FRTN10 Multivariable Control — Lecture 1

Anders Rantzer

Automatic Control LTH, Lund University

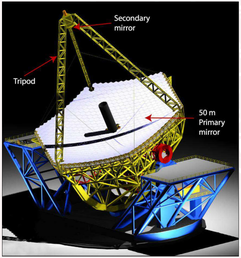
Todays lecture

- Introduction/examples
- Overview of course + feedback/feedforward
- Review linear systems
  - Review of time-domain models
  - Review of frequency-domain models
  - Norm of signals
  - Gain of systems

Many actuators and measurements

Example: Control of Large Deformable Telescope Mirror

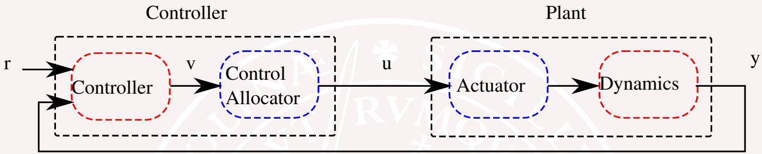
- Large number of sensors and actuators (500-3000)
- Computational limitations (1kHz)
- Tolerance ≈ 1 nano-meter
- Control accuracy crucial for telescope performance!

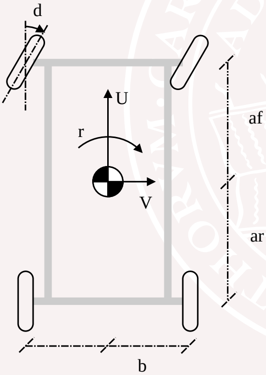


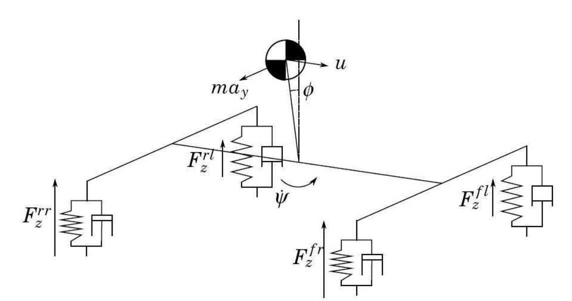
See more at e.g., `http://www.tmt.org/`

Example: Rollover protection needed

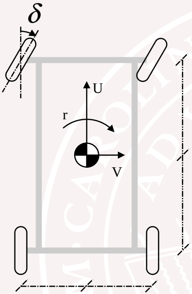
Rollover Control

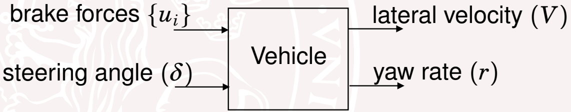






Car dynamics





State space model

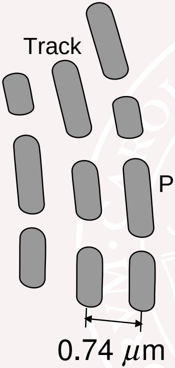
$$\begin{bmatrix} \dot{V} \\ \dot{r} \end{bmatrix} = A \begin{bmatrix} V \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ b_1 \end{bmatrix} (u_1 + u_2 - u_3 - u_4) + \begin{bmatrix} b_2 \\ b_3 \end{bmatrix} \delta$$

Fredrik Arp (Volvo) on Environmental Issues

[Sydsvenskan 2007]:
“Genom effektivisering av de konventionella bensin- och dieselmotorerna kan vi hämta hem en besparing på 20 procent i emissioner och bränsleekonomi de närmaste fem-sex åren”

Med andra ord: Bättre reglering gör skillnad!

The DVD reader tracking problem

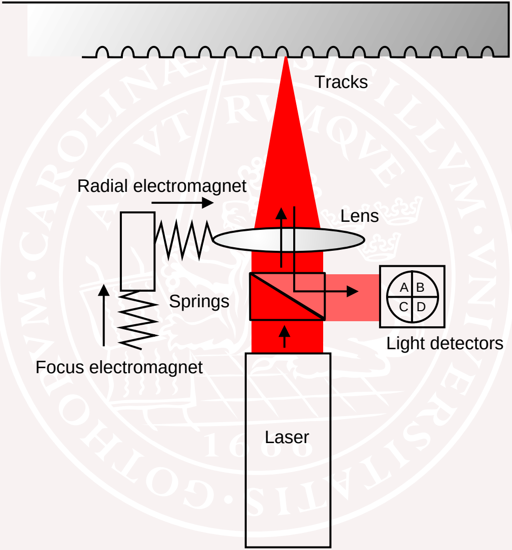


- 3.5 m/s speed along track
- 0.022 μm tracking tolerance
- 100 μm deviations at 23 Hz due to asymmetric discs

DVD Digital Versatile Disc, 4.7 Gb

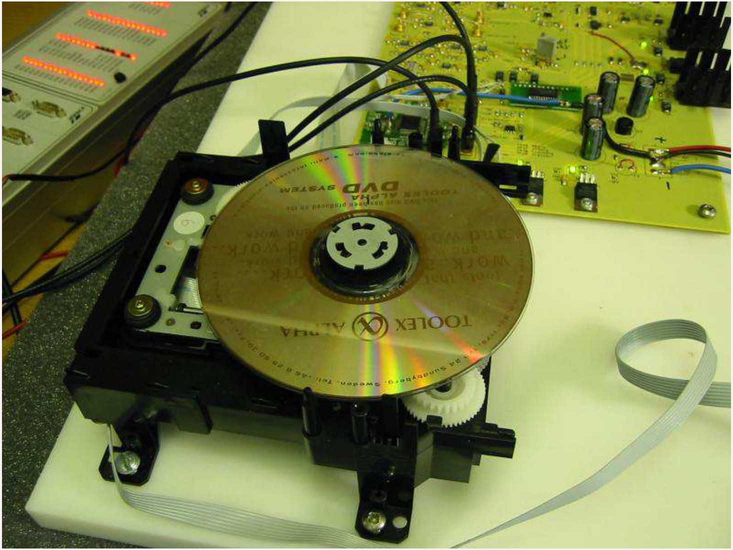
CD Compact Disc, 650 Mb, mostly audio and software

The DVD pick-up head

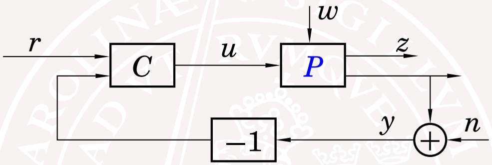


Input-output diagram for DVD control

The DVD reader in our lab



Control problem

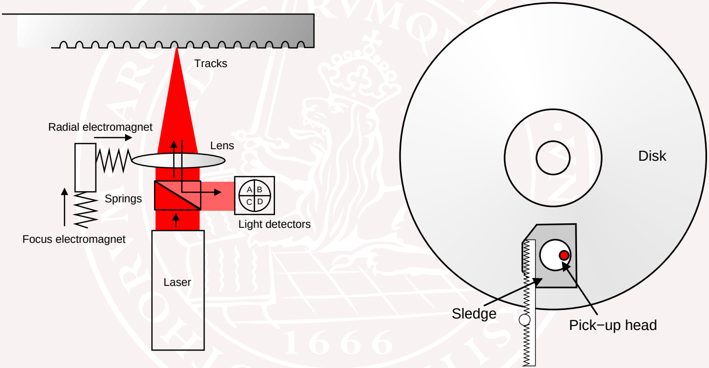


Given the system P and measurement signals y, determine the control signals u such that the control objective z follows the reference r as “close as possible” despite disturbances w, measurement errors n (noise etc.) and uncertainties of the real process.

For closed-loop ctrl $\Longrightarrow$ determine controller C.

Mid-ranging control

Example: Radial control of pich-up-head of DVD-player



The pick-up-head has two electromagnets for fast positioning of the lens (left). Larger radial movements are taken care of by the sledge (right).

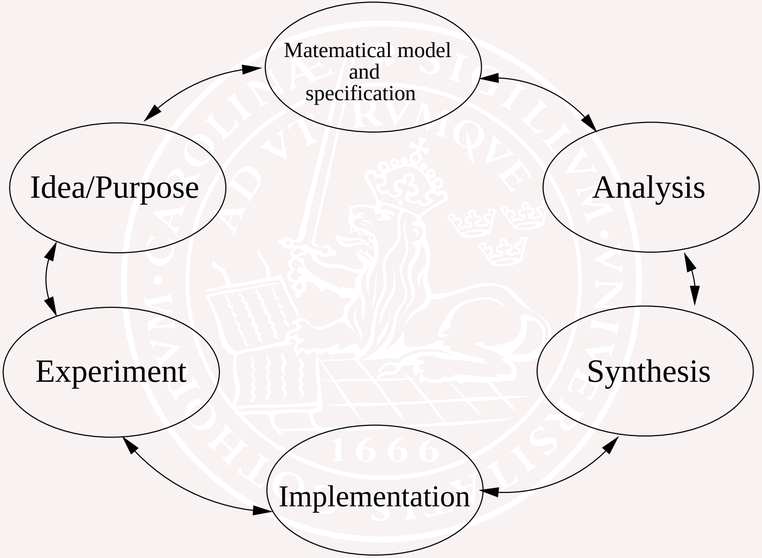
DVD in the course

- Focus control and tracking control lectured as a design example (Case study lecture 5)

What do we learn?

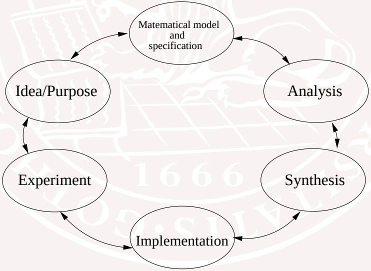
- Challenging design excercises
- Respect fundamental limitations
- Sampling frequency critical
- The use of observers

The design process



Contents of the course

L1-L5	Specifications, models and loop-shaping by hand
L6-L8	Limitations on achievable performance
L9-L11	Controller optimization: Analytic approach
L12-L14	Controller optimization: Numerical approach

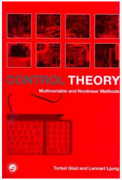


Course home page

`http://www.control.lth.se/Education/EngineeringProgram/FRTN10.html`

Literature

- T. Glad and L. Ljung:
  - Svensk utgåva: *Reglerteori – Flervariabla och olinjära metoder , 2nd ed* Studentlitteratur, 2004
  - English translation: *Control Theory – Multivariable and Nonlinear Methods*, Taylor and Francis
- Lecture Slides/Notes on the web
- Exercise problems with solutions on the web
- Laboratory PMs
- Swedish-English control dictionary on homepage



KFS sells the book

Course web page:

`http://www.control.lth.se/course/FRTN10`

Lectures

The lectures (30 hours) are given as follows:

Mondays Sep 2, 9, 16, 23 and Oct 14	8.15 in MH:A except today!
Wednesdays Sep 4, 11, 18 and Oct 2, 9, 16	8.15 in M:B
Thursdays Sep 5, 19, 26 and Oct 3	15.15 in M:B

All course material is in English.

The lectures are given by

Anders Rantzer and Per Hagander

Exercise sessions and TAs

The exercises (28 hours) are taught according to the schedule

First session	Monday 13–15	Monday 15–17	lab A and B
Second session	Thursday 13–15	Friday 13–15	lab A and B

They are all held in the department laboratory on the bottom floor in the south end of the Mechanical Engineering building.

Fredrik Magnusson Jerker Nordh Josefin Berner Ola Johnsson




Laboratory experiments

The three laboratory experiments are mandatory.

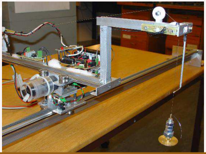
Sign-up lists are posted on the web at least one week before the first laboratory experiment. The lists close one day before the first session.

The Laboratory PMs are available at the course homepage.

Before the lab sessions some home assignments have to be done. No reports after the labs.

Lab	Week	Booking Starts	Responsible	Content
Lab 1	w 38-39	Sep 11	Josefin Berner	Flex-servo
Lab 2	w 41	Sep 23	Josefin Berner	Quad-tank
Lab 3	w 42	Oct 7	Fredrik Magnusson	Crane





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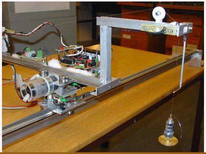

Exam

The exam (5 hours) will be given

- Wednesday Oct 23.

Lecture notes and text book are allowed, but no exercises material or extra hand-written notes.

Next time January 8, 2013 (pre-register on web `http://www.control.lth.se/Education/EngineeringProgram`).

Use of computers in the course

- Use personal student-account or a common course account
- Matlab in exercises and laboratories (!!)
- Web page:

  `http://www.control.lth.se/Education/EngineeringProgram/FRTN10`

Feedback is important

For each course LTH use the following feedback mechanisms

- CEQ (reporting / longer time scale)
- Student representatives (fast feedback)
  - Election of student representative ("kursombud")
- Email to `anders.rantzer@control.lth.se` `per.hagander@control.lth.se`

Help us close the loop for better performance!

Registration

You **must register for the course by signing the form available** upfront during the break (will be passed around also during the 2nd hour).

If your name is not in the form please fill in an empty row.

LADOK registration will be done immediately.

If you decide to abort/skip the course within three weeks from today you should inform me and then the LADOK registration will be removed.

Course Outline

L1-L5 Specifications, models and loop-shaping by hand

1. Introduction and system representations
2. Stability and robustness
3. Specifications and disturbance models
4. Control synthesis in frequency domain
5. Case study

L6-L8 Limitations on achievable performance

L9-L11 Controller optimization: Analytic approach

L12-L14 Controller optimization: Numerical approach

Lecture 1

- Description of linear systems (different representations)
  - Review of time-domain models
  - Review of frequency-domain models
- Norm of signals
- Gain of systems

State Space Equations

State-space and time-solution

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

$$y(t) = Ce^{At}x(0) + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$$

Example

$$\dot{x}_1 = -x_1 + 2x_2 + u_1 + u_2 - u_3$$
$$\dot{x}_2 = -5x_2 + 3u_2 + u_3$$
$$y_1 = x_1 + x_2 + u_3$$
$$y_2 = 4x_2 + 7u_1$$

How many states, inputs and outputs?

$$\dot{x} = Ax + Bu \qquad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} * & * \\ * & * \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$y = Cx + Du \qquad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} * & * \\ * & * \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

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$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & -1 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 7 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

State space form cont'd

Exampel:
2nd order differential equation

$$\ddot{y} + 3\dot{y} + 2y = 5u$$

Write on state space form.

How to chose states?
What if derivatives of input signal appears?

- Superposition
- Canonical forms
- Collection of formulae
- ...

State space form cont'd

Exampel:
2nd order differential equation

$$\ddot{y} + 3\dot{y} + 2y = 5u$$

Write on state space form.

How to chose states?
What if derivatives of input signal appears?

- Superposition
- Canonical forms
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- ...

Change of coordinates

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

Change of coordinates

$$z = Tx$$

$$\begin{cases} \dot{z} = T\dot{x} = T(Ax + Bu) & = T(AT^{-1}z + Bu) = TAT^{-1}z + TBu \\ y = Cx + Du & = CT^{-1}z + Du \end{cases}$$

Note: There are many different state-space representations for the same transfer function and system!

Change of coordinates

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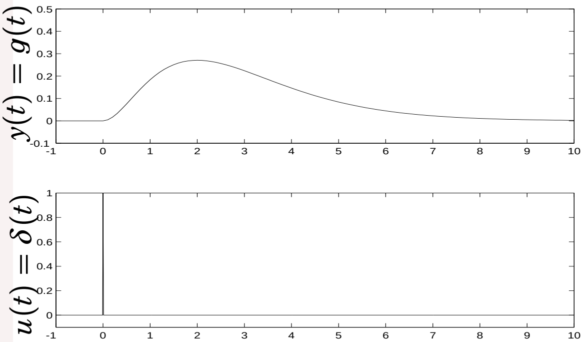
Impulse response

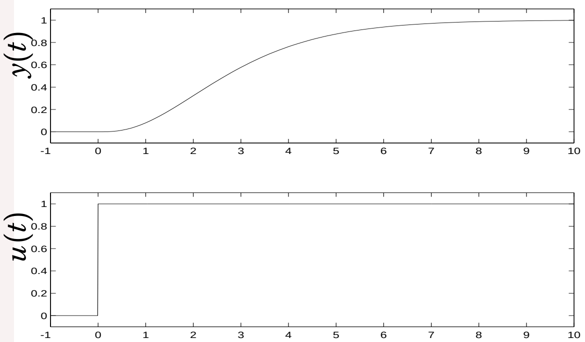


Common experiment in medicin and biology

$$g(t) = \int_0^t Ce^{A(t-\tau)}B\delta(\tau)d\tau + D\delta(t) = Ce^{At}B + D\delta(t)$$

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau = [g * u](t)$$

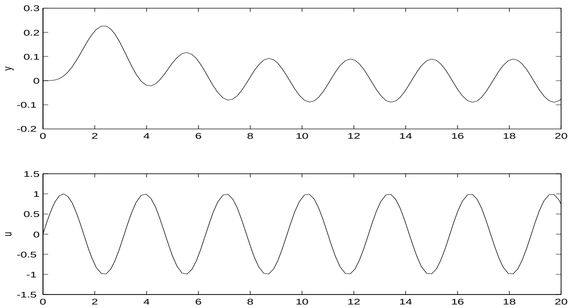
Step response



Common experiment in process industry

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau$$

Frequency response



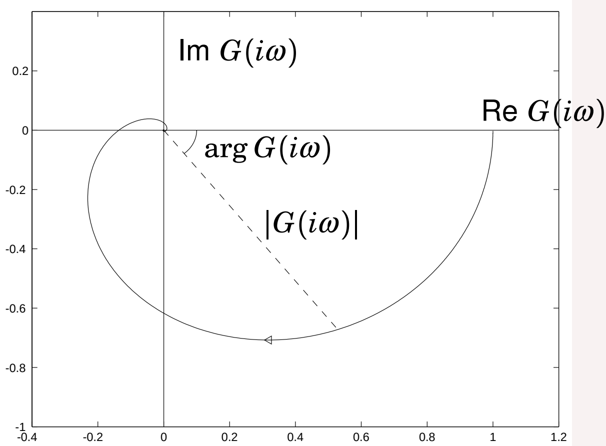
The transfer function $G(s)$ is the Laplace transform of the impulse response $G = \mathcal{L}g$. The input $u(t) = \sin \omega t$ gives

$$y(t) = \int_0^t g(\tau)u(t-\tau)d\tau = \text{Im}\left[\int_0^t g(\tau)e^{-i\omega\tau}d\tau \cdot e^{i\omega t}\right]$$

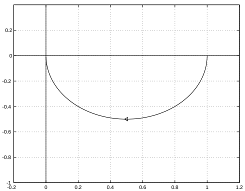
$$[t \to \infty] = \text{Im}\left(G(i\omega)e^{i\omega t}\right) = |G(i\omega)| \sin\left(\omega t + \arg G(i\omega)\right)$$

After a transient, also the output becomes sinusoidal

The Nyquist Diagram



Asymptotic formulas for first order system



$$G(s) = \frac{1}{s+1}$$

$$G(i\omega) = \frac{1}{i\omega + 1} = \frac{1 - i\omega}{\omega^2 + 1}$$

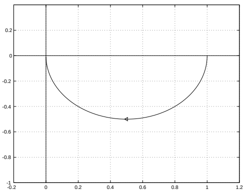
Small ω : $\quad G(i\omega) \approx 1$

Large ω : $\quad G(i\omega) \approx \frac{1}{\omega^2} - i\frac{1}{\omega}$

Matlab:

```
» s=tf('s');
» G=1/(s+1);
» nyquist(G)
```

Asymptotic formulas for first order system



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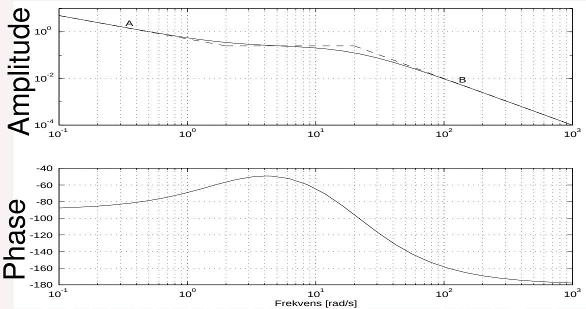
The Bode Diagram



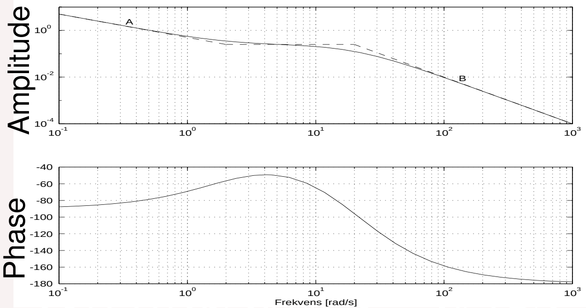
$$G = G_1 G_2 G_3 \qquad \begin{cases} \log|G| = \log|G_1| + \log|G_2| + \log|G_3| \\ \arg G = \arg G_1 + \arg G_2 + \arg G_3 \end{cases}$$

Each new factor enter additively!

Hint: Set matlab-scales

» `ctrlpref`

The Bode Diagram



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The L_2-norm of a signal

For $y(t) \in \mathbf{R}^n$ the "L_2-norm"

$$\|y\|_2 := \sqrt{\int_0^\infty |y(t)|^2 dt} \quad \text{is equal to} \quad \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} |\mathcal{L}y(i\omega)|^2 d\omega}$$

The equality is known as Parseval's formula

The L_2-gain of a system For a system $\mathcal{S}$ with input u and output $\mathcal{S}(u)$, the L_2-gain is defined as

$$\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2}$$

Miniproblem

What are the gains of the following systems?

1. $y(t) = -u(t)$ (a sign shift)
2. $y(t) = u(t-T)$ (a time delay)
3. $y(t) = \int_0^t u(\tau)d\tau$ (an integrator)
4. $y(t) = \int_0^t e^{-(t-\tau)}u(\tau)d\tau$ (a first order filter)

The L_2-gain from frequency data

Consider a stable system $\mathcal{S}$ with input u and output $\mathcal{S}(u)$ having the transfer function $G(s)$. Then, the system gain

$$\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2} \quad \text{is equal to} \quad \|G\|_\infty := \sup_\omega |G(i\omega)|$$

Proof. Let $y = \mathcal{S}(u)$. Then

$$\|y\|^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\mathcal{L}y(i\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(i\omega)|^2 \cdot |\mathcal{L}u(i\omega)|^2 d\omega \leq \|G\|_\infty^2 \|u\|^2$$

The inequality is arbitrarily tight when $u(t)$ is a sinusoid near the maximizing frequency.

W. Wright at Western Society of Engineers 1901

“Men already know how to construct wings or airplanes, which when driven through the air at sufficient speed, will not only sustain the weight of the wings themselves, but also that of the engine, and of the engineer as well. Men also know how to build engines and screws of sufficient lightness and power to drive these planes at sustaining speed ... **Inability to balance and steer still confronts students of the flying problem.** ... When this one feature has been worked out, the age of flying will have arrived, for all other difficulties are of minor importance.”

Wright was right!

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Smart Grid Gotland

SURF - exchange program with Caltech

Example: DARPA Grand Challenge and Team Caltech

- Autonomously Los Angeles to Las Vegas in < 10 h in 2004
- Lund students in SURF programme at Caltech every year
- `http://www.control.lth.se/SURF`

Course Outline

L1-L5 Specifications, models and loop-shaping by hand

1. Introduction and system representations
2. Stability and robustness
3. Disturbance models
4. Control synthesis in frequency domain
5. Case study

L6-L8 Limitations on achievable performance

L9-L11 Controller optimization: Analytic approach

L12-L14 Controller optimization: Numerical approach