Welcome to

FRTN10 Multivariable Control

Anton Cervin

Department of Automatic Control
Lund University

Department of Automatic Control

- Founded 1965 by Karl Johan Åström (IEEE Medal of Honor)
- Approx. 50 employees
- Education for B, BME, C, D, E, F, I, K, M, N, Pi, W
- Research in autonomous systems, distributed control, robotics, process control, automotive systems, biomedicine, . . .

Lecture 1 – Outline

1. Course program
2. Course introduction
3. Signals and systems
   - System representations
   - Signal norm and system gain

Administration

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Mika Nishimura
Course administrator

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Prerequisites

FRT010 Automatic Control, Basie Course or FRTN25 Automatic Process Control is required prior knowledge.

It is assumed that you have taken the basic courses in mathematics, including linear algebra and calculus in several variables, and preferably also systems & transforms or linear systems.

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EXTRA

Review lecture tomorrow (Tuesday) at 17.15–20.00 (ca) in M:2112b

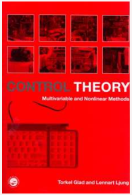
Review lecture

- System representations
  - State-space form, transfer function, pole/zero map, impulse and step responses, Bode diagram, Nyquist diagram
- Analysis
  - Block diagram
  - Stability: characteristic equation, Nyquist criterion
  - Stationary errors, robustness
- State-space design
  - State feedback, controllability
  - Kalman filtering, observability
- Frequency-domain design
  - Lead and lag filters

Course material

All course material is available in English. Most lectures are covered by the following textbook sold by KFS AB:

- Glad & Ljung: *Reglerteori – Flervariabla och olinjära metoder*, (2 uppl.), Studentlitteratur, 2003.
- English edition: Glad & Ljung: *Control Theory – Multivariable and Nonlinear Methods*, Taylor & Francis Ltd / CRC Press



All other material on the homepage:

- Lecture slides (also handed out)
- Lecture notes (so far only for Lectures 1–8, 13)
- Exercise problems with solutions
- Laboratory assignments

`http://www.control.lth.se/course/FRTN10`

Lectures

The lectures (30 hours in total) are given by Anton Cervin on Mondays (w. 35–39, 41), Tuesdays (w. 35–36), and Thursdays (w. 35–41).

See the LTH schedule generator for details.

Exercise sessions and TAs

The exercise sessions (28 hours in total) are arranged in two groups (free choice):

Group	*Times*	*Room*
1	Wednesday 10–12, Friday 10–12	Lab A
2	Wednesday 13–15, Friday 13–15	Lab A

Hamed Sedaghi

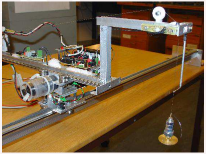
Olof Troeng

Martin Morin

Laboratory experiments

The three laboratory sessions (12 hours in total) are mandatory. A link to the booking system (SAM) is posted on the course homepage. You must sign up before the first session starts. Before each session there are pre-lab assignments that must be completed. No reports are required afterwards.

Lab	*Weeks*	*Booking*	*Room*	*Responsible*	*Process*
1	37–38	Aug 30	Lab C	Hamed Sedaghi	Flexible linear servo
2	39–40	Sep 13	Lab C	Olof Troeng	Quadruple tank
3	41–42	Sep 27	Lab B	Mattias Fält	Rotating crane



Exam

The exam is given on October 27 at 14:00–19:00.

Retake exams are offered in April and August, 2018.

The textbook, lecture notes, and lecture slides (with markings/small notes) are allowed on the exam. You may also bring an *Automatic Control—Collection of Formulae*, standard mathematical tables (TEFYMA), and a pocket calculator.

Use of computers in the course

- In our lab rooms, use your personal student account or a common course account
- Matlab is used in both exercise sessions and laboratory sessions
  - Control System Toolbox
  - Simulink
  - CVX (`http://cvxr.com/cvx`, used in exercise session 12)
  - (Symbolic Math Toolbox)

Feedback and Q&A

For each course LTH uses the following feedback mechanisms

- CEQ (reporting / longer time scale)
- Student representatives (fast feedback)
  - Election of student representative ("kursombud")

We will be using Piazza for Q&A:

`https://piazza.com/lu.se/fall2017/frtn10/home`

Please post your questions here!

Course registration

Course registration in Ladok will be performed on Thursday.

Put a mark next to your name on the registration list (or fill in your details on an empty row at the end).

If you decide to drop out during the first three weeks of the course, you should notify us so that we can unregister you in Ladok.

Do not forget to do “terminsregistrering”!!

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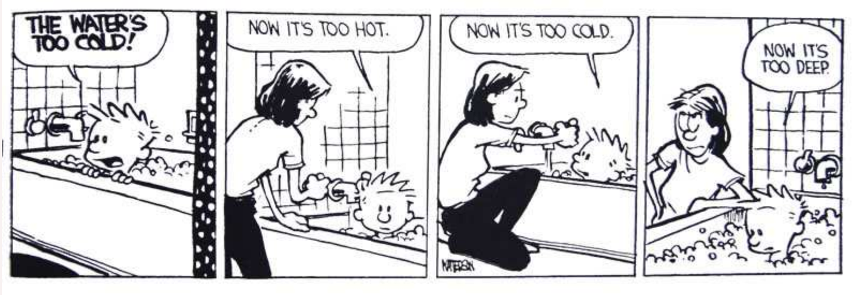
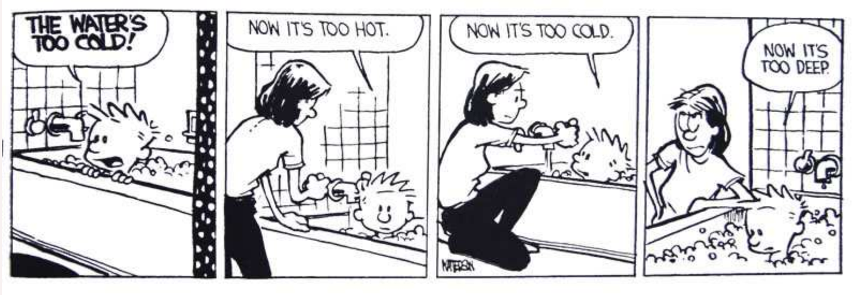
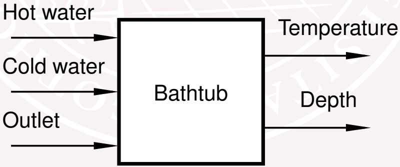
Multivariable control – Example 1



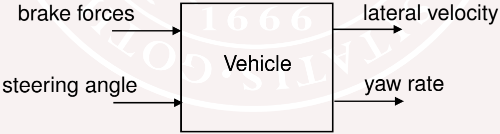
Multivariable control – Example 1



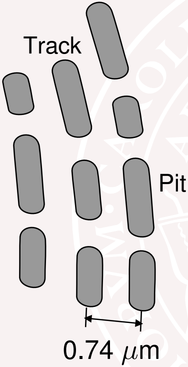


Example 2: Rollover control

Example 2: Rollover control

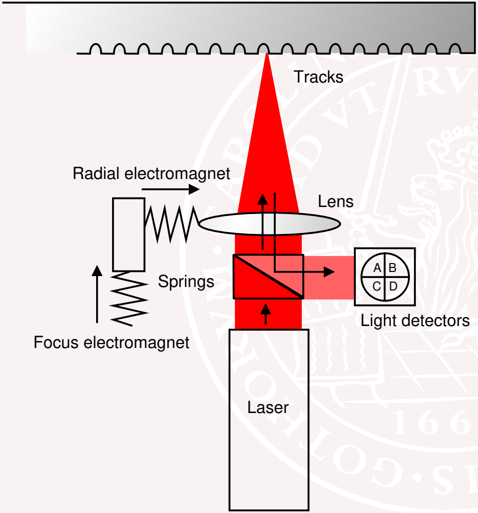


Example 3: DVD player

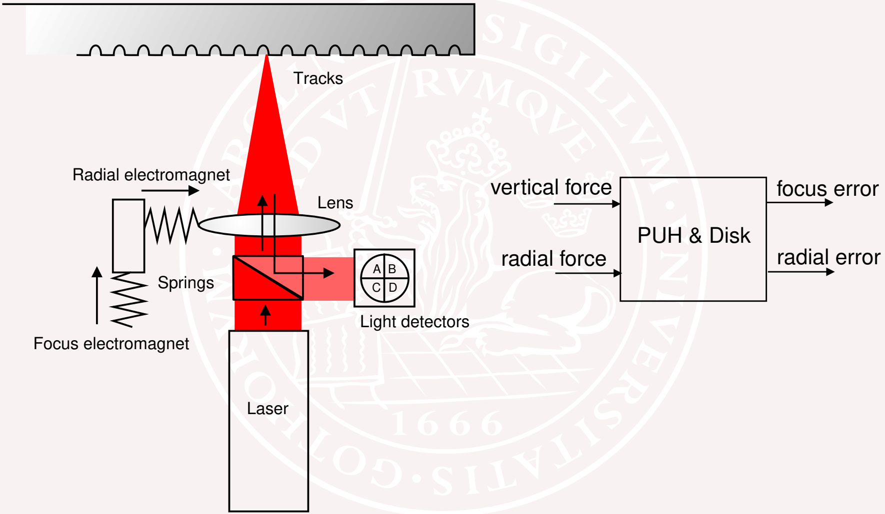


- 3.5 m/s speed along track
- 0.022 μm tracking tolerance
- 100 μm deviations at ~23 Hz due to asymmetric discs

Focus and tracking control

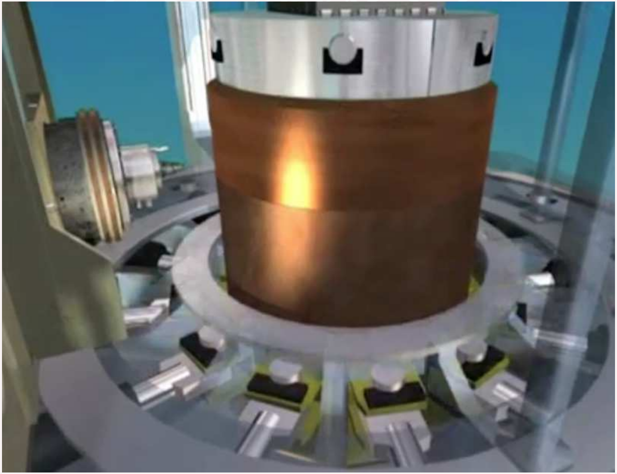


Focus and tracking control



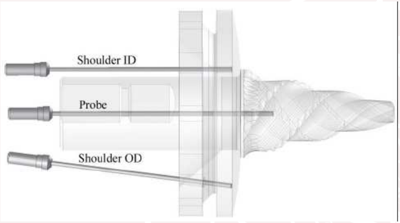
Example 4: Control of friction stir welding

Prototype FSW machine at the Swedish Nuclear Fuel and Waste Management Company (SKB) in Oskarshamn



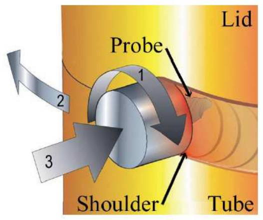
Control of friction stir welding

Measurement variables:



- Temperatures (3 sensors)
- Motor torque
- Shoulder depth

Control variables:

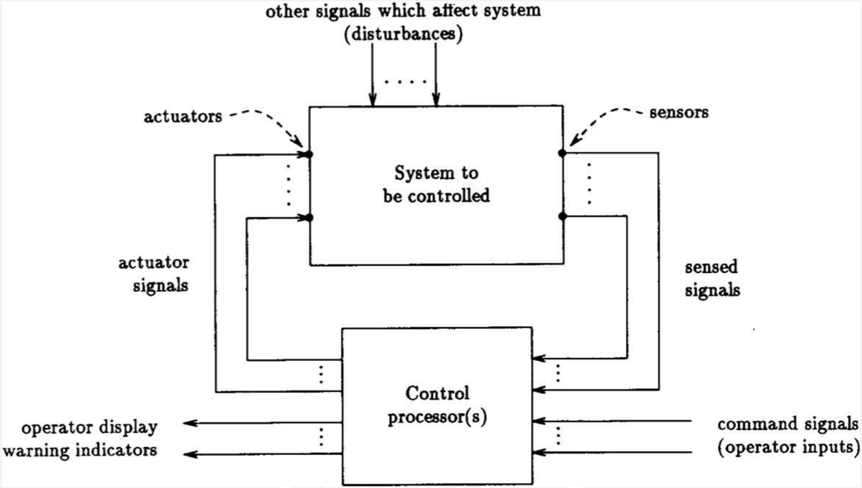


- Tool rotation speed
- Weld speed
- Axial force

Control objectives:

- Keep weld temperature at 845 °C
- Keep shoulder depth at 1 mm

A general control system



[Boyd *et al.*: "Linear Controller Design: Limits of Performance via Convex Optimization", *Proceedings of the IEEE*, 78:3, 1990]

Contents of the course

Despite its name, this course is **not only about multivariable control**. You will also learn about:

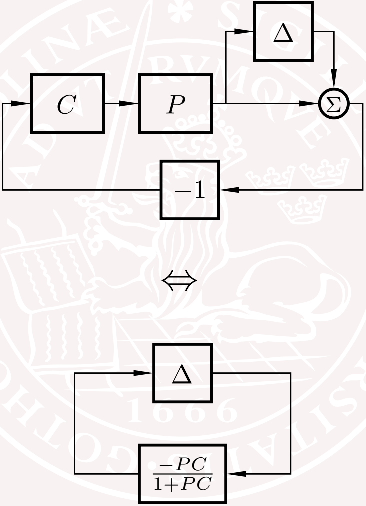
- sensitivity and robustness
- design trade-offs and fundamental limitations
- stochastic control
- optimization of controllers

Outline of lectures

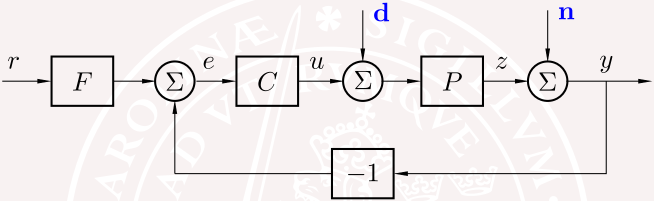
- **L1–L5** Specifications, models and loop-shaping by hand
- **L6–L8** Limitations on achievable performance
- **L9–L11** Controller optimization: analytic approach
- **L12–L14** Controller optimization: numerical approach
- **L15** Course review

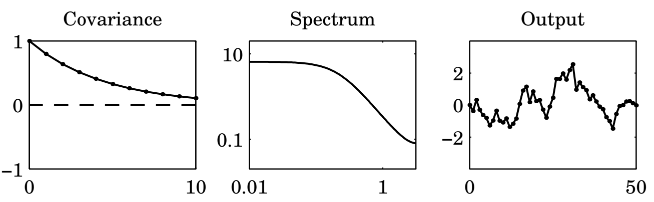
Lecture 1: Systems and signals

Lecture 2: Stability and robustness

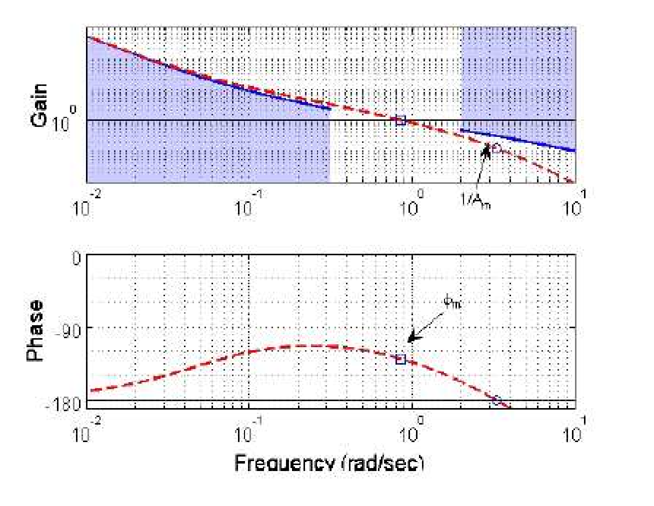


Lecture 3: Specifications and disturbance models

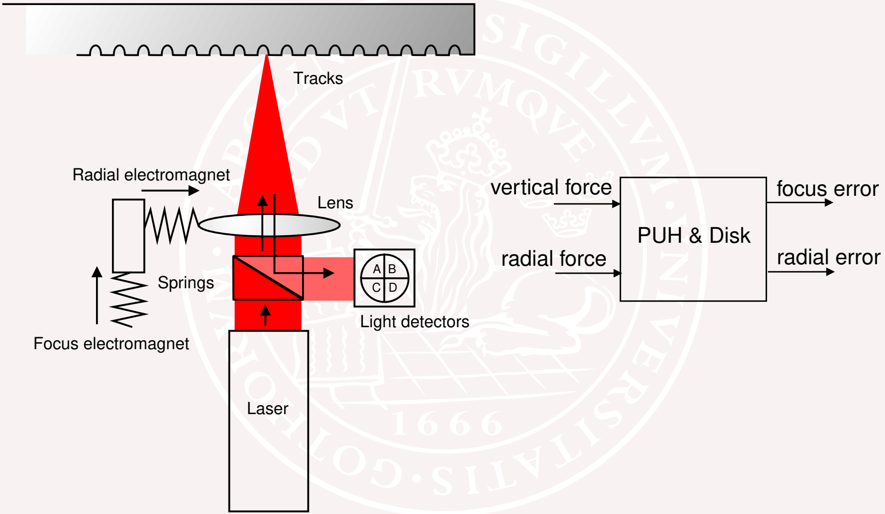




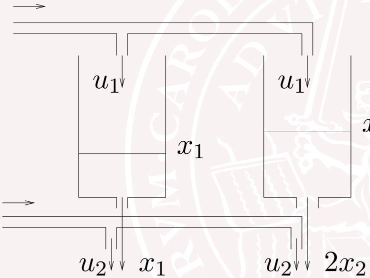
Lecture 4: Control synthesis in frequency domain



Lecture 5: Case study – DVD player

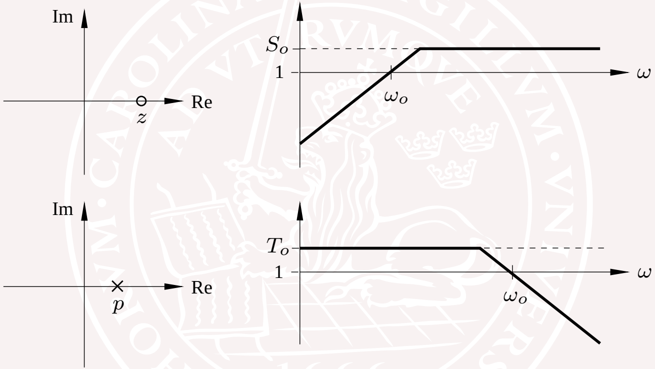


Lec. 6: Controllability/observability, multivar. poles/zeros

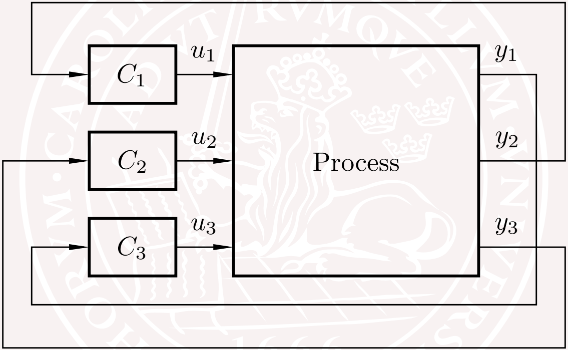


$$G(s) = \begin{pmatrix} \frac{1}{s+2} & 1 \\ \frac{2}{s+2} & 1 \end{pmatrix}$$

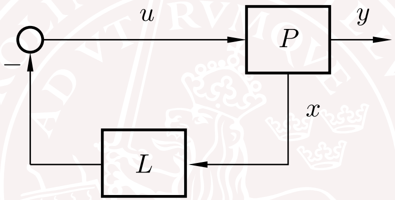
Lecture 7: Fundamental limitations



Lecture 8: Multivariable and decentralized control

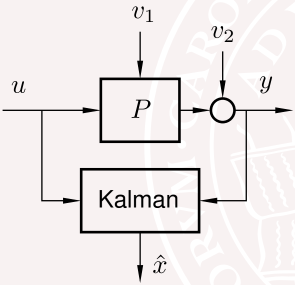


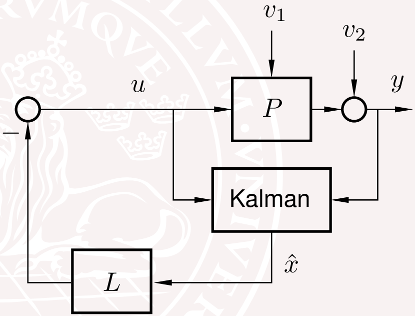
Lecture 9: Linear-quadratic control



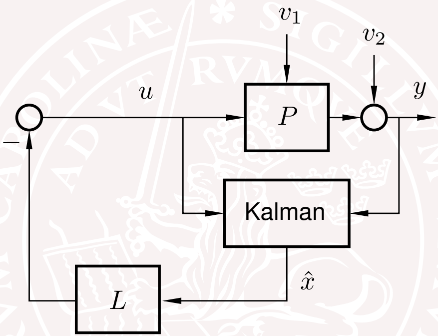
$$\min_L \int_0^\infty \left(x^T Q_1 x + u^T Q_2 u \right) dt$$

Lecture 10: Kalman filtering, LQG



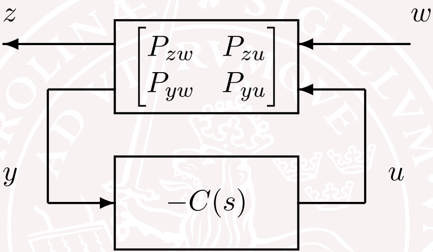


Lecture 11: More on LQG



$$\min_{L,K} \underset{v_1,v_2}{\mathrm{E}} \left\{ x^T Q_1 x + u^T Q_2 u \right\}$$

Lecture 12: Youla parameterization, internal model control



ALL stabilizing controllers:

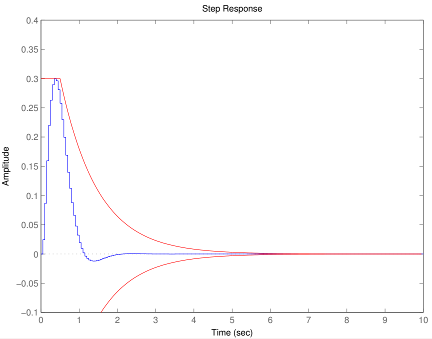
$$C(s) = \left[I - Q(s)P_{yu}(s)\right]^{-1}Q(s)$$

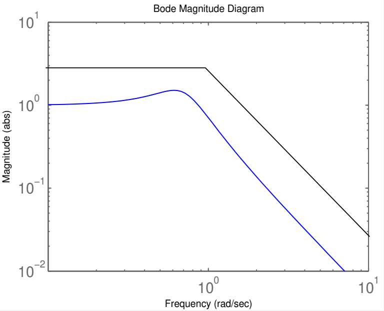
Lecture 13: Synthesis by convex optimization

Minimize

$$\int_{-\infty}^{\infty} \left| P_{zw}(i\omega) + P_{zu}(i\omega) \overbrace{\sum_k Q_k \phi_k(i\omega)}^{Q(i\omega)} P_{yw}(i\omega) \right|^2 d\omega \quad \}$$

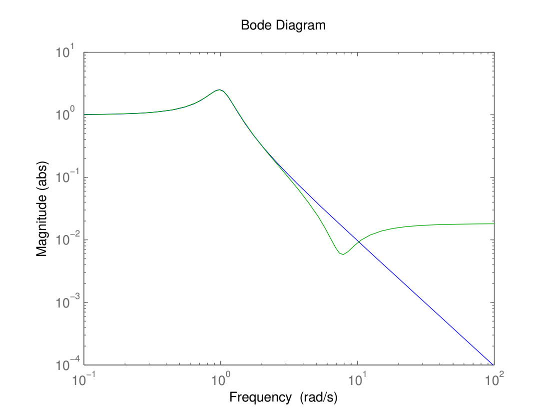
subject to constraints





Lecture 14: Controller simplification

$$C(s) = \frac{(s/1.3+1)(s/45+1)}{(s/1.2+1)(s^2+0.4s+1.04)(s/50+1)} \approx \frac{s^2-2.3s+57}{s^2+0.41s+1.1}$$



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Systems

A **system** is a mapping from the input signal $u(t)$ to the output signal $y(t)$, $-\infty < t < \infty$:

$$y = \mathcal{S}(u)$$

System properties

A system $\mathcal{S}$ is

- **causal** if $y(t_1)$ only depends on $u(t)$, $-\infty < t \leq t_1$,
  non-causal otherwise

- **static** if $y(t_1)$ only depends on $u(t_1)$,
  dynamic otherwise

- **discrete-time** if $u(t)$ and $y(t)$ are only defined for a countable set of discrete time instances $t = t_k,\ k = 0, \pm 1, \pm 2, \ldots$,
  continuous-time otherwise

System properties (cont'd)

A system $\mathcal{S}$ is

- **single-variable** or **scalar** if $u(t)$ and $y(t)$ are scalar signals, **multivariable** otherwise
- **time-invariant** if $y(t) = \mathcal{S}(u(t))$ implies $y(t+\tau) = \mathcal{S}(u(t+\tau))$, **time-varying** otherwise
- **linear** if $\mathcal{S}(\alpha_1 u_1 + \alpha_2 u_2) = \alpha_1 \mathcal{S}(u_1) + \alpha_2 \mathcal{S}(u_2)$, **nonlinear** otherwise

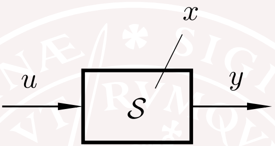
LTI system representations

We will mainly deal with continuous-time **linear time-invariant** (LTI) systems in this course

For LTI systems, the same input–output mapping $\mathcal{S}$ can be represented in a number of equivalent ways:

- linear ordinary differential equation
- linear state-space model
- transfer function
- impulse response
- step response
- frequency response
- ...

State-space models



Linear state-space model:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

Solution:

$$y(t) = Ce^{At}x(0) + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$$

Mini-problem 1

$$
\begin{aligned}
\dot{x}_1 &= -x_1 + 2x_2 + u_1 + u_2 - u_3 \\
\dot{x}_2 &= -5x_2 + 3u_2 + u_3 \\
y_1 &= x_1 + x_2 + u_3 \\
y_2 &= 4x_2 + 7u_1
\end{aligned}
$$

How many state variables, inputs and outputs?

Determine the matrices A, B, C, D to write the system as

$$
\begin{cases}
\dot{x} = Ax + Bu \\
y = Cx + Du
\end{cases}
$$

Change of coordinates

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

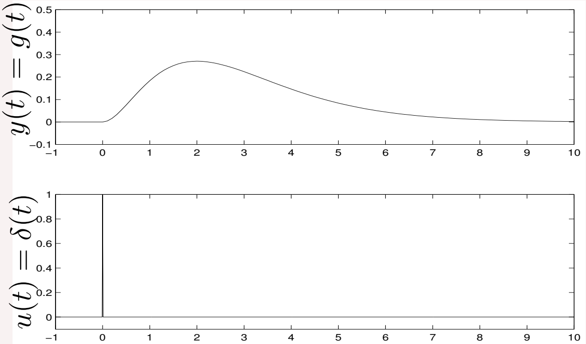
Change of coordinates

$$z = Tx, \quad T \text{ invertible}$$

$$\begin{cases} \dot{z} = T\dot{x} = T(Ax + Bu) & = T(AT^{-1}z + Bu) = TAT^{-1}z + TBu \\ y = Cx + Du & = CT^{-1}z + Du \end{cases}$$

Note: There are infinitely many different state-space representations of the same input–output mapping $y = \mathcal{S}(u)$

Impulse response

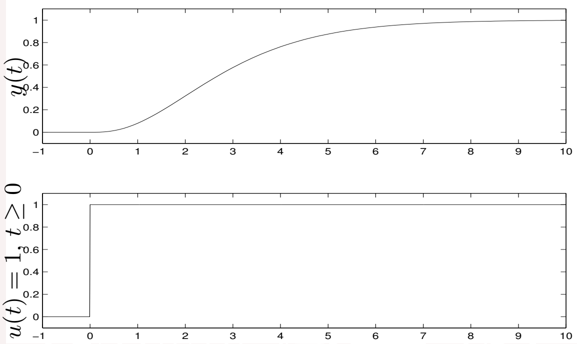


Common experiment in medicine and biology

$$g(t) = \int_0^t Ce^{A(t-\tau)}B\delta(\tau)d\tau + D\delta(t) = Ce^{At}B + D\delta(t)$$

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau = (g * u)(t)$$

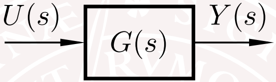
Step response



Common experiment in process industry

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau = \int_0^t g(\tau)d\tau$$

Transfer function



$$G(s) = \mathcal{L}\{g(t)\}$$

$$y(t) = (g * u)(t) \quad \Leftrightarrow \quad Y(s) = G(s)U(s)$$

Conversion from state-space form to transfer function:

$$G(s) = C(sI - A)^{-1}B + D$$

Transfer function

A transfer function is **rational** if it can be written as

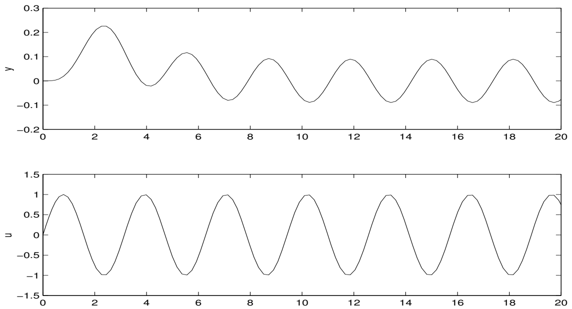
$$G(s) = \frac{B(s)}{A(s)}$$

where $B(s)$ and $A(s)$ are polynomials in s

It is **proper** if $\deg B \leq \deg A$ and **strictly proper** if $\deg B < \deg A$

A rational and proper transfer function can be converted to state-space form (see Collection of Formulae)

Frequency response



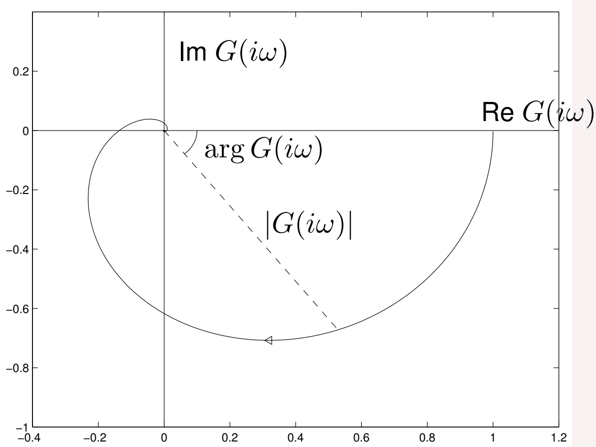
Assume stable transfer function $G = \mathcal{L}g$. Input $u(t) = \sin \omega t$ gives

$$y(t) = \int_0^t g(\tau)u(t-\tau)d\tau = \text{Im}\left[\int_0^t g(\tau)e^{-i\omega\tau}d\tau \cdot e^{i\omega t}\right]$$

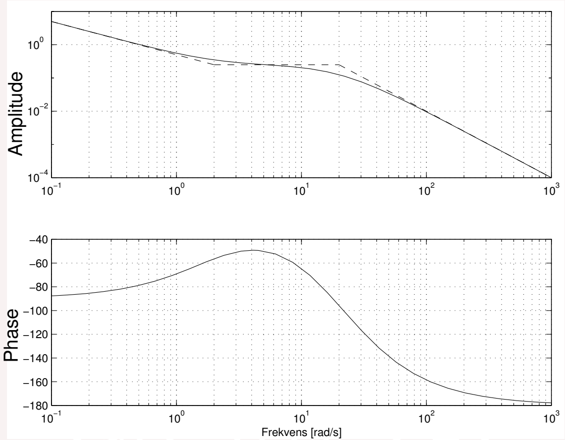
$$[t \to \infty] = \text{Im}\left(G(i\omega)e^{i\omega t}\right) = |G(i\omega)|\sin\left(\omega t + \arg G(i\omega)\right)$$

After a transient, also the output becomes sinusoidal

The Nyquist diagram



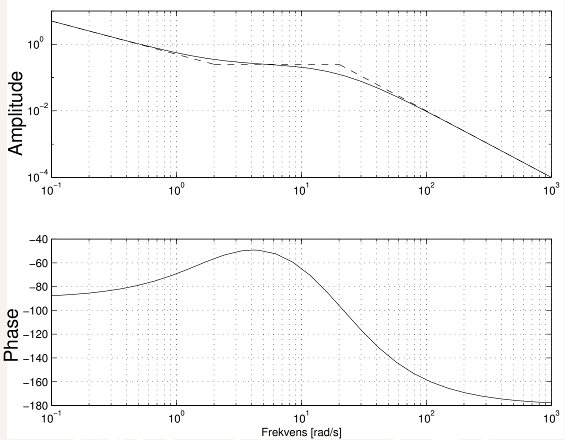
The Bode diagram



$$G = G_1 G_2 G_3 \qquad \begin{cases} \log|G| = \log|G_1| + \log|G_2| + \log|G_3| \\ \arg G = \arg G_1 + \arg G_2 + \arg G_3 \end{cases}$$

Each new factor enters additively!

The Bode diagram



$$G = G_1 G_2 G_3 \qquad \begin{cases} \log|G| = \log|G_1| + \log|G_2| + \log|G_3| \\ \arg G = \arg G_1 + \arg G_2 + \arg G_3 \end{cases}$$

Each new factor enters additively!

Hint: Set Matlab scales

» `ctrlpref`

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Signal norm and system gain

[G&L: Ch 1.5]

How to quantify

- the "size" of the signals u and y
- the "maximum amplification" between u and y

Signal norm

The L_2 norm of a signal $y(t) \in \mathbf{R}^n$ is defined as

$$\|y\|_2 = \sqrt{\int_0^\infty |y(t)|^2 dt}$$

By Parseval's theorem it can also be expressed as

$$\|y\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^\infty |Y(i\omega)|^2 d\omega}$$

System gain

The (L_2) gain of a system $\mathcal{S}$ with input u and output $\mathcal{S}(u)$ is defined as

$$\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2}$$

Mini-problem 2

What are the gains of the following scalar LTI systems?

1. $y(t) = -u(t)$ (a sign shift)
2. $y(t) = u(t-T)$ (a time delay)
3. $y(t) = \int_0^t u(\tau)d\tau$ (an integrator)
4. $y(t) = \int_0^t e^{-(t-\tau)}u(\tau)d\tau$ (a first order filter)

L_2 gain of LTI systems

Consider a stable LTI system $\mathcal{S}$ with input u and output $\mathcal{S}(u)$ having the transfer function $G(s)$. Then

$$\|\mathcal{S}\| = \sup_{\omega} |G(i\omega)| := \|G\|_\infty$$

Proof. Let $y = \mathcal{S}(u)$. Then

$$\|y\|^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(i\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(i\omega)|^2 \cdot |U(i\omega)|^2 d\omega \le \|G\|_\infty^2 \|u\|^2$$

The inequality is arbitrarily tight when $u(t)$ is a sinusoid near the maximizing frequency.

(How to interpret $|G(i\omega)|$ for matrix transfer functions will be explained in Lecture 2.)

Lecture 1 – Summary

- Course overview
- Review of LTI system descriptions
- L_2 norm of signals
  - Definition: $\|y\|_2 := \sqrt{\int_0^\infty |y(t)|^2 dt}$
- L_2 gain of systems
  - Definition: $\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2}$
  - Special case—stable LTI systems: $\|\mathcal{S}\| = \sup_\omega |G(i\omega)|$