms for improving the course:

- CEQ (reporting / longer time scale)
- Student representatives (short time scale)
  - Election of student representatives ("kursombud")
- Mid-course evaluation

Two weeks before the exam we open up a Piazza site for Q&A

CEQ 2017


+100
+50
0
-50
-100
God undvisn.
Tydliga mål
Förstå. examination
Lämplig arbbel.
Fr17 Angelägen f. utb.
Fr26 Överlag nöjd.

Course development

Major improvements in the 2018 version:

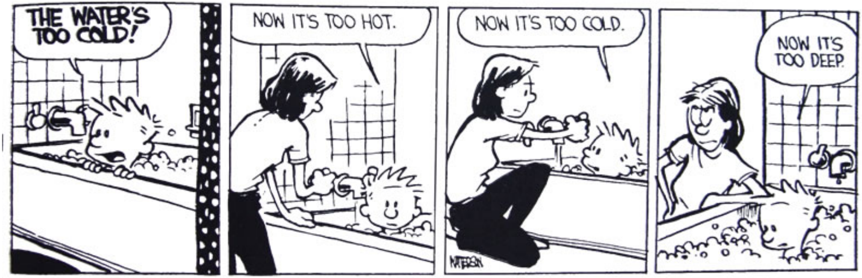
- New Exercise 1 with review of basic course material
- Five dedicated computer exercises; the rest in ordinary classrooms
- Completion of the lecture notes
- New Lab 3 based on the MinSeg process

Course registration

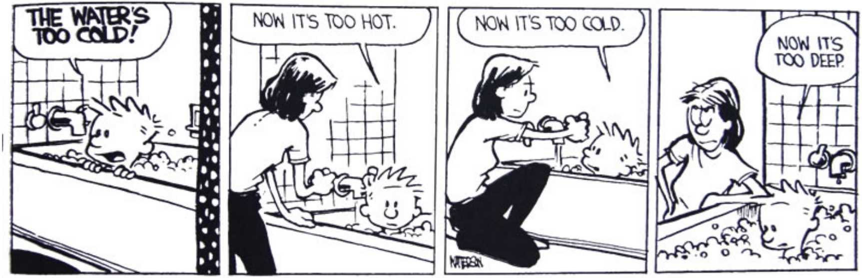
- Please do the course registration in Ladok as soon as possible!
- If you have not signed up for the course in advance, you need to contact your program planner for late sign-up.
- Put a mark next to your name on the attendance list (or fill in your details on an empty row at the end).
- If you decide to drop out during the first three weeks, you should notify us.

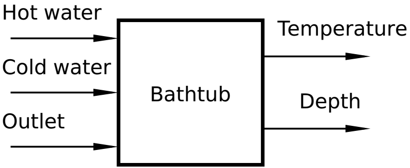
Lecture 1 – Outline

Multivariable control – Example 1



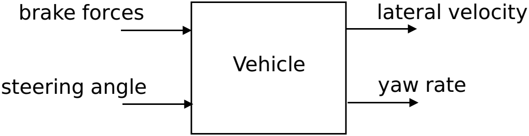
Multivariable control – Example 1



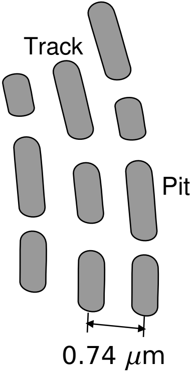


Example 2: Rollover control

Example 2: Rollover control

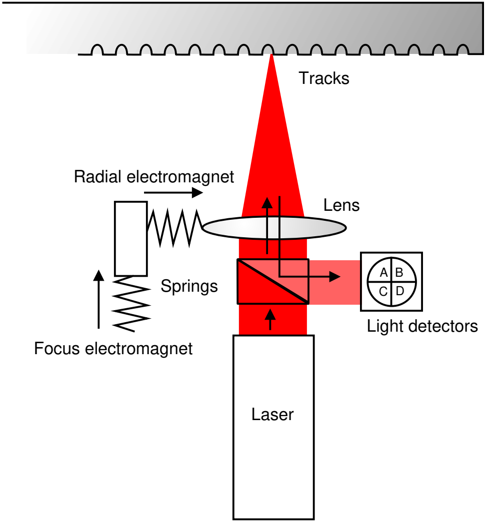


Example 3: DVD player

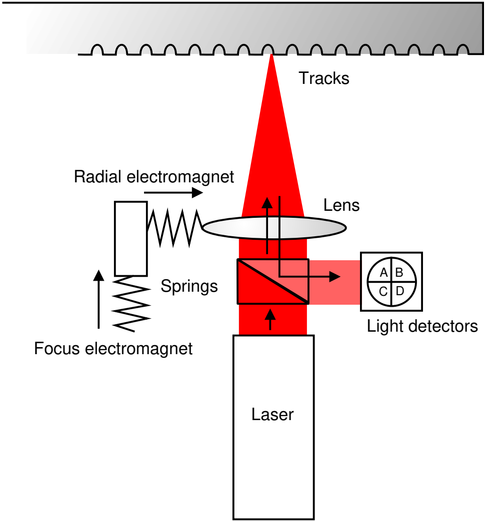


- 3.5 m/s speed along track
- 0.022 μm tracking tolerance
- 100 μm deviations at ~23 Hz due to asymmetric discs

Focus and tracking control

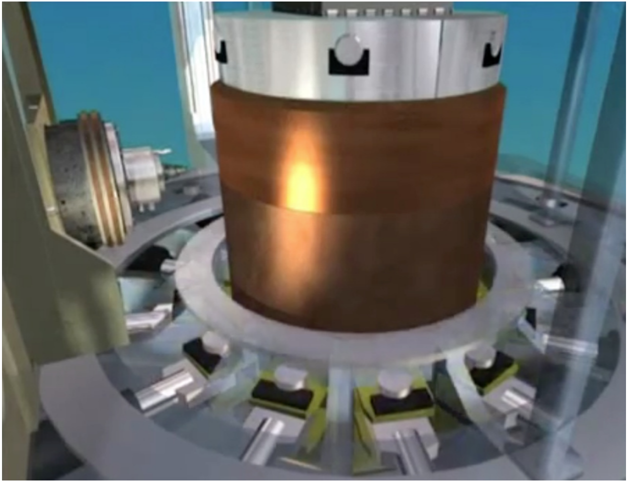


Focus and tracking control



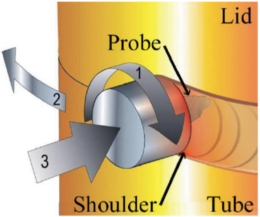
Example 4: Control of friction stir welding

Prototype FSW machine at the Swedish Nuclear Fuel and Waste Management Company (SKB) in Oskarshamn



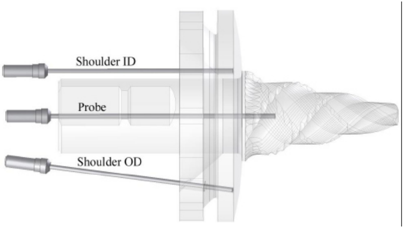
Control of friction stir welding

Control variables:



- Tool rotation speed
- Weld speed
- Axial force

Measurement variables:

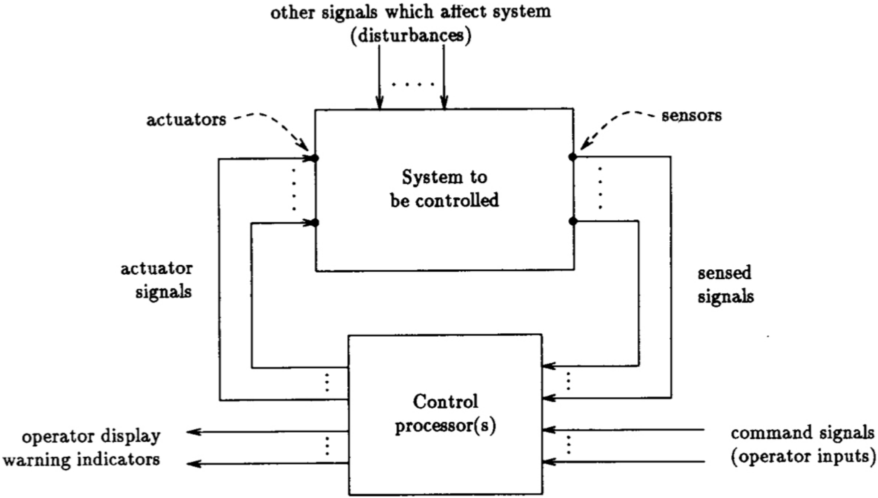


- Temperatures (3 sensors)
- Motor torque
- Shoulder depth

Control objectives:

- Keep weld temperature at 845 °C
- Keep shoulder depth at 1 mm

A general control system



[Boyd *et al.*: "Linear Controller Design: Limits of Performance via Convex Optimization", *Proceedings of the IEEE*, 78:3, 1990]

Contents of the course

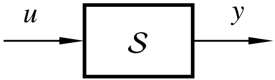
Despite its name, this course is **not only about multivariable control**. You will also learn about:

- sensitivity and robustness
- design trade-offs and fundamental limitations
- stochastic control
- optimization of controllers

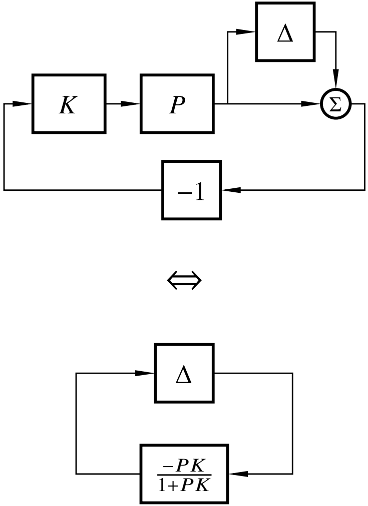
Outline of lectures

L1–L5	Specifications, models and loop-shaping by hand
L6–L8	Limitations on achievable performance
L9–L11	Controller optimization: analytic approach
L12–L14	Controller optimization: numerical approach
L15	Course review

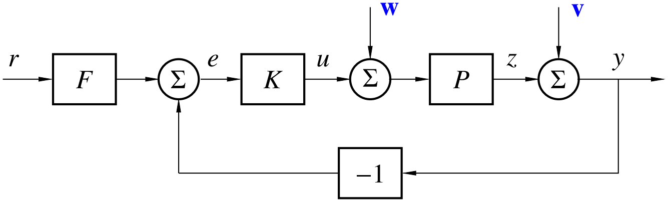
Lecture 1: Introduction

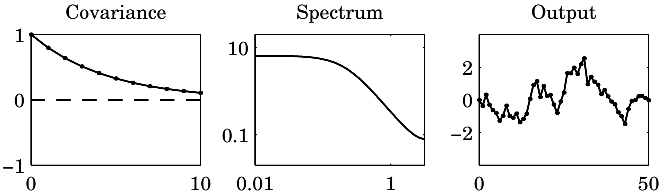


Lecture 2: Stability and robustness

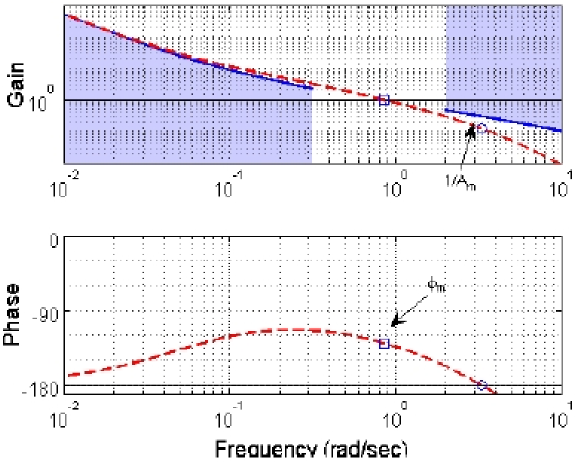


Lecture 3: Specifications and disturbance models

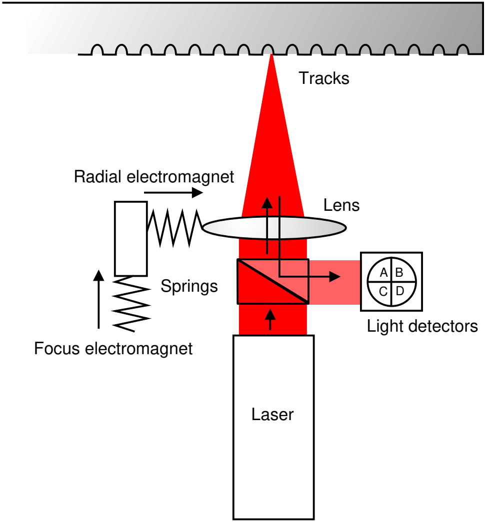


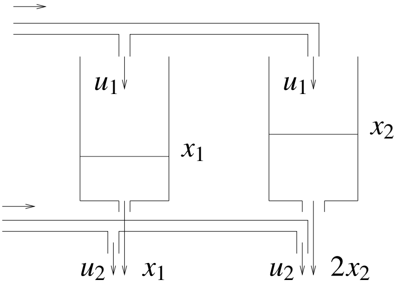


Lecture 4: Control synthesis in frequency domain



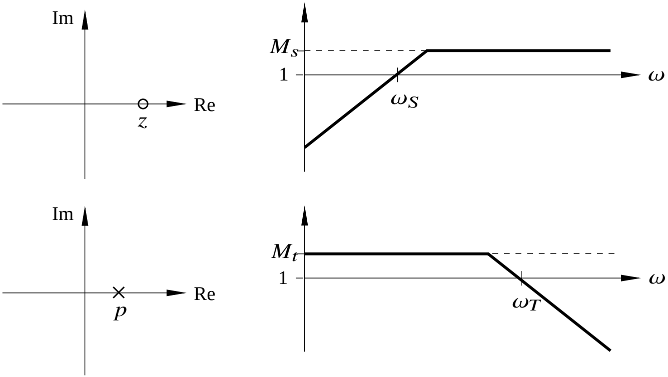
Lecture 5: Case study: DVD player



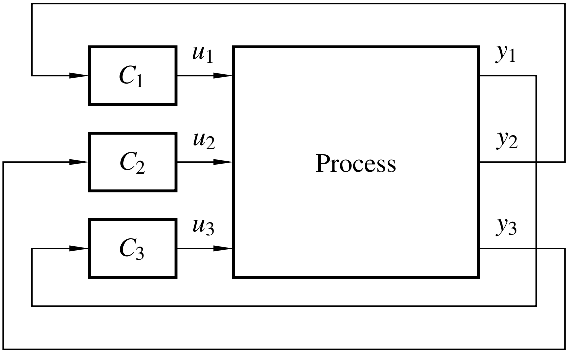


$$G(s) = \begin{pmatrix} \frac{1}{s+2} & 1 \\ \frac{2}{s+2} & 1 \end{pmatrix}$$

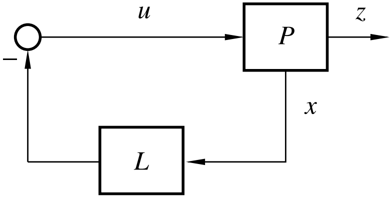
Lecture 7: Fundamental limitations



Lecture 8: Decentralized control

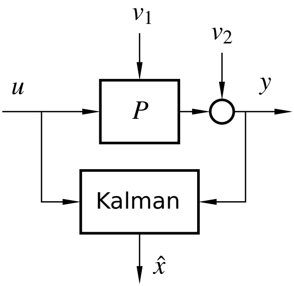


Lecture 9: Linear-quadratic control

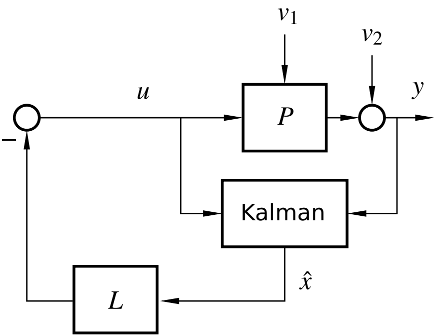


$$\min_L \int_0^\infty \left(x^T Q_1 x + u^T Q_2 u \right) dt$$

Lecture 10: Kalman filtering

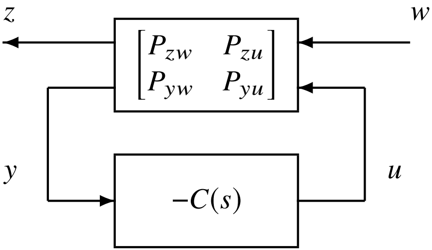


Lecture 11: LQG control



$$\min_{K,L} \mathop{\mathrm{E}}_{v_1,v_2} \left\{ x^T Q_1 x + u^T Q_2 u \right\}$$

Lec. 12: Youla parameterization, internal model control



ALL stabilizing controllers:

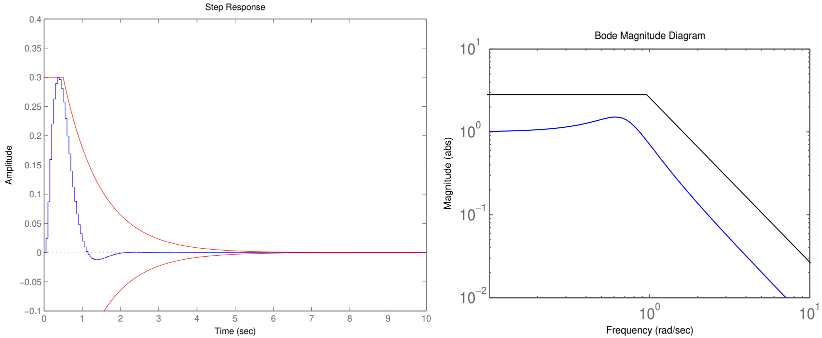
$$C(s) = \left[I - Q(s)P_{yu}(s)\right]^{-1} Q(s)$$

Lecture 13: Synthesis by convex optimization

Minimize e.g.

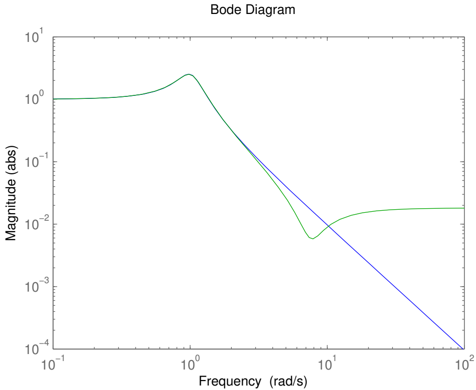
$$\int_{-\infty}^{\infty} \Big|P_{zw}(i\omega) + P_{zu}(i\omega) \overbrace{\sum_k Q_k \phi_k(i\omega)}^{Q(i\omega)} P_{yw}(i\omega)\Big|^2 d\omega$$

subject to constraints



Lecture 14: Controller simplification

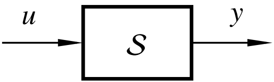
$$C(s) = \frac{(s/1.3+1)(s/45+1)}{(s/1.2+1)(s^2+0.4s+1.04)(s/50+1)} \approx \frac{s^2-2.3s+57}{s^2+0.41s+1.1}$$



Lecture 1 – Outline

1. Course program
2. Course introduction
3. Signals and systems
   - System representations
   - Signal norm and system gain

Systems



A **system** is a mapping from the input signal $u(t)$ to the output signal $y(t)$, $-\infty < t < \infty$:

$$y = \mathcal{S}(u)$$

System properties

A system $\mathcal{S}$ is

- **causal** if $y(t_1)$ only depends on $u(t)$, $-\infty < t \leq t_1$,
  non-causal otherwise

- **static** if $y(t_1)$ only depends on $u(t_1)$,
  dynamic otherwise

- **discrete-time** if $u(t)$ and $y(t)$ are only defined for a countable set of discrete time instances $t = t_k,\ k = 0, \pm 1, \pm 2, \ldots$,
  continuous-time otherwise

System properties (cont'd)

A system $\mathcal{S}$ is

- **single-variable** or **scalar** if $u(t)$ and $y(t)$ are scalar signals, **multivariable** otherwise
- **time-invariant** if $y(t) = \mathcal{S}(u(t))$ implies $y(t+\tau) = \mathcal{S}(u(t+\tau))$, **time-varying** otherwise
- **linear** if $\mathcal{S}(\alpha_1 u_1 + \alpha_2 u_2) = \alpha_1 \mathcal{S}(u_1) + \alpha_2 \mathcal{S}(u_2)$, **nonlinear** otherwise

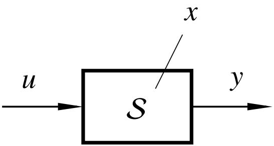
LTI system representations

We will mainly deal with continuous-time **linear time-invariant** (LTI) systems in this course

For LTI systems, the same input–output mapping $\mathcal{S}$ can be represented in a number of equivalent ways:

- linear ordinary differential equation
- linear state-space model
- transfer function
- impulse response
- step response
- frequency response
- ...

State-space models



Linear state-space model:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

Solution:

$$y(t) = Ce^{At}x(0) + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$$

Mini-problem 1

$$\dot{x}_1 = -x_1 + 2x_2 + u_1 + u_2 - u_3$$
$$\dot{x}_2 = -5x_2 + 3u_2 + u_3$$
$$y_1 = x_1 + x_2 + u_3$$
$$y_2 = 4x_2 + 7u_1$$

How many states, inputs and outputs?

Determine the matrices A, B, C, D to write the system as

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

Change of coordinates

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

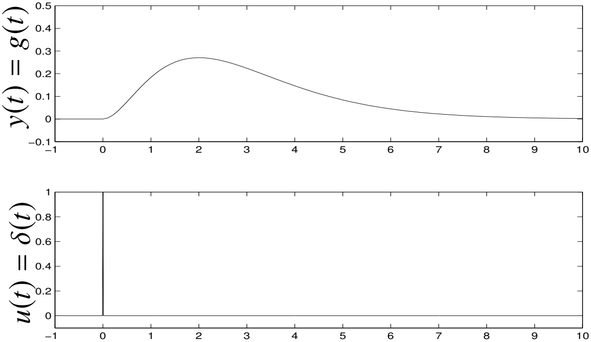
Change of coordinates

$$z = Tx, \quad T \text{ invertible}$$

$$\begin{cases} \dot{z} = T\dot{x} = T(Ax + Bu) & = T(AT^{-1}z + Bu) = TAT^{-1}z + TBu \\ y = Cx + Du & = CT^{-1}z + Du \end{cases}$$

Note: There are infinitely many different state-space representations of the same input–output mapping $y = \mathcal{S}(u)$

Impulse response

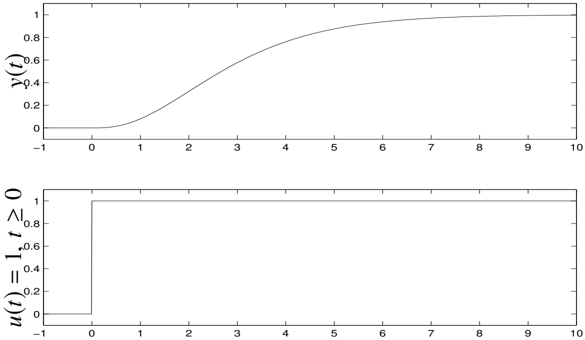


Common experiment in medicine and biology

$$g(t) = \int_0^t Ce^{A(t-\tau)}B\delta(\tau)d\tau + D\delta(t) = Ce^{At}B + D\delta(t)$$

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau = (g * u)(t)$$

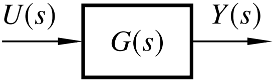
Step response



Common experiment in process industry

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau = \int_0^t g(\tau)d\tau$$

Transfer function



$$G(s) = \mathcal{L}\{g(t)\}$$

$$y(t) = (g * u)(t) \quad \Leftrightarrow \quad Y(s) = G(s)U(s)$$

Conversion from state-space form to transfer function:

$$G(s) = C(sI - A)^{-1}B + D$$

Transfer function

A transfer function is **rational** if it can be written as

$$G(s) = \frac{B(s)}{A(s)}$$

where $B(s)$ and $A(s)$ are polynomials in s

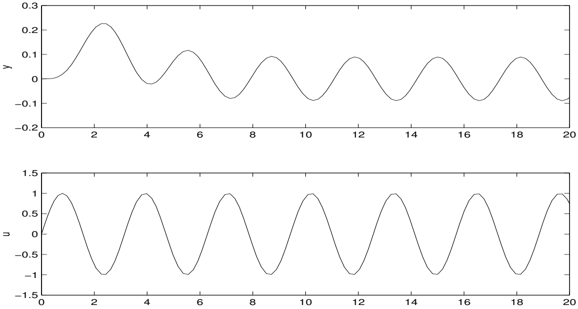
- Example of non-rational function: Time delay e^{-sL}

It is **proper** if $\deg B \leq \deg A$ and **strictly proper** if $\deg B < \deg A$

- Example of non-proper function: Pure derivative s

A rational and proper transfer function can be converted to state-space form (see Collection of Formulae)

Frequency response



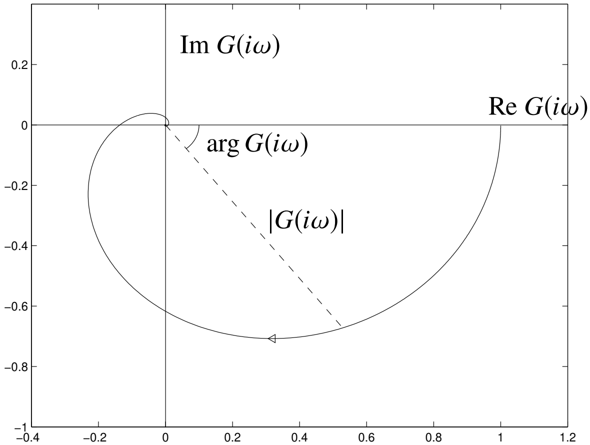
Assume stable transfer function $G = \mathcal{L}g$. Input $u(t) = \sin \omega t$ gives

$$y(t) = \int_0^t g(\tau)u(t-\tau)d\tau = \text{Im}\left[\int_0^t g(\tau)e^{-i\omega\tau}d\tau \cdot e^{i\omega t}\right]$$

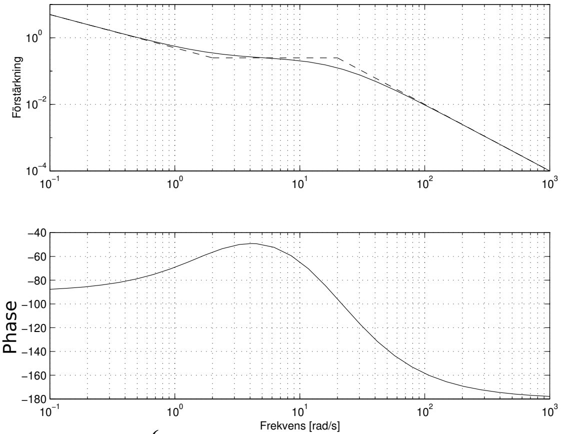
$$[t \to \infty] = \text{Im}\left(G(i\omega)e^{i\omega t}\right) = |G(i\omega)| \sin\left(\omega t + \arg G(i\omega)\right)$$

After a transient, also the output becomes sinusoidal

The Nyquist diagram



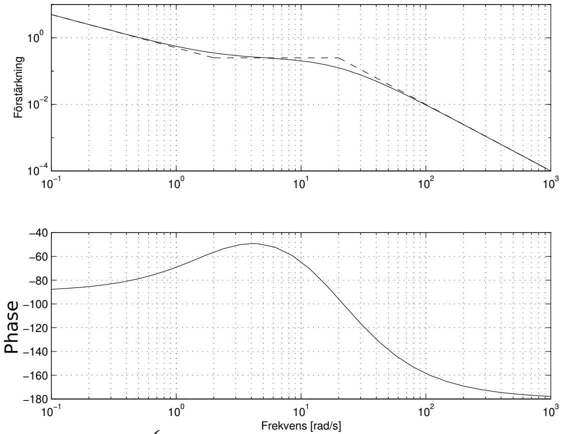
The Bode diagram



$$G = G_1 G_2 G_3 \qquad \begin{cases} \log|G| = \log|G_1| + \log|G_2| + \log|G_3| \\ \arg G = \arg G_1 + \arg G_2 + \arg G_3 \end{cases}$$

Each new factor enters additively!

The Bode diagram



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Each new factor enters additively!

Hint: Set Matlab units

```
>> ctrlpref
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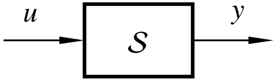
Signal norm and system gain



How to quantify

- the “size” of the signals u and y
- the “maximum amplification” between u and y

Signal norm

The L_2 norm of a signal $y(t) \in \mathbb{R}^n$ is defined as

$$\|y\| = \sqrt{\int_0^\infty |y(t)|^2 dt}$$

By Parseval's theorem it can also be expressed as

$$\|y\| = \sqrt{\frac{1}{2\pi} \int_{-\infty}^\infty |Y(i\omega)|^2 d\omega}$$

System gain

The L_2 (or “L_2-induced”) gain of a general system $\mathcal{S}$ with input u and output $\mathcal{S}(u)$ is defined as

$$\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|}{\|u\|}$$

L_2 gain of LTI systems

THEOREM 1.1
Consider a stable LTI system $\mathcal{S}$ with transfer function $G(s)$. Then

$$\|\mathcal{S}\| = \sup_{\omega} |G(i\omega)| := \|G\|_\infty$$

Proof. Let $y = \mathcal{S}(u)$. Then

$$\|y\|^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(i\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(i\omega)|^2 |U(i\omega)|^2 d\omega \leq \|G\|_\infty^2 \|u\|^2$$

The inequality is arbitrarily tight when $u(t)$ is a sinusoid near the maximizing frequency.

(How to interpret $|G(i\omega)|$ for matrix transfer functions will be explained in Lecture 2.)

Mini-problem 2

What are the L_2 gains of the following scalar LTI systems?

1. $y(t) = -u(t)$ (a sign shift)
2. $y(t) = u(t-T)$ (a time delay)
3. $y(t) = \int_0^t u(\tau)d\tau$ (an integrator)
4. $y(t) = \int_0^t e^{-(t-\tau)}u(\tau)d\tau$ (a first order filter)

Lecture 1 – Summary

- Course overview
- Review of LTI system descriptions (see also Exercise 1)
- L_2 norm of signals
  - Definition: $\|y\| := \sqrt{\int_0^\infty |y(t)|^2 dt}$
- L_2 gain of systems
  - Definition: $\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|}{\|u\|}$
  - Special case—stable LTI systems: $\|\mathcal{S}\| = \sup_\omega |G(i\omega)| := \|G\|_\infty$ (also known as the "H_∞ norm" of the system)