Recall Example: Wind Farm Control

A wind farm is controlled to minimize structural loads subject to fixed power production:

$$\text{Minimize} \quad \mathbf{E} \sum_k (x_k^2 + u_k^2)$$

subject to $u_1 + \ldots + u_n = 0$ and

$$\begin{cases} \dot{x}_1 = -x_1 + u_1 + w_1 \\ \vdots \\ \dot{x}_n = -x_n + u_n + w_n \end{cases}$$

Compare the solutions for $n = 1$, $n = 2$, $n = 10$ and $n = 100$.

Wind Farm Example Revisited

Define the average structural load $x_0 = \frac{1}{n}(x_1 + \cdots + x_n)$ and the deviation from average $z_k = x_k - x_0$. Then

$$\dot{x}_0 = -x_0 + \frac{1}{n}(w_1 + \cdots + w_n) \qquad \mathbf{E}x_0^2 = \frac{1}{2n}$$

$$\dot{z}_k = -z_k + u_k + \frac{1}{n}(w_1 + \cdots + w_n)$$

with the optimal control law $u_k = -\ell z_k = -\ell(x_k - x_0)$.

Hence every turbine should compute the optimal control $-\ell x_k$ without constraint, then subtract the average over all turbines!

As a result

$$\dot{x}_k = -(1 + \ell)x_k + \ell x_0 + w_k$$

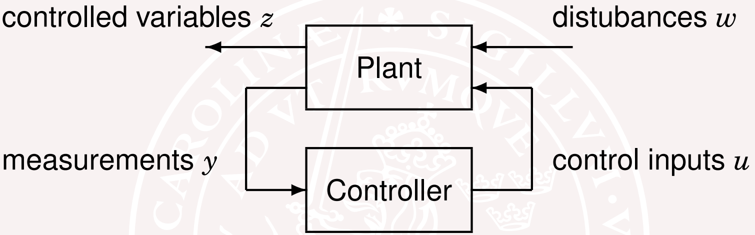
The variance of the term ℓx_0 decreases with n, so for large farms the constraint $u_1 + \ldots + u_n = 0$ is negligible. On the other hand, for a farm with just one turbine, it would imply that $u_1 = 0$.

Lecture 12: Internal Model Control

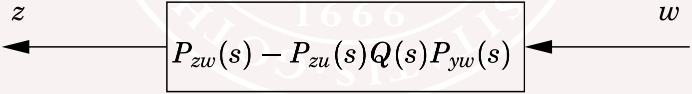
- Youla Parametrization
- Internal Model Control
- Dead Time Compensation

Section 8.4 in Glad/Ljung.

The Q-parametrization (Youla)

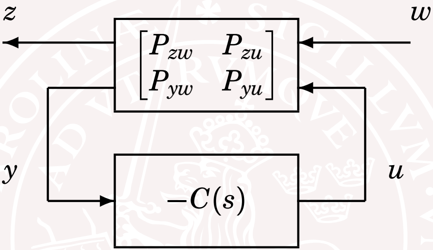


Idea for lecture 12-14:
The choice of controller generally corresponds to finding $Q(s)$, to get desirable properties of the map from w to z:



Once $Q(s)$ is determined, a corresponding controller is found.

The Youla Parametrization



The closed loop transfer matrix from w to z is

$$G_{zw}(s) = P_{zw}(s) - P_{zu}(s)Q(s)P_{yw}(s)$$

where

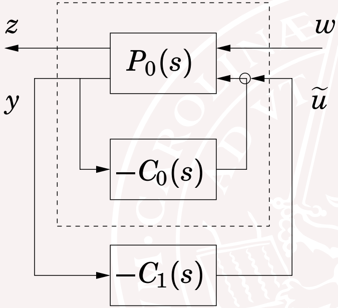
$$\begin{aligned} Q(s) &= C(s)\left[I + P_{yu}(s)C(s)\right]^{-1} \\ C(s) &= Q(s) + Q(s)P_{yu}(s)C(s) \\ C(s) &= \left[I - Q(s)P_{yu}(s)\right]^{-1}Q(s) \end{aligned}$$

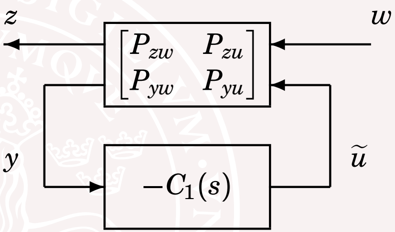
Closed loop maps for stable plants

Suppose the original plant P is stable. Then

- Stabilty of $Q(s)$ implies stability of $P_{zw}(s) - P_{zu}(s)Q(s)P_{yw}(s)$
- If $Q = C\left[I + P_{yu}C\right]^{-1}$ is unstable, then the closed loop is unstable.

Closed loop maps for unstable plants





In case $P_0(s)$ is unstable, let $C_0(s)$ be a stabilizing controller. Then the previous argument can be applied with P_{zw}, P_{zu} and P_{yw} representing the stabilized closed loop system.

Next lecture: Synthesis by convex optimization

A general control synthesis problem can be stated as a convex optimization problem in the variable $Q(s)$. The problem could have a quadratic objective, with linear/quadratic constraints:

Minimize $\int_{-\infty}^{\infty} |P_{zw}(i\omega) + P_{zu}(i\omega) \overbrace{\sum_k Q_k\phi_k(i\omega)}^{Q(i\omega)} P_{yw}(i\omega)|^2 d\omega$ } quadratic objective

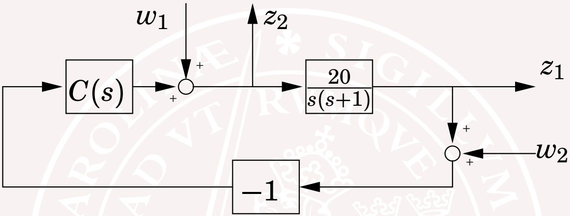
subject to
- step response $w_i \to z_j$ is smaller than f_{ijk} at time t_k
- step response $w_i \to z_j$ is bigger than g_{ijk} at time t_k

} linear constraints

- Bode magnitude $w_i \to z_j$ is smaller than h_{ijk} at ω_k } quadratic constraints

Here $Q(s) = \sum_k Q_k\phi_k(s)$, where $\phi_1, \dots, \phi_m$ are fixed "basis functions" and $Q_0, \dots, Q_m$ are optimization variables. Once $Q(s)$ has been determined, the controller is obtained as
$C(s) = \left[I - Q(s)P_{yu}(s)\right]^{-1} Q(s)$

Example — DC-motor



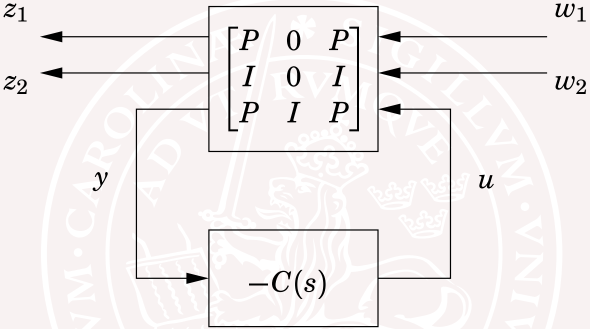
The transfer matrix from (w_1, w_2) to (z_1, z_2) is

$$G_{zw}(s) = \begin{bmatrix} \frac{P}{1+PC} & \frac{-PC}{1+PC} \\ \frac{1}{1+PC} & \frac{-C}{1+PC} \end{bmatrix}$$

where $P(s) = \frac{20}{s(s+1)}$. How should we choose stable P_{zw}, P_{zu}, P_{yw} and Q to get

$$G_{zw}(s) = P_{zw}(s) - P_{zu}(s)Q(s)P_{yw}(s) \quad ?$$

Stabilizing nominal feedback for DC-motor

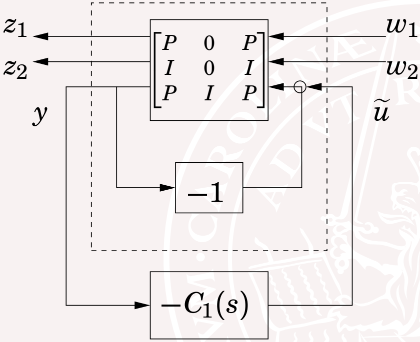


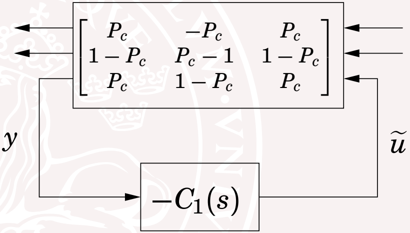
The plant $P(s) = \frac{20}{s(s+1)}$ is not stable, so write

$$C(s) = C_0(s) + C_1(s)$$

where $C_0(s) \equiv 1$ is a stabilizing controller.

Redraw diagram for DC motor example





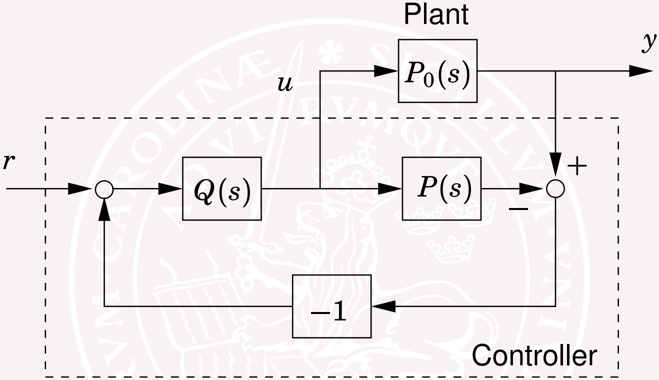
$$G_{zw}(s) = \begin{bmatrix} P_c & -P_c \\ 1-P_c & P_c-1 \end{bmatrix} + \begin{bmatrix} P_c \\ 1-P_c \end{bmatrix} Q \begin{bmatrix} P_c & 1-P_c \end{bmatrix}$$

where $P_c(s) = (1+P(s))^{-1}P(s) = \frac{20}{s^2+s+20}$ is stable.

Outline

- ◦ Youla Parametrization
- • **Internal Model Control**
- ◦ Dead Time Compensation

Internal Model Control

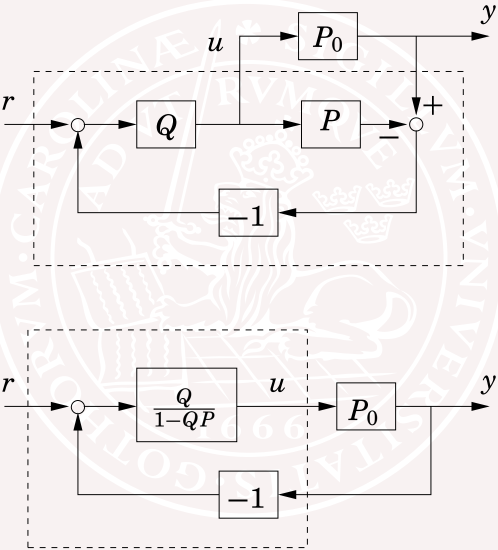


Feedback is used only as the real process deviates from $P(s)$.

The transfer function $Q(s)$ defines how the desired input depends on the reference signal.

When $P = P_0$, the transfer function from r to y is $P(s)Q(s)$.

Two equivalent diagrams



Internal Model Control — Strictly proper plants

When $P = P_0$, the transfer function from r to y is $P(s)Q(s)$.

Hence, ideally, one would like to put $Q(s) = P(s)^{-1}$. For several reasons this is not possible for accurate process models:

- If $P(s)$ is strictly proper, the inverse would have more zeros than poles. Alternatively, one could choose

$$Q(s) = \frac{1}{(\lambda s + 1)^n} P(s)^{-1}$$

  where n is large enough to make Q proper. The parameter λ influences the speed of control.

Internal Model Control — Zeros and delays

Once again, ideally, one would like to put $Q(s) = P(s)^{-1}$.

Here are other reasons why this is often not possible:

- If $P(s)$ has unstable zeros, the inverse would be unstable. Alternatively, one could either remove every unstable factor $(-\beta s + 1)$ from the plant numerator before inverting, or replace it by $(\beta s + 1)$. With the latter alternative, only the phase is modified, not the amplitude function.
- If $P(s)$ includes a time delay, its inverse would have to predict the future. Instead, the time delay is removed before inverting.

Example 1 — First order plant model

$$P(s) = \frac{1}{\tau s + 1}$$

$$Q(s) = \frac{1}{\lambda s + 1} P(s)^{-1} = \frac{\tau s + 1}{\lambda s + 1}$$

$$C(s) = \frac{Q(s)}{1 - Q(s)P(s)} = \frac{\frac{\tau s+1}{\lambda s+1}}{1 - \frac{1}{\lambda s+1}} = \underbrace{\frac{\tau}{\lambda}\left(1 + \frac{1}{s\tau}\right)}_{\text{PI controller}}$$

Example 2 — Non-minimum phase plant

$$P(s) = \frac{-\beta s + 1}{\tau s + 1}$$

$$Q(s) = \frac{(-\beta s + 1)}{(\beta s + 1)} P(s)^{-1} = \frac{\tau s + 1}{\beta s + 1}$$

$$C(s) = \frac{Q(s)}{1 - Q(s)P(s)} = \frac{\frac{\tau s+1}{\beta s+1}}{1 - \frac{(-\beta s+1)}{(\beta s+1)}} = \underbrace{\frac{\tau}{2\beta}\left(1 + \frac{1}{s\tau}\right)}_{\text{PI controller}}$$

Outline

- ◦ Youla Parametrization
- ◦ Internal Model Control
- • **Dead Time Compensation**

Dead Time Compensation

Consider the plant model

$$P(s) = P_1(s)e^{-s\tau}$$

Let $C_0 = Q/(1 - QP_1)$ be the controller we would have used without delays. Then $Q = C_0/(1 + C_0P_1)$.

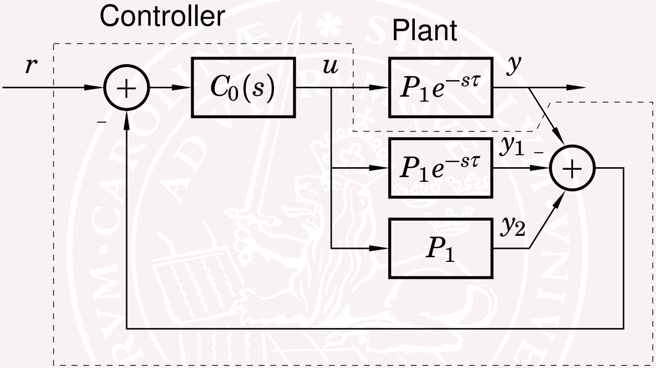
The rule of thumb tell us to use the same Q also for systems with delays. This gives

$$C(s) = \frac{Q(s)}{1 - Q(s)P_1(s)e^{-s\tau}} = \frac{C_0/(1 + C_0P_1)}{1 - e^{-s\tau}P_1C_0/(1 + C_0P_1)}$$

$$C(s) = \frac{C_0(s)}{1 + (1 - e^{-s\tau})C_0(s)P_1(s)}$$

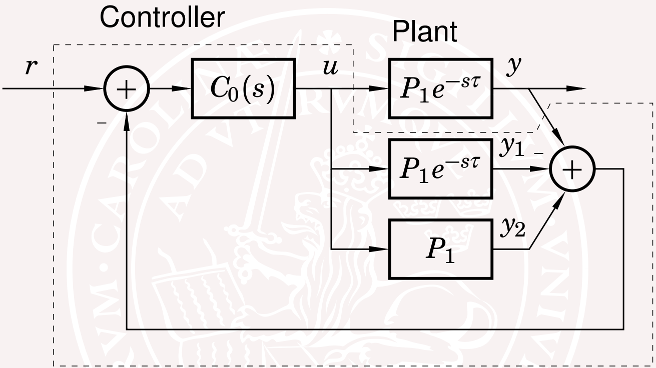
This modification of the $C_0(s)$ to account for time delays is known as dead time compensation according to Otto Smith.

Smith Compensator



Idea: Make an internal model of the process (with and without the delay) in the controller. Ideally Y and Y_1 cancel each other and use feedback from Y_2 "without delay".

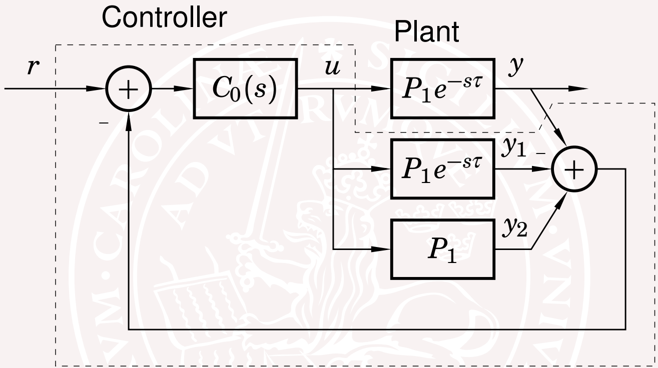
Smith Compensator



$$Y(s) = e^{-s\tau}\frac{C_0(s)P_1(s)}{1 + C_0(s)P_1(s)}R(s)$$

- Delay eliminated from denominator!
- Reference response greatly simplified!

Smith Compensator — A Success Story!

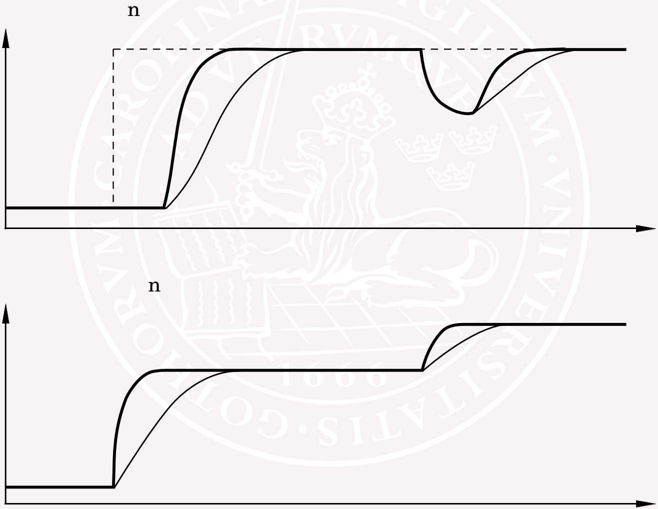


- Intriguing properties
- Numerous modifications
- Many industrial applications

Otto J.M. Smith listed in the ISA "Leaders of the Pack" list (2003) as one of the 50 most influential innovators since 1774.

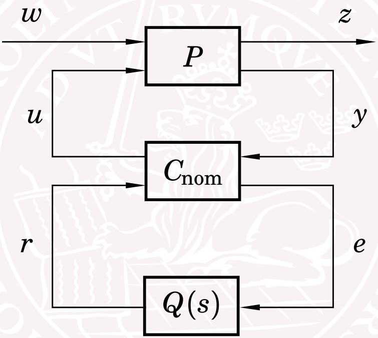
Example: Dead Time Compensation

Otto Smith compensator (thick) and standard PI controller (thin)



Youla parametrization revisited

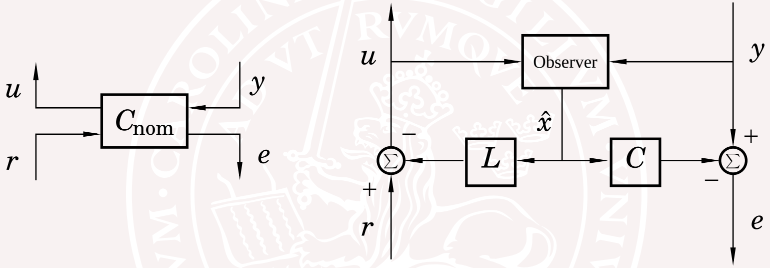
The Youla-parametrization:



where C_{nom} stabilizes the $[P, C]$-system and $Q(s)$ is any stable transfer function.

Nominal Controller

Linear system $\dot{x} = Ax + Bu + B_w w$, $y = Cx + D_w w$



with observer

$$\dot{\hat{x}} = A\hat{x} + Bu + Ke$$
$$u = r - L\hat{x}$$
$$e = y - C\hat{x}$$

Summary of Internal Model Control

- $Q(s)$ can be designed by hand for simple plants
- Ideas applicable also to multivariable plants
- Warning:
  Cancellation of slow poles gives poor disturbance rejection