## Lecture 4:

Todays lecture: Loop shaping design

- Specifications in frequency domain
- Loop shaping design

Continuing from lecture 3...

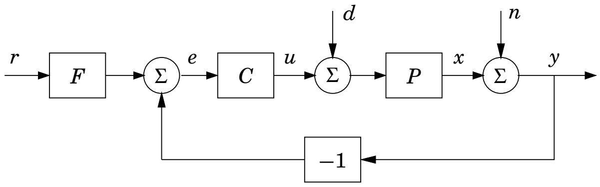
- The closed-loop system
  - Look at all transfer functions in the loop! (Gang of Four / Gang of six)
  - Robustness

New today

- Loop shaping

[Glad & Ljung] Ch. 6.4–6.6, 8.1–8.2 + AK

## Relations between signals




$$X = \frac{P}{1+PC}D - \frac{PC}{1+PC}N + \frac{PCF}{1+PC}R$$
$$Y = \frac{P}{1+PC}D + \frac{1}{1+PC}N + \frac{PCF}{1+PC}R$$
$$U = -\frac{PC}{1+PC}D - \frac{C}{1+PC}N + \frac{CF}{1+PC}R$$

## Designing System with Two Degrees of Freedom

Design procedure:

- Design the feedback $C$ to achieve
  - Small sensitivity to load disturbances $d$
  - Low injection of measurement noise $n$
  - High robustness to process variations
- Then design the feedforward $F$ to achieve desired response to command signals $r$

For many problems in process control the load disturbance response is much more important than the set point response. The set point response is more important in motion control. Few textbooks and papers show more than set point responses.

## Gang of Four / Gang of Six

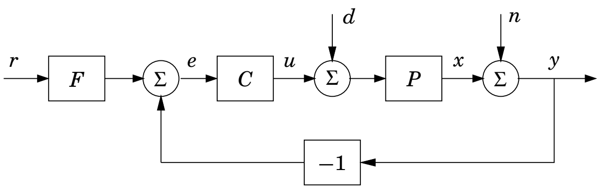
Six transfer functions are required to show the properties of a basic feedback loop. Four characterize the response to load disturbances and measurement noise.

$$\frac{PC}{1+PC} \qquad \frac{P}{1+PC}$$
$$\frac{C}{1+PC} \qquad \frac{1}{1+PC}$$

Two more are required to describe the response to set point changes.

$$\frac{PCF}{I+PC} \qquad \frac{CF}{1+PC}$$

## A Basic Control System




Ingredients:

- Controller: feedback $C$, feedforward $F$
- Load disturbance $d$: Drives the system from desired state
- Measurement noise $n$: Corrupts information about $x$
- Process variable $x$ should follow reference $r$

## Key Issues

Find a controller that

**A:** Reduces effects of load disturbances

**B:** Does not inject to much measurement noise into the system

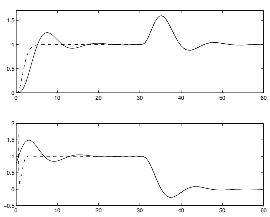
**C:** Makes the closed loop insensitive to variations in the process

**D:** Makes output follow command signals

Convenient to use a controller with two degrees of freedom, i.e. separate signal transmission from $y$ to $u$ and from $r$ to $u$. This gives a complete separation of the problem: Use feedback to deal with A, B, and C. Use feedforward to deal with D!

## Time domain specifications

- Step response (w.r.t reference and/or load disturbance)
  - rise-time $T_r$
  - overshoot
  - settling time $T_s$
  - static error $e_0$
- ...



step responce    load disturbance

## Frequency domain specifications

Closed loops specs.

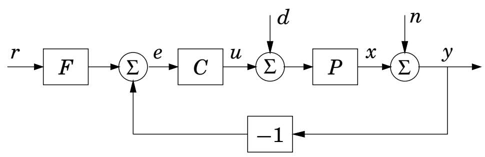
- resonace peak $M_p$
- bandwidth $\omega_B$ (see definition!!)

Open-loop measures

- $M_S$ and $M_T$-circles
- Amplitude margin $A_m$, phase margin $\phi_m$
- cross-over frequency $\omega_c$
- ...

Note: Often the design is made in Bode/Nyquist/Nichols diagrams for loop-gain $L = PC$ (open loop system)

Specifications on closed loop system




Would like:

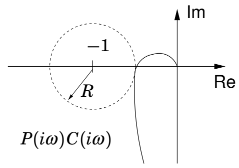
- Small influence of low-frequency disturbance $d$ on $z$
- Limited amplification of high-frequency noise $n$ in control $u$
- Robust stability despite high-frequency uncertainty

[Lecture 2]:

Different interpretations of the *Sensitivity function* $S = \dfrac{1}{1+PC}$

1. $S = G_{n\to y}(s) = G_{r\to e}(s)$ [See previous slide]
   - Note: $S = G_{r\to e}(s)$; Want low gain for low fq's...
2. $S = \frac{d(\log H)}{d(\log P)} = \frac{dH/H}{dP/P}$
   - ("How sensitive is the closed loop $H$ wrt process variations")
3. $S$ measures the distance from the Nyquist plot to $(-1+0i)$.

$$R^{-1} = \sup_{\omega} \left| \frac{1}{1+P(i\omega)C(i\omega)} \right|$$




## Frequency domain specs.

Closed-loop:
Find specifications $W_T$ and $W_S$ for closed-loops transfer functions s.t

$$|T(i\omega)| \le |W_T^{-1}(i\omega)|$$
$$|S(i\omega)| \le |W_S^{-1}(i\omega)|$$

(Magnitude transfers to singular values for MIMO-systems)

Examples:

- $|S(i\omega)| < 1.5$ for $\omega < 5$ Hz
- $|S| < |W_S^{-1}|$ = s/(s+10)
- $|T| < |W_T^{-1}|$ = 10/(s+10)
- "The closed loop system should have a bandwidth of at least ... rad/s"

These specifications can not be chosen independently of each other.

$$S + T = 1$$

Limiting factors:

- Fundamental limitations [Lecture 7/Ch 7]:
  - RHP zero at $z \Longrightarrow \omega_{BS} \le z/2$
  - Time delay $T \Longrightarrow \omega_{BS} \le 1/T$
  - RHP pole at $p \Longrightarrow \omega_{OT} \ge 2p$
- Bode's integral theorem
  - The "waterbed effect"
- Bode's relation
  - good phase margin requires certain distance between $\omega_{BS}$ and $\omega_{0T}$
- Model uncertainty:
  - Robust stability gives new "forbidden area"
  - Robust performance somewhat more complicated

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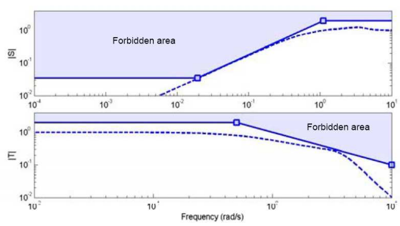



## Design: Consider open loop system

Try to look at ***loop-gain*** $L = PC$ for design and to translate specifications of $S$ & $T$ into specs of $L$

$$S = \frac{1}{1+L} \approx 1/L \quad \text{if } L \text{ is Large}$$
$$T = \frac{L}{1+L} \approx L \quad \text{if } L \text{ is small}$$

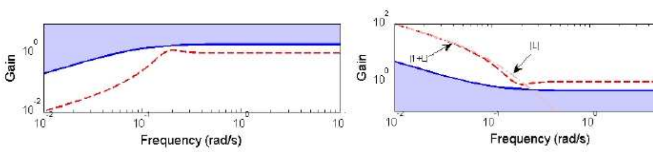
Classical loop shaping:

- design $C$ so that $L = PC$ satisfies constraints on $S$ and $T$
- how are the specifications related?
- what to do with the regions around cross-over frequency $\omega_c$ (where $|L| = 1$)?

## Sensitivity vs Loop Gain

$$S = \frac{1}{1+L}$$

$$|S(i\omega)| \le |W_S^{-1}(i\omega)| \Longleftrightarrow |1+L(i\omega)| > |W_S(i\omega)|$$




small frequencies, $W_S$ large $\Longrightarrow 1+L$ large, and $|L| \approx |1+L|$.

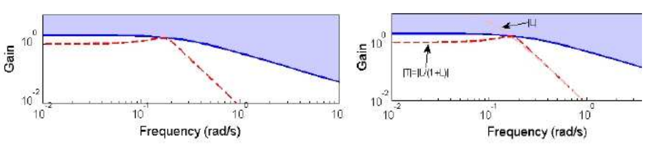
$$|L(i\omega)| \ge |W_S(i\omega)| \quad (approx.)$$

(typically valid for $\omega < \omega_{BS}$)

## Complementary Sensitivity vs Loop Gain

$$T = \frac{L}{1+L}$$

$$|T(i\omega)| \le |W_T^{-1}(i\omega)| \Longleftrightarrow \frac{|L(i\omega)|}{|1+L(i\omega)|} \le |W_T^{-1}(i\omega)|$$

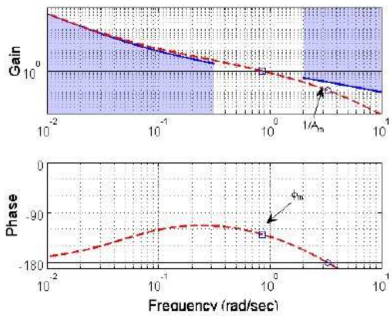



large frequencies, $W_T^{-1}$ small $\Longrightarrow |T| \approx |L|$

$$|L(i\omega)| \le |W_T^{-1}(i\omega)| \quad (approx.)$$

(typically valid for $\omega > \omega_{OT}$)

Resulting constraints on loop-gain $L$:

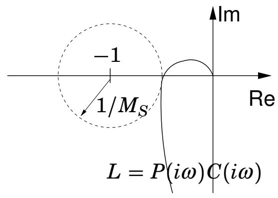


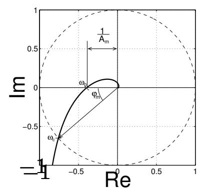

Remark: approximations inexact around cross-over frequency $\omega_c$. In this region, focus is on stability margins $A_m$, $\phi_m$.

## $M_S$ and $M_T$ and stability margins

Specifying $|T(i\omega)| \le M_T$ and $|S(i\omega)| \le M_S$ gives bounds for the amplitude and phase margins (but not the other way round!)

$$|S(i\omega)| \le M_S \quad \Longrightarrow \quad A_m > \frac{M_S}{M_S - 1}, \quad \phi_m > 2 \arcsin \frac{1}{M_S}$$




Q: Why does not $A_m$ and $\phi_m$ give bounds on $M_T$ and $M_S$?

## Lead-lag compensation

Shape loop gain $L = PC$ using a compensator $C$ composed of

- Lag (phase retarding) elements

$$C_{lag} = \frac{s+a}{s+a/M}, \quad M > 1$$

- Lead (phase advancing) elements

$$C_{lead} = N\frac{s+b}{s+bN}, \quad N > 1$$

- Gain

$$K$$

Typically

$$C = K\frac{s+a}{s+a/M} \cdot N\frac{s+b}{s+bN}$$

## Iterative lead-lag design

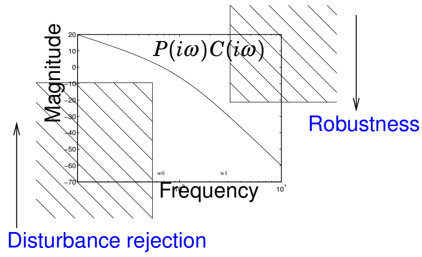
- Step 1: Lag (phase retarding) element
  - Add phase retarding element to get low-frequency asymptote right
- Step 2: Phase advancing element
  - Use phase advancing element to obtain correct phase margin
- Step 3: Adjust gain
  - Usually need to "lift up" or "push down" amplitude curve to obtain the desired cross-over frequency.

Adjusting the gain in Step 3 leaves the phase unaffected, but may ruin low-frequency asymptote (need to revise lag element) $\Longrightarrow$ An iterative method!

These requirements is to say that the *loop transfer matrix*

$$L = P(i\omega)C(i\omega)$$

should have small norm $\|P(i\omega)C(i\omega)\|$ at high frequencies, while at low the frequencies instead $\|[P(i\omega)C(i\omega)]^{-1}\|$ should be small.




## Classical loop shaping

Map specifications on requirements on loop gain L.

- Low-frequency specifications from $W_S$
- High-frequency specifications from $W_T^{-1}$
- Around cross-over frequency, mapping is crude
  - Position cross-over frequency (constrained by $W_S$, $W_T$)
  - Adjust phase margin (e.g. from $M_S$, $M_T$ specifications)

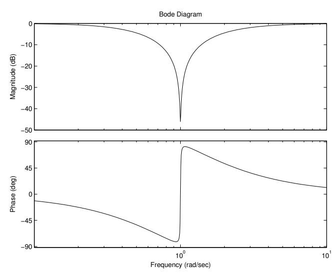
## Properties of leads-lag elements

- Lag (phase retarding) elements
  - Reduces static error
  - Reduces stability margin
- Lead (phase advancing) elements
  - Increased speed by increased $\omega_c$
  - Increased phase
    $\Longrightarrow$ May improve stability
- Gain
  - Translates magnitude curve
  - Does not change phase curve

See "Collection of Formulae" for lead-lag link diagrams

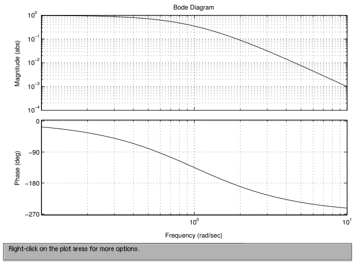
Example of other compensation-link:

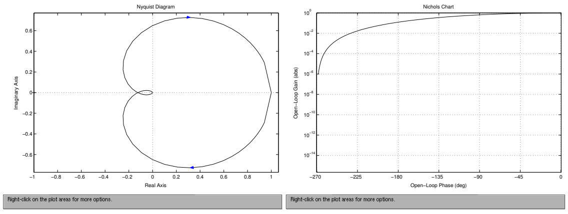
Notch-filter $\dfrac{s^2 + 0.01s + 1}{s^2 + 2s + 1}$



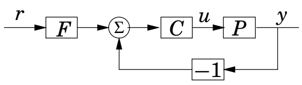
Bode, Nyquist and Nichols diagrams

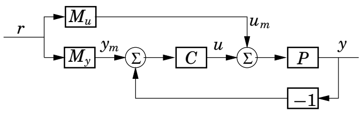
$log|PC| = log|P| + log|C|$
$arg\{PC\} = \arg\{P\} + arg\{C\}$




## Feedforward design




The reference signal $r$ specifies the desired value of $y$.

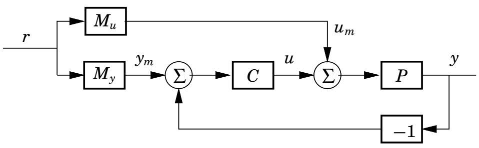
Ideally

$$\frac{P(s)C(s)}{1+P(s)C(s)}F(s) \approx 1$$

A more advanced option is

$$F(s) = \frac{1+P(s)C(s)}{P(s)C(s)(sT+1)^d}$$

for some suitable time constant $T$ and $d$ large enough to make $F$ proper and implementable.




Notice that $M_u$ and $M_y$ can be viewed as generators of the desired output $y_m$ and the inputs $u_m$ which corresponds to $y_m$.

## QFT

Quatitative Feedback Design Theory was developed by Horowitz *et. al.* to ensure desired loop-gain properties despite model uncertainties.

Basic principle: Let the (uncertain) system be represented by several transfer functions and at each frequency we get a corresponding set (template) of points which all should satisfy the constraints.

Equivalently

$$F(s) \approx \frac{1+P(s)C(s)}{P(s)C(s)}$$

Exact equality is generally impossible because of pole excess in $P$.

The simplest and most common approximation is to use a constant gain

$$F = \frac{1+P(0)C(0)}{P(0)C(0)}$$

## Example

$$P(s) = \frac{1}{(s+1)^4} \qquad F(s) = \frac{1+P(s)C(s)}{P(s)C(s)(sT+1)^d}$$

The closed loop transfer function from $r$ to $u$ then becomes

$$\frac{C(s)}{1+P(s)C(s)}F(s) = \frac{(s+1)^4}{(sT+1)^4}$$

which has low-fq gain 1, but gain $1/T^4$ for $\omega \longrightarrow \infty$.

## Design of Feedforward revisited

The transfer function from $r$ to $e = y_m - y$ is $(M_y - PM_u)S$

Ideally, $M_u$ should satisfy $M_u = M_y/P$. This condition does not depend on $C$!

Since $M_u = M_y/P$ should be stable, causal and not include derivatives we find that

- Unstable process zeros must be zeros of $M_y$
- Time delays of the process must be time delays of $M_y$
- The pole excess of $M_y$ must be greater than the pole excess of $P$

Take process limitations into account!

## Example of Feedforward Design revisited

If

$$P(s) = \frac{1}{(s+1)^4} \qquad M_y(s) = \frac{1}{(sT+1)^4}$$

then

$$M_u(s) = \frac{M_y(s)}{P(s)} = \frac{(s+1)^4}{(sT+1)^4} \qquad \frac{M_u(\infty)}{M_u(0)} = \frac{1}{T^4}$$

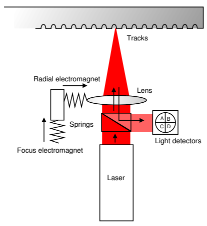
Fast response ($T$ small) requires high gain of $M_u$.

Bounds on the control signal limit how fast response we can obtain.

## Next lecture

Case study DVD-player

- Use **loop-shaping techniques from this lecture** for focus control design in DVD-player
- track following (modelling of disturbances, control)





## Summary

Frequency design;

- Good mapping between $S$,$T$ and $L = PC$ at low and high frequencies (mapping around cross-over frequency less clear)
- Simple relation between $C$ and $L \implies$ easy to shape $L$!
- Lead-lag control: iterative adjustment procedure
- What if closed-loop specifications are not satisfied?
  - we made a poor design (did not iterate enough), or
  - the specifications are not feasible (fundamental limitations in Lecture 7)
- Alternatives:
  - $H_\infty$-optimal control: finds stabilizing controller that satisfies constraints, if such a controller exists

Feedforward design