FRTN10 Multivariable Control, Lecture 4

Automatic Control LTH, 2016

Course Outline

L1-L5 Specifications, models and loop-shaping by hand

1. Introduction
2. Stability and robustness
3. Specifications and disturbance models
4. **Control synthesis in frequency domain**
5. Case study

L6-L8 Limitations on achievable performance

L9-L11 Controller optimization: Analytic approach

L12-L14 Controller optimization: Numerical approach

Lecture 4 – Outline

1. Frequency domain specifications
2. Loop shaping
3. Feedforward design

[Glad & Ljung] Ch. 6.4–6.6, 8.1–8.2

Lecture 4 – Outline

1. Frequency domain specifications
2. Loop shaping
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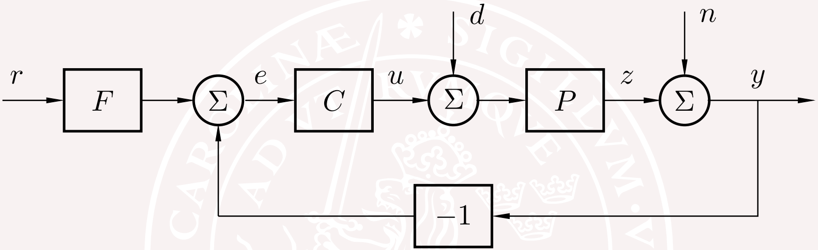
Relations between signals



$$Z = \frac{P}{1+PC}D - \frac{PC}{1+PC}N + \frac{PCF}{1+PC}R$$

$$Y = \frac{P}{1+PC}D + \frac{1}{1+PC}N + \frac{PCF}{1+PC}R$$

$$U = -\frac{PC}{1+PC}D - \frac{C}{1+PC}N + \frac{CF}{1+PC}R$$

Design specifications

Find a controller that

- **A:** reduces the effect of load disturbances
- **B:** does not inject too much measurement noise into the system
- **C:** makes the closed loop insensitive to process variations
- **D:** makes the output follow the setpoint

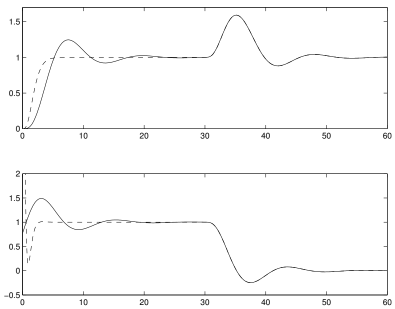
If possible, use a controller with **two degrees of freedom**, i.e. separate signal transmission from y to u and from r to u. This gives a nice separation of the design problem:

1. Design feedback to deal with A, B, and C
2. Design feedforward to deal with D

Time domain specifications

E.g. specifications on step responses
(w.r.t. reference, load disturbance)

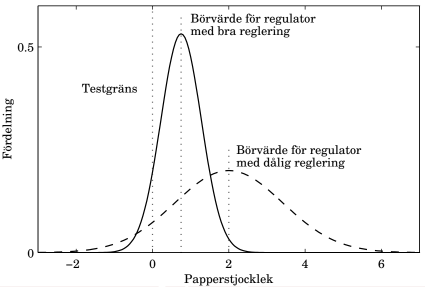
- Rise-time T_r
- Overshoot M
- Settling time T_s
- Static error e_0
- $\dots$



reference change   load disturbance

Stochastic signal specifications

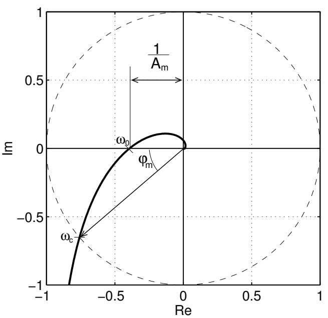
- Output variance
- Control signal variance
- ...



Frequency domain specifications

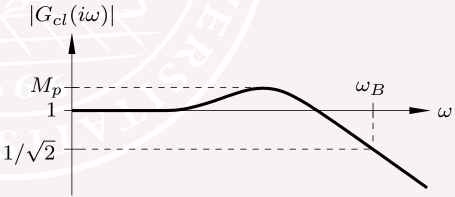
Open-loop specifications

- Amplitude margin A_m, phase margin φ_m
- Cross-over frequency ω_c
- M_s and M_t circles in Nyquist diagram
- …



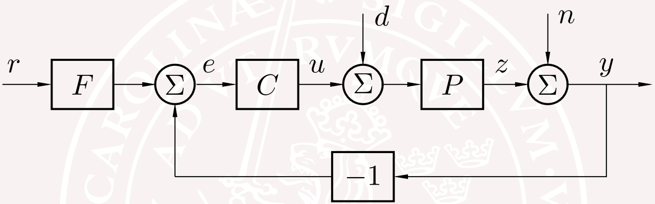
Closed-loop specifications, e.g.

- resonance peak M_p
- bandwidth ω_B
- …



Frequency domain specifications

Closed-loop specifications, cont'd:



Desired properties:

- Fast tracking of setpoint r
- Small influence of load disturbance d on z
- Small influence of model errors on z
- Limited amplification of noise n in control u
- Robust stability despite model errors

Frequency domain specifications

Ideally, we would like to design the controller (C and F) so that

- $\dfrac{PCF}{1+PC} = 1$
- $\underbrace{\dfrac{P}{1+PC}}_{=PS} = \underbrace{\dfrac{1}{1+PC}}_{=S} = \underbrace{\dfrac{C}{1+PC}}_{=P^{-1}T} = \underbrace{\dfrac{PC}{1+PC}}_{=T} = 0$

$S + T = 1$ and other constraints makes this is impossible to achieve.

Typical compromise:

- Make T small at high frequencies ($\omega > \omega_B$)
- Make S small at low frequencies (+ possibly other disturbance dominated frequencies)

Expressing specifications on S and T

Maximum sensitivity specifications, e.g.,

- $\|S\|_\infty \leq M_s$
- $\|T\|_\infty \leq M_t$

Frequency-weighted specifications, e.g.,

- $\|W_S S\|_\infty \leq 1$ or $|S(i\omega)| \leq |W_S^{-1}(i\omega)|, \ \forall\omega$
- $\|W_T T\|_\infty \leq 1$ or $|T(i\omega)| \leq |W_T^{-1}(i\omega)|, \ \forall\omega$

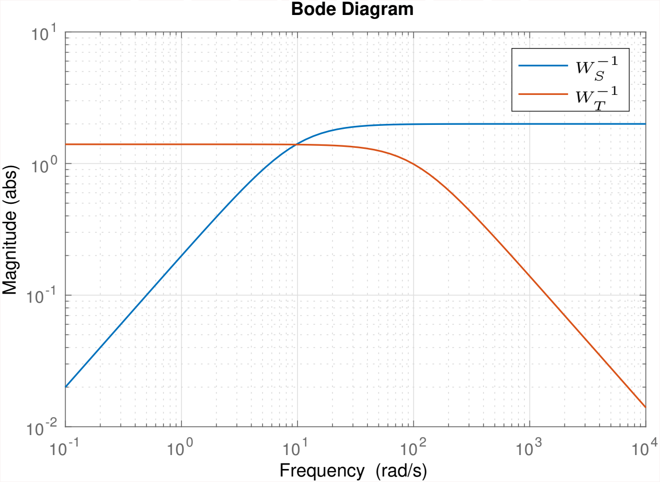
where $W_S(s)$ and $W_T(s)$ are stable transfer functions

Piecewise specifications, e.g.

- $|S(i\omega)| < \frac{0.2}{\omega}, \ \omega \leq 10$ and $|S(i\omega)| < 2, \ \omega > 10$

Specifications on S and T – example

$$W_S^{-1}(s) = \frac{2s}{s+10}, \quad W_T^{-1}(s) = \frac{140}{s+100}$$



Limitations on specifications

The specifications cannot be chosen independently of each other:

- $S + T = 1$

Fundamental limitations [Lecture 7]:

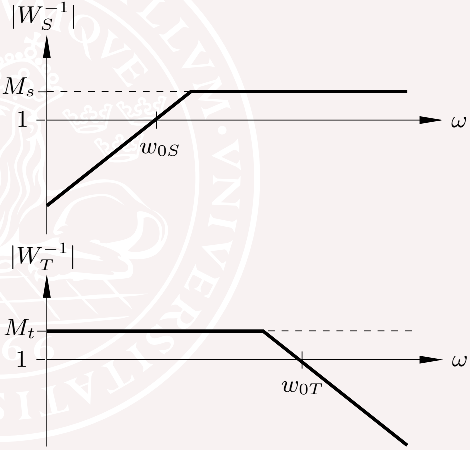
- RHP zero at $z \Rightarrow \omega_{0S} \leq z/2$
- Time delay $T \Rightarrow \omega_{0S} \leq 1/T$
- RHP pole at $p \Rightarrow \omega_{0T} \geq 2p$

Bode's integral theorem:

- The "waterbed effect"

Bode's relation:

- good phase margin requires certain distance between ω_{0S} and ω_{0T}



Lecture 4 – Outline

1. Frequency domain specifications
2. Loop shaping
3. Feedforward design

Loop shaping

Idea: Look at the **loop gain** $L = PC$ for design and to translate specifications on S and T into specifications on L

$$S = \frac{1}{1+L} \approx \frac{1}{L} \qquad \text{if } L \text{ is large}$$

$$T = \frac{L}{1+L} \approx L \qquad \text{if } L \text{ is small}$$

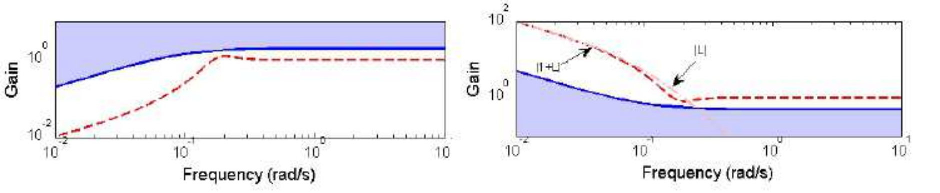
Classical loop shaping: Manually design C so that $L = PC$ satisfies constraints on S and T

- how are the specifications related?
- what to do with the region around cross-over frequency ω_c (where $|L| \approx 1$)?

Sensitivity vs loop gain

$$S = \frac{1}{1+L}$$

$$|S(i\omega)| \leq |W_S^{-1}(i\omega)| \iff |1 + L(i\omega)| > |W_S(i\omega)|$$



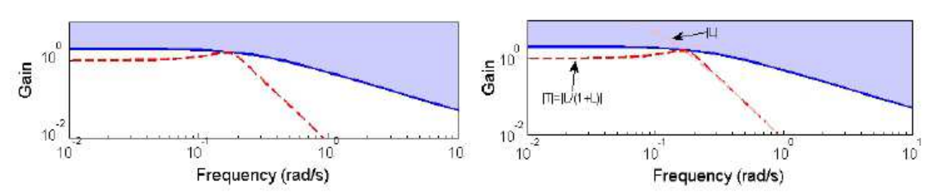
For small frequencies, W_S large $\implies 1 + L$ large, and $|L| \approx |1 + L|$.

$$|L(i\omega)| \geq |W_S(i\omega)| \quad (approx.)$$

Complementary sensitivity vs loop gain

$$T = \frac{L}{1+L}$$

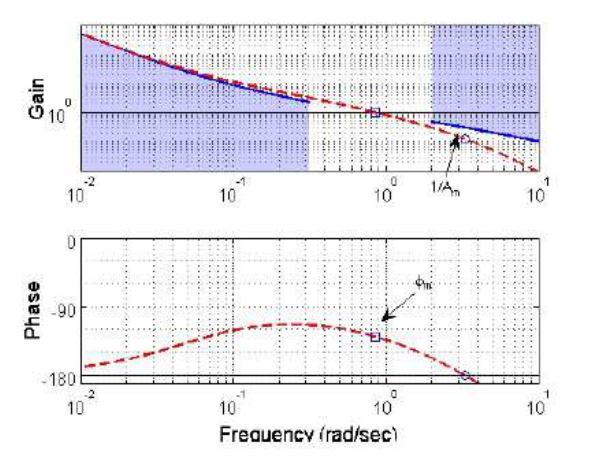
$$|T(i\omega)| \leq |W_T^{-1}(i\omega)| \Longleftrightarrow \frac{|L(i\omega)|}{|1+L(i\omega)|} \leq |W_T^{-1}(i\omega)|$$



For large frequencies, W_T^{-1} small $\Longrightarrow |T| \approx |L|$

$$|L(i\omega)| \leq |W_T^{-1}(i\omega)| \quad (approx.)$$

Resulting constraints on loop gain L:

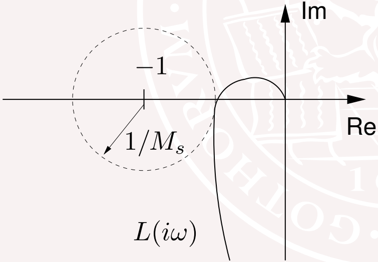


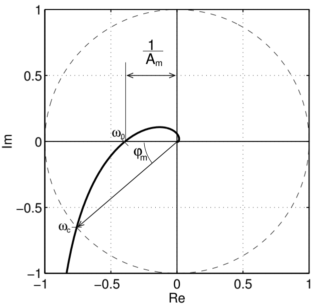
Approximations are inexact around cross-over frequency ω_c. In this region, focus is on stability margins (A_m, φ_m)

M_s and M_t vs gain and phase margins

Specifying $|S(i\omega)| \leq M_s$ and $|T(i\omega)| \leq M_t$ gives bounds for the gain and phase margins (but not the other way round!)

$$|S(i\omega)| \leq M_s \quad \Longrightarrow \quad A_m > \frac{M_s}{M_s - 1}, \quad \varphi_m > 2 \arcsin \frac{1}{M_s}$$





(Q: Why do not A_m and φ_m give bounds on M_s and M_t?)

M_s and M_t vs gain and phase margins

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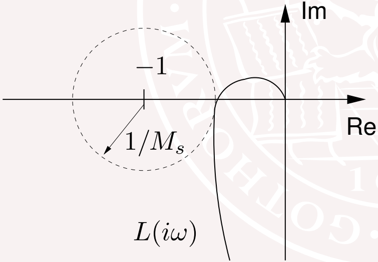
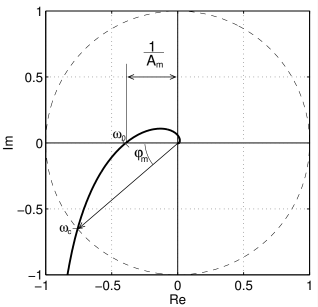




(Q: Why do not A_m and φ_m give bounds on M_s and M_t?)

Lead–lag compensation

Shape the loop gain $L = PC$ using a compensator C composed of

- Lag (phase retarding) elements

$$C_{lag}(s) = \frac{s+a}{s+a/M}, \quad M > 1$$

- Lead (phase advancing) elements

$$C_{lead}(s) = N\frac{s+b}{s+bN}, \quad N > 1$$

- Gain

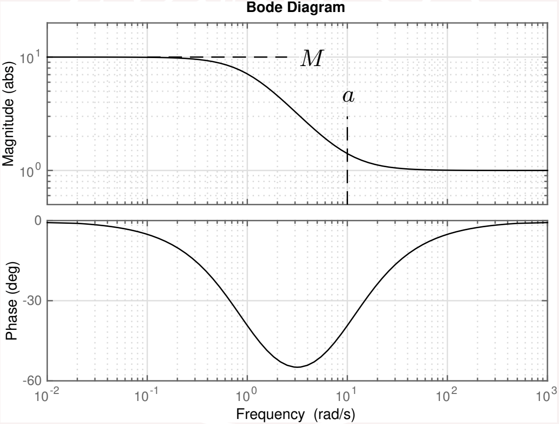
$$K$$

Example:

$$C(s) = K\frac{s+a}{s+a/M} \cdot N\frac{s+b}{s+bN}$$

Lag filter

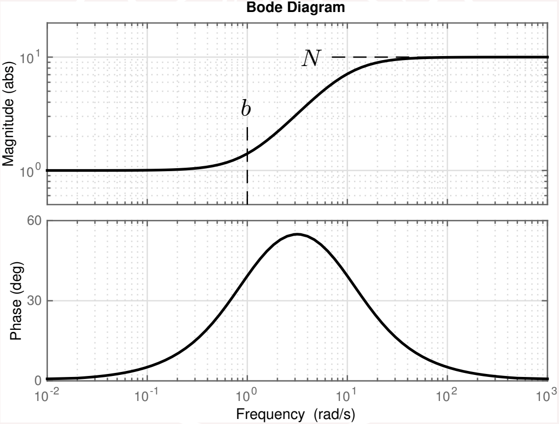
$$G_{lag}(s) = \frac{s+a}{s+a/M}, \quad M > 1$$



Special case: $M = \infty \;\Rightarrow$ integrator

Lead filter

$$G_{lead}(s) = N\frac{s+b}{s+bN}, \quad N > 1$$



Maximum phase advance for different N given in Collection of Formulae

Properties of lead–lag filters

- Lag element
  - Reduces static error
  - Reduces stability margin
- Lead element
  - Increases speed (by increasing ω_c)
  - Increased phase

    $\Longrightarrow$ May improve stability
- Gain
  - Translates the magnitude curve
  - Does not change phase curve

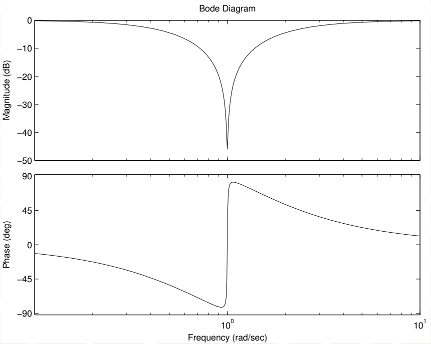
Iterative lead–lag design

- Step 1: Lag (phase retarding) element
  - Add phase retarding element to get low-frequency asymptote right
- Step 2: Phase advancing element
  - Use phase advancing element to obtain correct phase margin
- Step 3: Adjust gain
  - Usually need to amplitude curve to obtain the desired cross-over frequency.

Adjusting the gain in Step 3 leaves the phase unaffected, but may ruin low-frequency asymptote (need to revise lag element) $\Longrightarrow$ Need to iterate!

Example of other compensation link:

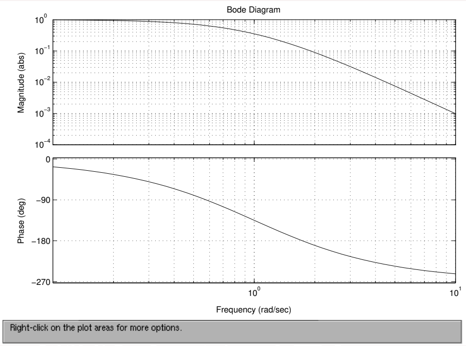
Notch filter $\dfrac{s^2 + 0.01s + 1}{s^2 + 2s + 1}$

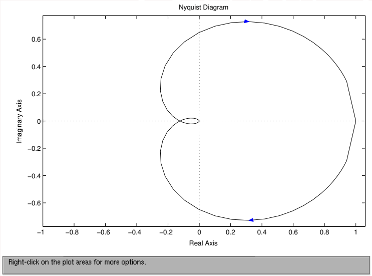


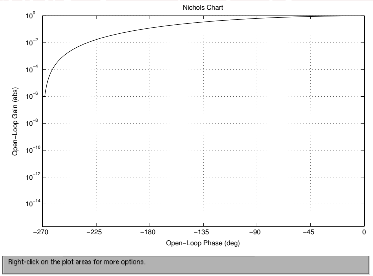
(E.g., supress measurement noise at specific frequency)

Bode, Nyquist and Nichols diagrams

$$\log|PC| = \log|P| + \log|C|$$
$$\arg\{PC\} = \arg\{P\} + \arg\{C\}$$







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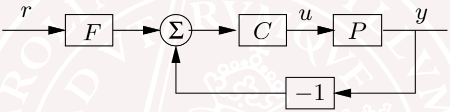
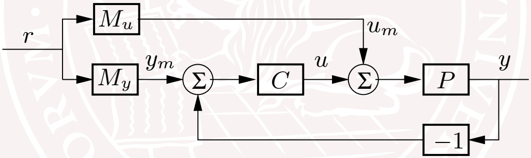
Feedforward design

Examples of 2-DOF configurations:

(1)

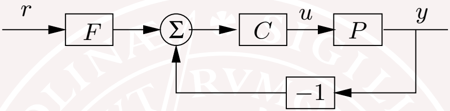


(2)



Ideally, we would like the output to follow the setpoint perfectly, i.e. $y = r$

Feedforward design (1)



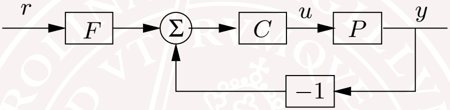
Perfect following requires

$$F = \frac{1 + PC}{PC} = T^{-1}$$

In general impossible because of pole excess in T. Also

- T might contain non-minimum-phase factors that can/should not be inverted
- u must typically satisfy some upper and lower limits

Feedforward design (1)

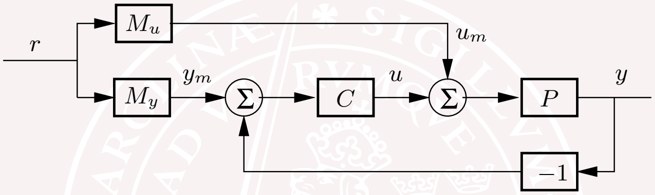


Assume T minimum phase. An implementable choice of F is then

$$F(s) = \frac{1 + P(s)C(s)}{P(s)C(s)(sT_f + 1)^d}$$

where d is large enough to make F proper and implementable

Feedforward design (2)

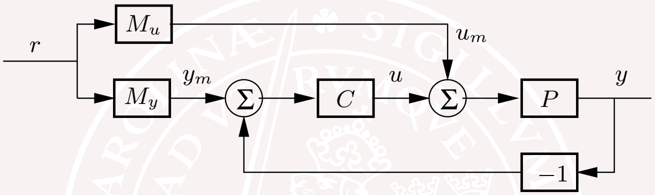


M_y and M_u can be viewed as generators of the desired output y_m and the input u_m that corresponds to y_m

For y to follow y_m, select

$$M_u = M_y/P$$

Feedforward design (2)



Since $M_u = M_y/P$ should be stable, causal and proper we find that

- Unstable zeros of P must be zeros of M_y
- Time delays of P must be time delays of M_y
- The pole excess of M_y must not be smaller than the pole excess of P

Take process limitations into account!

Feedforward design – example

Process:

$$P(s) = \frac{1}{(s+1)^4}$$

Selected reference model:

$$M_y(s) = \frac{1}{(sT+1)^4}$$

Then

$$M_u(s) = \frac{M_y(s)}{P(s)} = \frac{(s+1)^4}{(sT_f+1)^4} \qquad M_u(\infty) = \frac{1}{T^4}$$

Fast response (small T_f) requires high gain of M_u.

Bounds on the control signal limit how fast response we can obtain.