FRTN10 Exercise 4. Loop Shaping, Preparations for Lab 1

Exercises 4.1–4.4 are preparatory exercises for Laboratory Session 1. In these exercises, we will design a controller, step by step, for the process given by the transfer function

$$P(s) = \frac{1}{s^2 + 0.7s + 1}$$

4.1 Create a transfer function object in Matlab, and take a look at the Bode and Nyquist diagrams of the process. In the following exercises you will use a number of different controllers to shape the Bode diagram of the open-loop system.

The structure of the control system is given in Figure 4.1. As you may have already heard, several transfer functions should be studied in a design. Besides a nice step response from r to y, also a fast recovery from, e.g., load disturbances, d is required. Furthermore, it is important to see how the control signal responds to the different input signals.

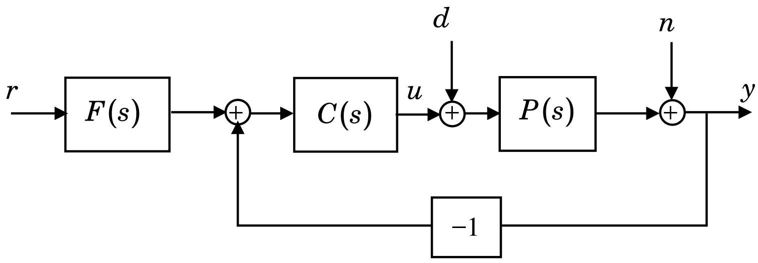

Figure 4.1 Our control loop with reference signal r, load disturbance d, measurement noise n and output y.

We have **two** degrees of freedom in designing our controller; the feedback part $C(s)$ and the prefilter $F(s)$.

We start by designing $C(s)$. For evaluation, we can look at the effect of a step load disturbance d (as this is only affected by the feedback loop). A good load step disturbance response goes quickly to zero. What is the closed-loop transfer function from d to y?

a. We will first try to control the system using a simple P-controller. Simulate **load step responses** for K=0.1, 1.0, 5.0, 10.0. Does the output go to zero? How much stationary error is left for different K's?

Tip: Use the Matlab command `figure(n)` to draw several plots. E.g.:

```
>> figure(1)
>> bode(P*C, P)     % Plot both the process and the
>>                  % compensated open-loop process
>> figure(2)
>> step(P/(1+C*P)) % Plot step load disturbance
>> figure(3)
>> bode(P/(1+C*P)) % Bode plot of closed loop from
>>                  % load disturbance
```

b. To remove the stationary error in the response to load disturbances, we need to add integral action to the controller. The transfer function of a PI controller is given as

$$C(s) = K\left(1 + \frac{1}{sT_i}\right) = K\left(\frac{sT_i + 1}{sT_i}\right)$$

Try some different values of K and T_i, and plot the step load response. Study the Bode diagram of the open-loop system. What effect does the integrator have on the phase curve? Try to find a controller that gives good performance. The error should vanish fast without too much oscillation.

4.2 To create a more advanced controller, we need to know the effect of adding additional poles and zeros to the controller, $C(s)$.

a. Let $C_a(s) = \frac{1}{s/a+1}$. How are the magnitude and the phase affected by a?. When and why would we add this kind of system to our controller?

b. Let $C_b(s) = \frac{s/b+1}{1}$. Again, how are magnitude and phase affected by b? When/why would we add this kind of system to our controller?

4.3 By combining a pole and a zero, we get a compensator in the form

$$C(s) = K\frac{(s/b + 1)}{(s/a + 1)}$$

A compensator where $b < a$ is called a *lead compensator*, and a compensator having $b > a$ is called a *lag compensator*. Plot the Bode diagrams for the two cases, and recall the properties of the lead and lag compensators from the basic course.

Now create a feedback controller

$$C(s) = K\frac{(sT_i + 1)}{sT_i} \cdot \frac{(s/b + 1)}{(s/a + 1)}$$

for the process $P(s)$, by adding a pole and a zero to the PI controller in Problem 4.1. Note that the added compensator will allow you to adjust the parameters, K and T_i, of the PI controller.

Requirements:

- The disturbance step response should settle within about 5 seconds. Specifically, $|y(t)| < 4 \cdot 10^{-3}$ for $t > 5$.
- No more than 20% overshoot in the step response from r to y.

Hints:

- Faster response is often tightly connected to a higher cut-off frequency ω_c.
- Oscillations are due to bad margins (being close to the -1 point in the Nichols or Nyquist diagrams).

If needed you can add more poles and zeros to your controller, but make sure that you keep the number of poles at least as many as the zeros. This will ensure that the system is *proper*, i.e. is not containing a pure derivative.

4.4 **a.** Calculate the closed-loop transfer function $G_{yr}(s)$ from reference r to output y with your controller in the loop. What would be the ideal frequency response for this transfer function?

b. The control signal $u(t)$ to the process is physically limited by

$$-10 \leq u(t) \leq 10,$$

which must be taken into account in the design. This limits how fast we can change the process.

Simulate the step response. Is the constraint on $u(t)$ satisfied? Improve $G_{yr}(s)$ by tuning the prefilter $F(s)$ so that the step response behaves nicely. How should $F(s)$ compensate $G_{yr}(s)$ in the frequency domain?

4.5 (*) A servo system has the transfer function:

$$G_o(s) = \frac{2.0}{s(s+0.5)(s+3)}$$

The closed system has a step response according to Figure 4.2. It is clear that the system is poorly damped and has a large overshoot. It is, however, fast enough. Create a lead controller that stabilizes the system by increasing the phase margin to $\phi_m = 50°$, without changing the cut-off frequency. ($\phi_m = 50°$ gives a relative damping of $\zeta \approx 0.5$ which achieves an overshoot of $M \approx 17\%$). The stationary error for the closed system is 0.75 for a ramp-shaped input signal. The error for a ramp function at the compensated system must not exceed 1.5.

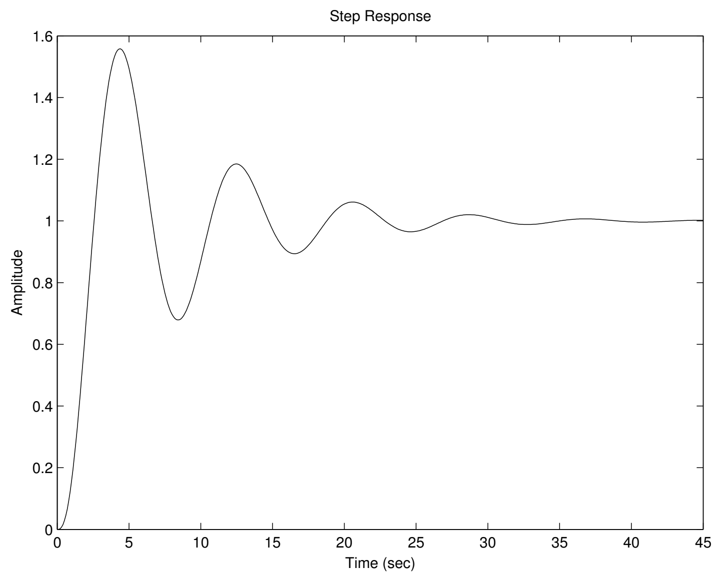

Figure 4.2 Step response from the closed servo system in 4.5.

Hint: It might be helpful to review the chapter in the basic course about lead/lag filters and the design process presented there.

4.6 (*) Consider the control system in Figure 4.1, where the plant is described by

$$P(s) = \frac{1}{(s+1)(s+0.02)}$$

and $F(s) = 1$. An unexperienced engineer has designed the controller

$$C(s) = \frac{(s+a)}{s}$$

with $a = 0.02$, but the resulting control system reacts extremely slowly to step disturbances in d. The reason is that the slow pole in -0.02 is canceled by the controller zero. The Bode diagrams of the plant, the controller, and the open-loop system are shown in Figure 4.3.

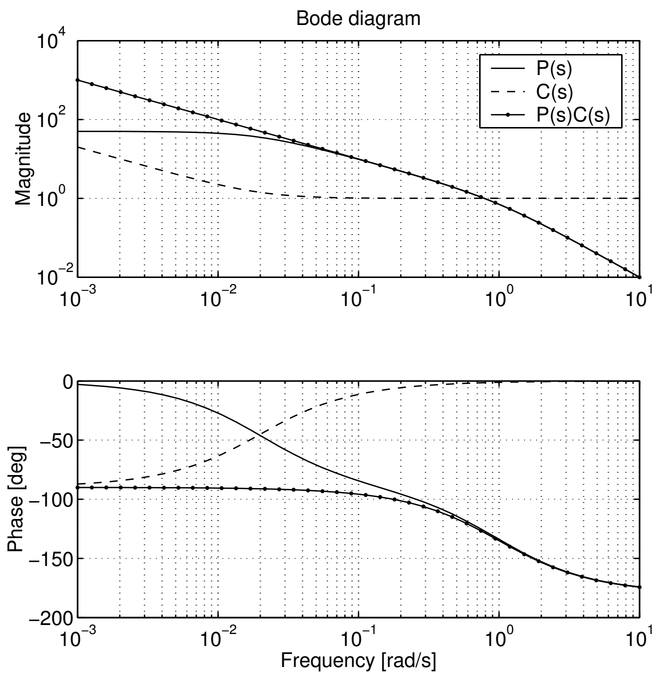

Figure 4.3 The Bode diagrams of $P(s)$, $C(s)$ and the open loop $P(s)C(s)$ when $a = 0.02$.

a. The load disturbance d is typically most significant at low frequencies, so we are interested in keeping the magnitude of the transfer function G_{yd} from d to y significantly smaller than 1 in a frequency range $[0, \omega_b]$. What is (approximately) ω_b if you use the given controller? Use the Bode diagram in Figure 4.3.

b. To reject the disturbance d faster, ω_b should be increased. For noise reasons, we want the cross-over frequency of the system to be the same.

How should the value of a in the controller be changed to achieve this? Motivate your design by showing that:

- The range $[0, \omega_b]$ where you get good disturbance rejection of d is increased.
- The cross-over frequency of the system is still approximately the same.

Exact proofs are not required; some Bode-diagram reasoning will do.

Solutions to Exercise 4. Loop Shaping, Preparations for Lab 1

4.1 *This is a preparatory exercise for the laboratory session. This is not a complete solution, just some helpful tips.*

a. Since we cannot change the phase of the system using a P-controller, higher gain will lead to lower phase margin (as the phase approaches -180 for high frequencies).

Higher gain will also decrease stationary errors, but increase the maximum peak in the sensitivity function (making the system very sensitive to measurement noise).

```
>> figure(1)
>> step(P/(1+0.1*P),P/(1+1*P),P/(1+5*P),P/(1+10*P));
>> title('Step responses')
>> figure(2)
>> bode(P/(1+0.1*P),P/(1+1*P),P/(1+5*P),P/(1+10*P));
>> title('Transfer functions from load disturbance');
>> figure(3)
>> bode(1/(1+0.1*P),1/(1+1*P),1/(1+5*P),1/(1+10*P));
>> title('Sensitivity functions');
```

b. It is not possible to achieve good behavior with a PI controller, but try to get it as good as possible:

```
>> figure(1)
>> K= ... ; Ti = ... ;
>> C = tf(K*[1 1/Ti],[1 0]);
>> step(P/(1+C*P);
```

4.2 *This is a preparatory exercise for the laboratory session. This is not a complete solution, just some helpful tips.*

a. From the basic course: We calculate the gain $\|C(i\omega)\| = 1/\sqrt{\omega^2/a^2+1}$ and use log scale. Then

$$\log|C(i\omega)| = -0.5\log(\omega^2/a^2+1) \approx \begin{cases} 0 & \omega << a \\ \log(a) - \log(\omega) & \omega >> a \end{cases}$$

and the two lines meet where $\omega = a$ (the breakpoint). Also, the phase is at -45° at $\omega = a$, starts at 0° and ends at -90°.

We can add a pole to the controller if we want to decrease gain for higher frequencies, e.g., to limit the cut-off frequency ω_c. It is often the case that we want to increase the gain at low frequencies, but keep it low at high frequencies. We can then use a controller of the type $C(s) = K/(s/a+1)$ with a pole to limit high frequency gain and a static gain larger than one to increase the low frequency gain.

```
>> C01 = tf([1],[1/0.1 1]);
>> C1 = tf([1],[1/1 1]);
>> C5 = tf([1],[1/5 1]);
>> bode(C01, C1, C5);
```

b. The same as in (a), except that a zero breaks the gain **up** at b.

$$\log|C(i\omega)| = 0.5\log(\omega^2/b^2+1) \approx \begin{cases} 0 & \omega << b \\ \log(\omega) - \log(b) & \omega >> b \end{cases}$$

We can add a zero to the controller to increase gain at high frequencies in order to increase the cut-off frequency ω_c. Also, since the phase of the zero goes to $+90°$, we increase the phase margin by adding a zero.

4.3 *This is a preparatory exercise for the laboratory session. This is not a complete solution, just some helpful tips.*

The following Matlab code shows some relevant plots for a design:

```
>> s = tf('s');
>> C = ...;       % Make up your own design

>> figure(1)
>> margin(C*P)    % Plot open-loop frequency response

>> figure(2)
>> % Plot step responses from load disturbance and reference signal to output signal y.
>> subplot(2,1,1)
>> step(P/(1+P*C));
>> title('Load step response');
>>
>> subplot(2,1,2)
>> step(P*C/(1+P*C))
>> title('Reference step response');
```

4.4 *This is a preparatory exercise for the laboratory session. This is not a complete solution, just some helpful tips.*

a. The ideal frequency response is $G_{yr} \equiv 1$. Then we would always have $y = r$. However, achieving something close to this would require very aggressive control, so that is not a good idea. (The controller would need to invert the process dynamics, resulting in second-order derivative action on the control error).

b. We want to shape $F(s)$ so that the constraints on the control signal are respected, for a step change in the reference. This may be achieved by reducing the bandwidth.

4.5 Plot the Bode diagram for $G_o(s)$ in Matlab or use the command

```
>> [Gm,Pm,Wcg,Wcp] = margin(G_o)
```

to calculate the cut-off frequency $\omega_c = 0.73$ and the phase margin $\phi_m = 20.7°$. To reach the aim of a $\phi_{m,desired} = 50°$, the controller has to increase the phase at the cut-off frequency with approx $30°$. We use the lead compensation given by

$$G_k(s) = KN\frac{s+b}{s+bN}$$

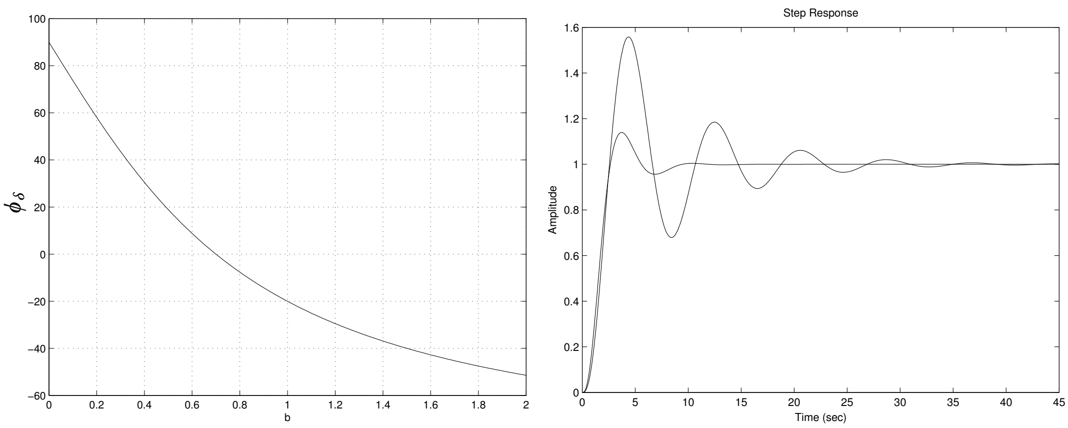

Figure 4.1 To the left: Plot of ϕ_δ against b. To the right: Step response from the original system and the compensated system in Problem 4.5.

with the phase

$$\phi = \arctan\left(\frac{s}{b}\right) - \arctan\left(\frac{s}{bN}\right)$$

The maximum of the phase compensation for the compensator is at the frequency $b\sqrt{N}$, which preferably should coincide with ω_c, hence $N = (\omega_c/b)^2$. Plot the phase addition of the compensator given by

$$\phi_\delta = \arctan\left(\frac{\omega_c}{b}\right) - \arctan\left(\frac{b}{\omega_c}\right)$$

and determine that the factor $b \approx 0.4$ for $\phi_\delta = 30°$ (see Figure 4.1). To keep the cut-off frequency invariant the gain of the compensator has to be calculated from $|G_k(i\omega_c)G_o(i\omega_c)| = K\sqrt{N} \cdot 1$ gives $K = \dfrac{1}{\sqrt{N}} = 0.55$. Plot the step response by the commands:

```
>> G_l=tf(K*N*[1 b],[1 b*N])
>> step(G_o*G_l/(1+(G_o*G_l))
```

The stationary error:

$$E(s) = \frac{1}{1 + G_k G_o} U(s) = \frac{s(s+0.5)(s+3)(s+bN)}{s(s+0.5)(s+3)(s+bN) + 2KN(s+b)} U(s)$$

The Laplace transform of a ramp function is $U(s) = 1/s^2$ and the error is

$$\lim_{s \to 0} sE(s) = \frac{1.5}{2K} = 1.37$$

which fulfills the specification.

4.6 **a.** The transfer function from d to y is given by

$$G_{yd}(s) = \frac{P}{1 + PC}$$

For frequencies $\omega \leq 0.5$ (approximately), it can be seen in the Bode diagram that both $|P(i\omega)| \gg 1$ and $|P(i\omega)C(i\omega)| \gg 1$. Therefore $G_{yd}(s) \approx \frac{1}{C}$, and $|C(i\omega)|$ becomes larger than 1 for frequencies $\omega \leq 0.02$.

The magnitude of $G_{yd}(s)$ is thus smaller than 1 in a frequency range of approximately $[0, 0.02]$, thus $\omega_b = 0.02$ rad/s.

This can also be seen as the frequency point where $|PC|$ becomes larger than $|P|$ in the bode diagram.

b. To increase ω_b, we would like to increase the gain of $C(i\omega)$ for frequencies $\omega > 0.02$. This is done by moving the zero in $C(s)$ (the break-point in the Bode diagram) from 0.02 to some higher frequency.

Choose, e.g., $a = 0.1$. Motivation:

- As $G_{yd}(s) \approx \frac{1}{C}$, and $|C(i\omega)|$ now becomes larger than 1 for frequencies $\omega \leq 0.1$, ω_b has been increased to about 0.1.
- The cut-off frequency for $a = 0.02$ is $\omega_c \approx 0.8$. As this frequency is higher than the new break-point 0.1, $C(i\omega_c) \approx 1$ still holds $\Rightarrow$ the cut-off frequency stays the same.