LUND UNIVERSITY

Department of
AUTOMATIC CONTROL

FRTN10 Multivariable Control

Exam 2017-01-03, 08:00–13:00

Points and grades

All answers must include a clear motivation and a well-formulated answer. Answers may be given in English or Swedish. The total number of points is 25. The maximum number of points is specified for each subproblem.

Accepted aid

The textbook *Glad & Ljung*, standard mathematical tables like TEFYMA, an authorized “Formelsamling i Reglerteknik”/”Collection of Formulas” and a pocket calculator. Handouts of lecture notes and lecture slides (including markings/notes) are also allowed.

Results

The result of the exam will be entered into LADOK. The solutions will be available on the course home page: `http://www.control.lth.se/course/FRTN10`

1. Consider the following system:

$$G(s) = \begin{bmatrix} \dfrac{2}{(s+10)(s+1)} & \dfrac{1}{s+1} \\ \dfrac{2}{s+2} & \dfrac{1}{s+2} \end{bmatrix}$$

a. Determine the poles and zeros of the system, including their multiplicity. (2 p)

b. Write the system in state-space form using a minimal number of state variables. (1.5 p)

c. Compute the RGA of the system in stationarity. Which inputs should be paired with which outputs in a decentralized control design? (1.5 p)

2. Design an Internal Model Controller for the process

$$P(s) = \frac{s+1}{(0.1s+1)^3}$$

Place the poles of the closed-loop system in the same location as the open-loop poles. Will the closed-loop system be able to follow a constant reference signal without stationary error? (3 p)

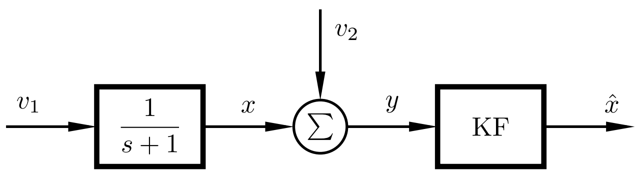

Figure 1 An open-loop system.

3. Consider the open-loop system in Figure 1. You should design the Kalman filter KF such that $\hat{x}$ is an optimal estimate of x. v_1 and v_2 are zero-mean white noise processes with intensities $R_1 = 6$ and $R_2 = 1$ respectively, and their cross-intensity is $R_{12} = 1$.

a. Show that the transfer function of the resulting Kalman filter is $\frac{2}{s+3}$. (2 p)

b. Calculate the stationary variance of x and the spectral density of x. (2 p)

4. A cascade control system is shown in the block diagram in Figure 2. We want to isolate the uncertainty as shown in Figure 3.

a. Find the transfer function G from n to u_2 expressed in terms of C_1, C_2, P_1, P_2 and F. (1 p)

b. The step response and the singular value plot of G are shown in Figures 4 and 5. For which of the following $\Delta(s)$ can you guarantee stability of the closed-loop system using the Small Gain Theorem?

- $\Delta_1(s) = \dfrac{2}{s+5}$

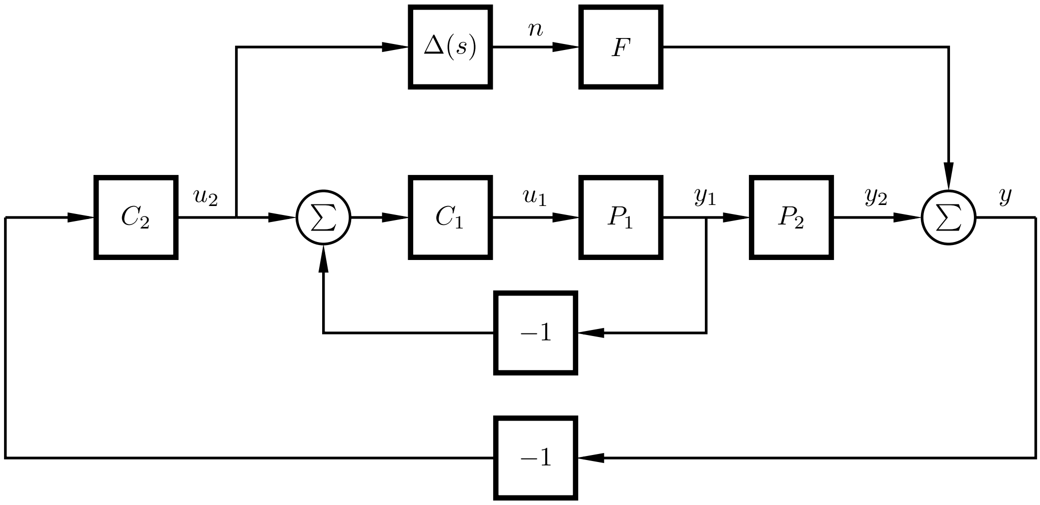

Figure 2 Block diagram for the system in Problem 4.

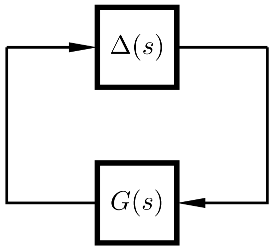

Figure 3 Desired block diagram for the system in Problem 4.

- $\Delta_2(s) = 0.8$
- $\Delta_3(s) = \dfrac{0.3s + 1}{s + 1}$

(1.5 p)

c. Is it possible that all of $\Delta_1(s)$, $\Delta_2(s)$ and $\Delta_3(s)$ can actually result in stable closed-loop systems? Motivate your answer. (0.5 p)

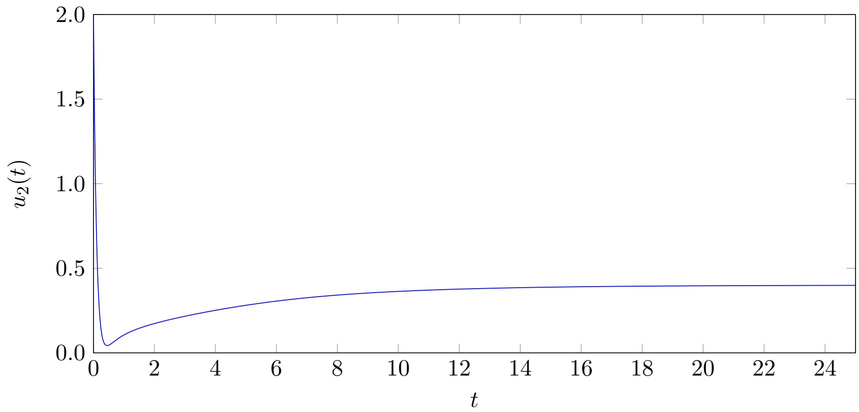

Figure 4 Step response of G.

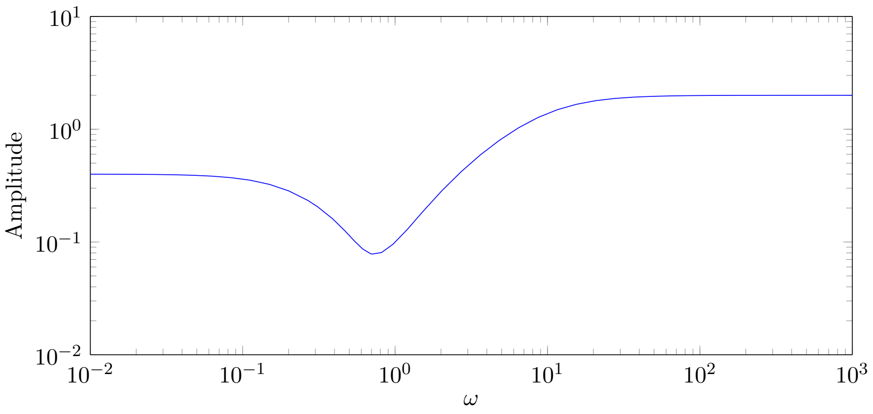

Figure 5 Singular value plot of G

5. You want to design an optimal controller that minimizes the cost function

$$J = \int \left(y^T(t)Q_1 y(t) + u^T(t)Q_2 u(t)\right) dt$$

with the weight matrices $Q_1 = \begin{pmatrix} 10 & 0 \\ 0 & 1 \end{pmatrix}$ and $Q_2 = 1$.

a. How many inputs u and outputs y does the system have? (0.5 p)

b. Explain in words how the closed-loop system behavior would change if the weight matrices were instead set to $Q_1^* = \begin{pmatrix} 1 & 0 \\ 0 & 0.1 \end{pmatrix}$ and $Q_2^* = 1$. (0.5 p)

c. The process is given by

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ \sqrt{10} \end{pmatrix} u$$
$$y = x$$

Design a state feedback law $u = -Lx$ that minimizes the cost function with the weight matrices Q_1 and Q_2. (1.5 p)

d. The result from subproblem c gives an optimal controller. But as you hopefully know, "optimal" is not the same as "good"; it depends on whether the cost function has been chosen wisely. With the given cost function, could you at least guarantee that the obtained closed-loop system will be stable? Motivate! (0.5 p)

6. You have been given the task to design an optimal controller $C(s)$ for a stable, single-input–single-output system $P(s)$, assuming a standard 1-degree-of-freedom controller structure. The controller should minimize the integrated error of the output signal y_{refstep} due to a reference step,

$$\int_0^\infty |y_{\text{refstep}}| dt.$$

subject to the following constraints,

1. The system should be robust to process variations, i.e. $|S(i\omega)| \leq 1.5$.
2. The impact of measurement noise on the control signal should be limited, i.e. $\sup_\omega |G_{un}(i\omega)| \leq 30$.
3. There should be less than 2% overshoot in the process output signal due to a reference step.
4. The control signal due to a reference step should be within $[-10, 10]$.

Since the plant is SISO and stable, the Youla parametrization

$$Q = C/(1 + PC), \tag{1}$$

has been chosen. For example, the following shows the affine relation between S and Q:

$$S = \frac{1}{1 + PC} = 1 - \frac{PC}{1 + PC} = 1 - PQ$$

A suitable finite basis $\{Q_k\}_{k=1}^{50}$ for the parametrization of Q was selected, and step responses and frequency responses for $\{Q_k\}$, $\{PQ_k\}$ were computed for the different basis functions at suitably chosen time points $\{t_1, t_2, \ldots, t_N\}$ and frequency points $\{\omega_1, \omega_2, \ldots, \omega_M\}$.

The results are have been stored in the Matlab variables `Q_sr`, `Q_fr`, `PQ_sr` and `PQ_fr`, where the columns correspond to different basis functions Q_k and the rows correspond to different time/frequency points. The variable b selects how much to use of each basis function, i.e. $Q = \sum Q_k b_k$.

Before falling asleep in front of the computer last night, you had started to code up the problem in `cvx` and had managed to specify the cost function and first constraint. The code that you woke up to was

```
M_s = 1.5;
max_overshoot = 1.02;
CS_max = 30;
umax = 10;

cvx_begin
variable b(50)

minimize sum( abs(ones(N,1) - PQ_sr*b ) )

subject to:
% 1. Maximum sensitivity constraint
max(abs(1 - PQ_fr*b)) <= M_s;

% 2. Constraint on transfer functionnnnnnnnnnnnnnnnnnnnnnnnnnnnnn

cvx_end
```

a. What is the missing code, required to enforce the constraints 2–4? You do not have to get the `cvx`/Matlab syntax exactly right. (2 p)

b. The convex optimization approach discussed in this problem makes it very easy to design controllers which are very close to optimal, for a wide range of different objective functions and constraints. Give one reason why the controllers designed with this method can be problematic for real-world implementation. (0.5 p)

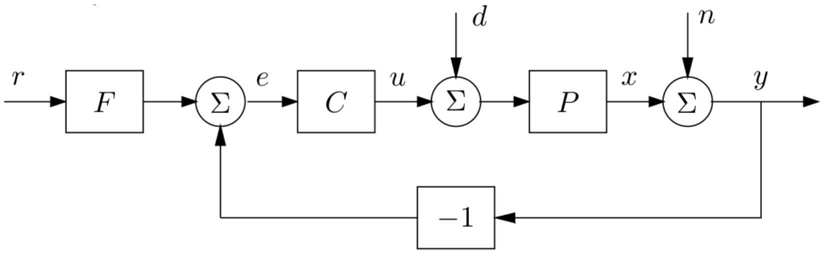

Figure 6 Two-degree-of-freedom controller structure for problem 7.

7. Let's do some loop shaping! The process is given by

$$P(s) = \frac{e^{-s}}{(s+1)(s+2)},$$

and the two-degree-of-freedom controller structure in Figure 6 is used to control it.

Your colleague has already designed a PID controller

$$C_1(s) = 1 + \frac{0.2}{s} + 0.2s.$$

The gang of four for this controller is plotted in Figure 7 and the effect of a step disturbance and measurement noise is plotted in Figure 8.

a. Mention one significant advantage of loop shaping compared to LQG when doing control design for single-input–single-output systems. (0.5 p)

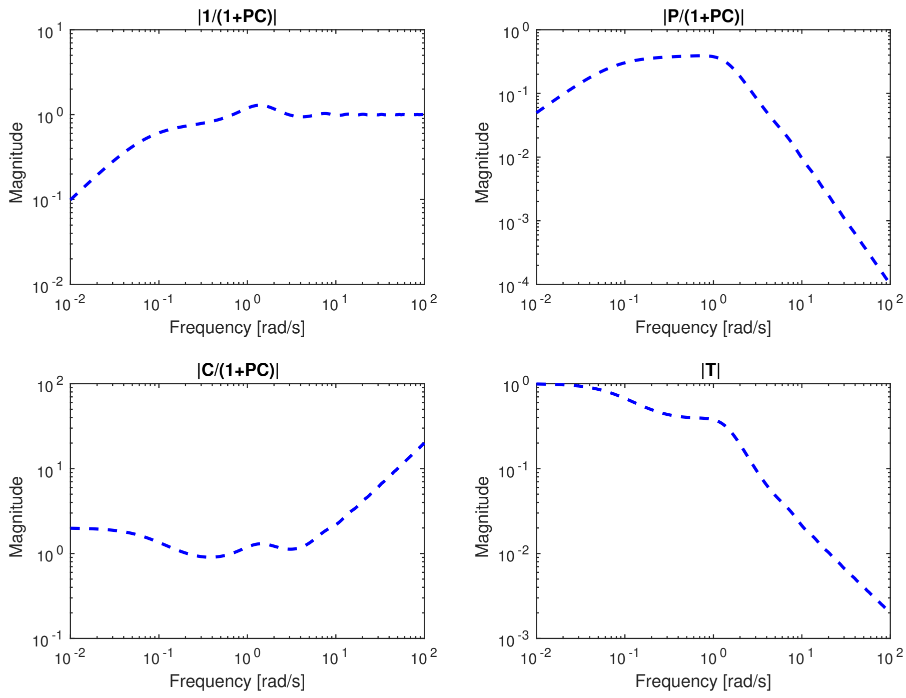

Figure 7 Gang of four for Problem 7.

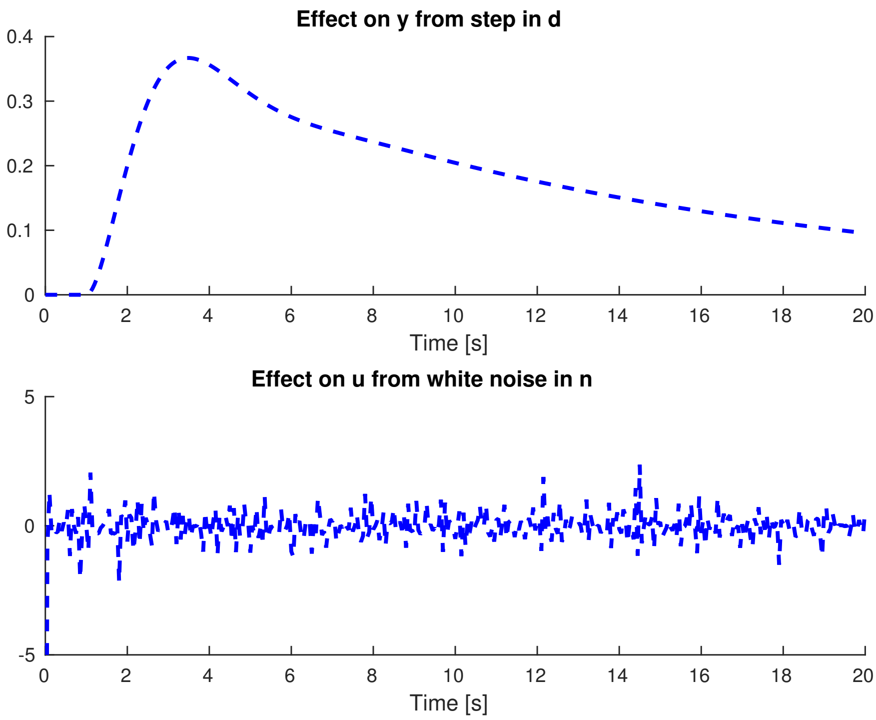

Figure 8 Effect of a step disturbance on the measured signal y and effect of white measurement noise on the control signal u, for Problem 7.

b. As seen in Figure 8 there is significant control signal activity due to measurement noise. Why is this typically bad? How should you change the controller to reduce the impact of measurement noise on the control signal? (1 p)

c. As seen in Figure 8, a step disturbance in d is attenuated too slowly. How should you change the controller so that the disturbances are rejected faster? Mention two alternatives. (1 p)

d. Can you conclude only from Figure 7 that the closed-loop system is stable? Motivate your answer. (0.5 p)

e. Is it typically best to design the controller C or the prefilter F first? Motivate! (0.5 p)

f. One way to design the prefilter F is to choose

$$F = \frac{1 + PC}{PC(1 + sT_f)^d}$$

where d is chosen large enough to make F proper. Why will this approach not work in our case? How could the approach be modified to make it work? (1 p)