FRTN10 Multivariable Control, Lecture 4

Automatic Control LTH, 2017

Course Outline

L1-L5 Specifications, models and loop-shaping by hand

1. Introduction
2. Stability and robustness
3. Specifications and disturbance models
4. **Control synthesis in frequency domain**
5. Case study

L6-L8 Limitations on achievable performance

L9-L11 Controller optimization: Analytic approach

L12-L14 Controller optimization: Numerical approach

Lecture 4 – Outline

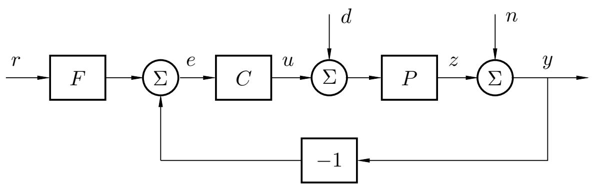
Frequency domain specifications

Loop shaping

Feedforward design

[Glad & Ljung] Ch. 6.4–6.6, 8.1–8.2

Relations between signals



$$Z = \frac{P}{1+PC}D - \frac{PC}{1+PC}N + \frac{PCF}{1+PC}R$$
$$Y = \frac{P}{1+PC}D + \frac{1}{1+PC}N + \frac{PCF}{1+PC}R$$
$$U = -\frac{PC}{1+PC}D - \frac{C}{1+PC}N + \frac{CF}{1+PC}R$$

Design specifications

Find a controller that

A: reduces the effect of load disturbances

B: does not inject too much measurement noise into the system

C: makes the closed loop insensitive to process variations

D: makes the output follow the setpoint

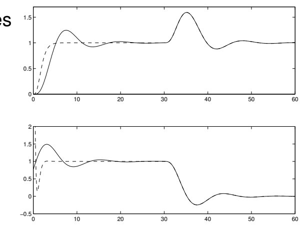
If possible, use a controller with **two degrees of freedom**, i.e. separate signal transmission from y to u and from r to u. This gives a nice separation of the design problem:

1. Design feedback to deal with A, B, and C
2. Design feedforward to deal with D

Time-domain specifications

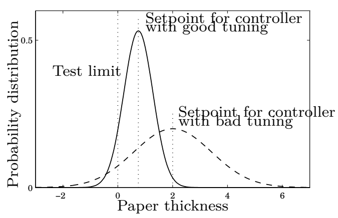
Specifications on e.g. step responses (w.r.t. reference, load disturbance)

- Rise-time T_r
- Overshoot M
- Settling time T_s
- Static error e_0
- ...



Stochastic time-domain specifications

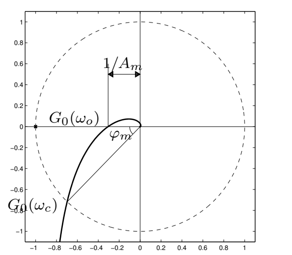
- Output variance
- Control signal variance
- ...



Frequency-domain specifications

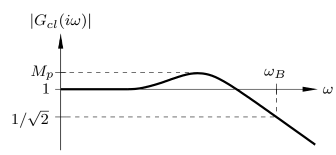
Open-loop specifications

- Amplitude margin A_m, phase margin φ_m
- Cross-over frequency ω_c
- M_s circle in Nyquist diagram
- ...



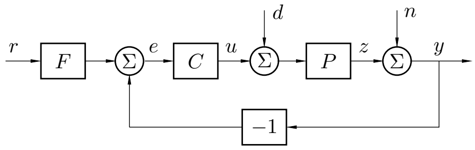
Closed-loop specifications, e.g.

- resonance peak M_p
- bandwidth ω_B
- ...



Frequency domain specifications

Closed-loop specifications, cont'd:



Desired properties:

- Fast tracking of setpoint r
- Small influence of load disturbance d on z
- Small influence of model errors on z
- Limited amplification of noise n in control u
- Robust stability despite model errors

Frequency domain specifications

Ideally, we would like to design the controller (C and F) so that

- $\frac{PCF}{1+PC} = 1$
- $\underbrace{\frac{P}{1+PC}}_{=PS} = \underbrace{\frac{1}{1+PC}}_{=S} = \underbrace{\frac{C}{1+PC}}_{=P^{-1}T} = \underbrace{\frac{PC}{1+PC}}_{=T} = 0$

$S + T = 1$ and other constraints makes this is impossible to achieve.

Typical compromise:

- Make T small at high frequencies ($\omega > \omega_B$)
- Make S small at low frequencies (+ possibly other disturbance dominated frequencies)

Expressing specifications on S and T

Maximum sensitivity specifications, e.g.,

- $\|S\|_\infty \le M_s$
- $\|T\|_\infty \le M_t$

Frequency-weighted specifications, e.g.,

- $\|W_S S\|_\infty \le 1$ or $|S(i\omega)| \le |W_S^{-1}(i\omega)|,\ \forall\omega$
- $\|W_T T\|_\infty \le 1$ or $|T(i\omega)| \le |W_T^{-1}(i\omega)|,\ \forall\omega$

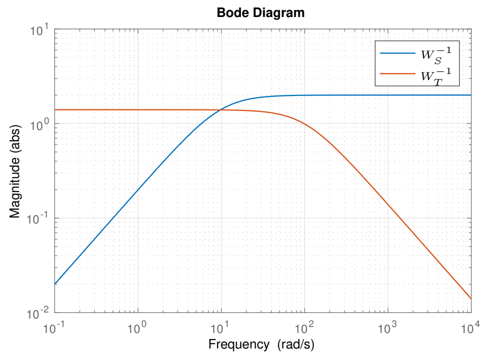
where $W_S(s)$ and $W_T(s)$ are stable transfer functions

Piecewise specifications, e.g.

- $|S(i\omega)| < \frac{0.2}{\omega},\ \omega \le 10$ and $|S(i\omega)| < 2,\ \omega > 10$

Specifications on S and T – example

$$W_S^{-1}(s) = \frac{2s}{s+10}, \quad W_T^{-1}(s) = \frac{140}{s+100}$$



Limitations on specifications

The specifications cannot be chosen independently of each other:

- $S + T = 1 \Rightarrow$
$$|S| + |T| \ge 1$$
$$\big||S| - |T|\big| \le 1$$

Fundamental limitations (Lecture 7):

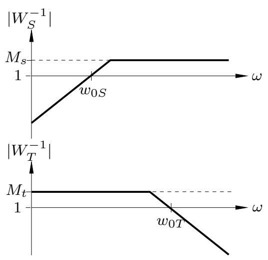
- RHP zero at $z \Rightarrow \omega_{0S} \le z/2$
- Time delay $T \Rightarrow \omega_{0S} \le 1/T$
- RHP pole at $p \Rightarrow \omega_{0T} \ge 2p$

Bode's integral theorem:

- The "waterbed effect"

Bode's relation:

- good phase margin requires certain distance between ω_{0S} and ω_{0T}



Lecture 4 – Outline

Frequency domain specifications

Loop shaping

Feedforward design

Loop shaping

Idea: Look at the **loop gain** $L = G_0 = PC$ for design and translate specifications on S and T into specifications on L

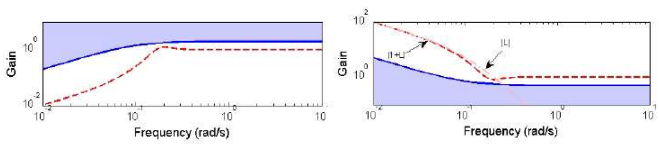
$$S = \frac{1}{1+L} \approx \frac{1}{L} \qquad \text{if } L \text{ is large}$$
$$T = \frac{L}{1+L} \approx L \qquad \text{if } L \text{ is small}$$

Classical loop shaping: Design C so that $L = PC$ satisfies constraints on S and T

- how are the specifications related?
- what to do with the region around cross-over frequency ω_c (where $|L| \approx 1$)?

Sensitivity vs loop gain

$$S = \frac{1}{1+L}$$
$$|S(i\omega)| \le |W_S^{-1}(i\omega)| \iff |1 + L(i\omega)| > |W_S(i\omega)|$$



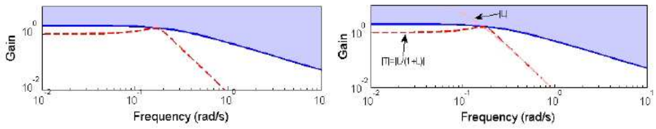
For small frequencies, W_S large $\Longrightarrow 1 + L$ large, and $|L| \approx |1 + L|$.

$$|L(i\omega)| \ge |W_S(i\omega)| \quad (approx.)$$

Complementary sensitivity vs loop gain

$$T = \frac{L}{1+L}$$

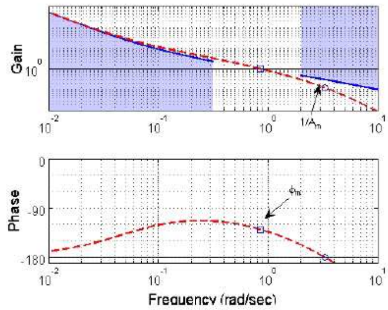
$$|T(i\omega)| \le |W_T^{-1}(i\omega)| \iff \frac{|L(i\omega)|}{|1+L(i\omega)|} \le |W_T^{-1}(i\omega)|$$



For large frequencies, W_T^{-1} small $\Longrightarrow |T| \approx |L|$

$$|L(i\omega)| \le |W_T^{-1}(i\omega)| \quad (approx.)$$

Resulting constraints on loop gain L:



Approximations are inexact around cross-over frequency ω_c. In this region, focus is on stability margins (A_m, φ_m)

Lead–lag compensation

Shape the loop gain $L = PC$ using a compensator $C = C_1C_2C_3\ldots$ composed of

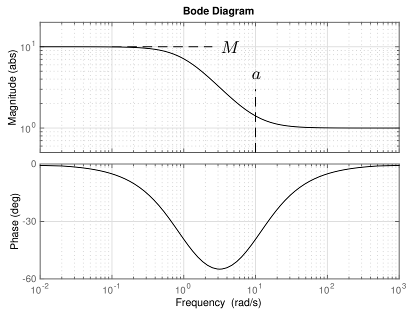
- Gain
$$K$$
- Lag (phase retarding) elements
$$C_{lag}(s) = \frac{s+a}{s+a/M}, \quad M > 1$$
- Lead (phase advancing) elements
$$C_{lead}(s) = N\frac{s+b}{s+bN}, \quad N > 1$$

Example:

$$C(s) = K\frac{s+a}{s+a/M} \cdot N\frac{s+b}{s+bN}$$

Lag filter

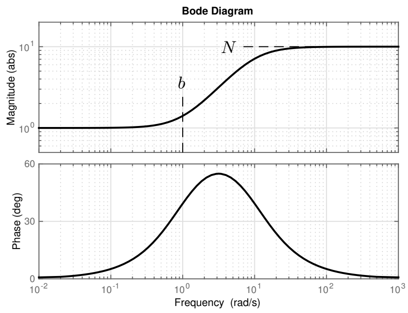
$$G_{lag}(s) = \frac{s+a}{s+a/M}, \quad M > 1$$



Special case: $M = \infty \Rightarrow$ integrator

Lead filter

$$G_{lead}(s) = N\frac{s+b}{s+bN}, \quad N > 1$$



Maximum phase advance for different N given in Collection of Formulae

Properties of lead–lag filters

- Gain
  - Translates the magnitude curve
  - Does not change phase curve
- Lag element
  - Reduces static error
  - Reduces stability margin
- Lead element
  - Increases speed (by increasing ω_c)
  - Increased phase
    $\Rightarrow$ May improve stability

Iterative lead–lag design

Typical workflow:

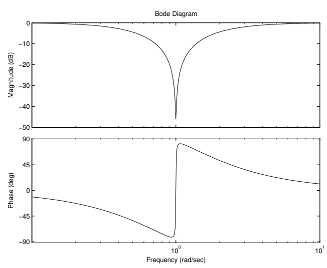
- Adjust gain to obtain the desired cross-over frequency
- Add lag element to improve the low-frequency gain
- Add lead element to improve the phase margin

Adding a lead element and adjusting the gain affect the cross-over frequency

Need to iterate!

Example of other compensation link:

Notch filter $\dfrac{s^2 + 0.01s + 1}{s^2 + 2s + 1}$



(E.g., supress measurement noise at specific frequency)

Lecture 4 – Outline

Frequency domain specifications

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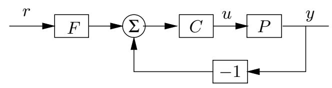
Feedforward design

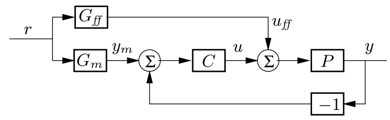
Feedforward design

Two common 2-DOF configurations:

(1)

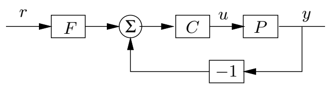


(2)



Ideally, we would like the output to follow the setpoint perfectly, i.e. $y = r$

Feedforward design (1)



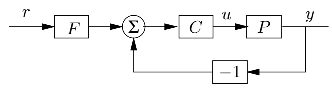
Perfect following requires

$$F = \frac{1 + PC}{PC} = T^{-1}$$

In general impossible because of pole excess in T. Also

- T might contain non-minimum-phase factors that can/should not be inverted
- u must typically satisfy some upper and lower limits

Feedforward design (1)

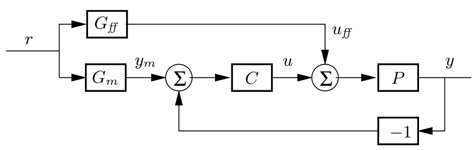


Assume T minimum phase. An implementable choice of F is then

$$F(s) = \frac{1 + P(s)C(s)}{P(s)C(s)(sT_f + 1)^d}$$

where d is large enough to make F proper

Feedforward design (2)

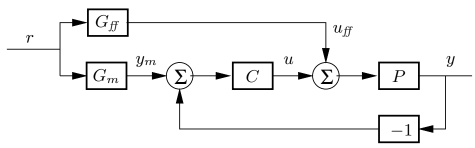


G_m and G_{ff} can be viewed as generators of the desired output y_m and the feedforward u_{ff} that corresponds to y_m

For y to follow y_m, select

$$G_{ff} = G_m/P$$

Feedforward design (2)



Since $G_{ff} = G_m/P$ should be stable, causal and proper we find that

- Unstable zeros of P must be zeros of G_m
- Time delays of P must be time delays of G_m
- The pole excess of G_m must not be smaller than the pole excess of P

Take process limitations into account!

Feedforward design – example

Process:

$$P(s) = \frac{1}{(s+1)^4}$$

Selected reference model:

$$G_m(s) = \frac{1}{(sT_m + 1)^4}$$

Then

$$G_{ff}(s) = \frac{G_m(s)}{P(s)} = \frac{(s+1)^4}{(sT_m+1)^4} \qquad G_\infty(\infty) = \frac{1}{T_m^4}$$

Fast response (small T_m) requires high gain in G_{ff}.

Bounds on the control signal limit how fast response we can obtain in practice

Lecture 4 – summary

Frequency domain design:

- Good mapping between S, T and $L = PC$ at low and high frequencies (mapping around cross-over frequency less clear)
- Simple relation between C and $L \Longrightarrow$ easy to shape L
- Lead–lag design: iterative adjustment procedure
- What if specifications are not satisfied?
  - we made a poor design (did not iterate enough), or
  - the specifications are not feasible (see Lecture 7)
- Later in the course:
  - Use optimization to find stabilizing controller that satisfies constraints, if such a controller exists

Feedforward design