FRTN10 Multivariable Control — Lecture 1

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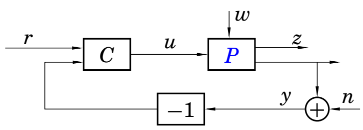
Todays lecture

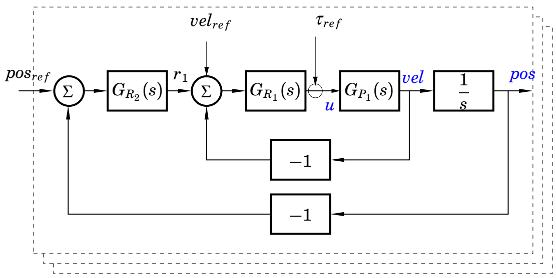
- Introduction/examples
- Overview of course + feedback/feedforward
- Review linear systems
  - Review of time-domain models
  - Review of frequency-domain models
  - Norm of signals
  - Gain of systems

Control problem



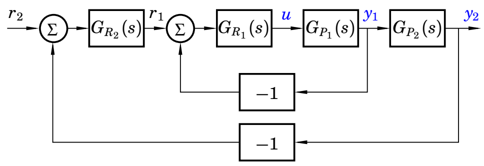
Given the system P and measurement signals y, determine the control signals u such that the control objective z follows the reference r as "close as possible" despite disturbances w, measurement errors n (noise etc.) and uncertainties of the real process.

For closed-loop ctrl $\Longrightarrow$ determine controller C.



Cascade control

For systems with one control signal and many outputs:

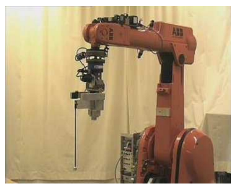


- $G_{R_1}(s)$ controls the subsystem $G_{P_1}(s)$ $(\Rightarrow G_{y_1 r_1}(s) \approx 1)$
- $G_{R_2}(s)$ controls the subsystem $G_{P_2}(s)$

Often used in motion control, e.g., robotics, with cascaded velocity and position controllers, BUT should have velocity reference feedforward!!

Example of couplings

Example: Couplings and interaction: "good"/"bad"



[movie]

"Robot Furuta pendulum": coupling as control action

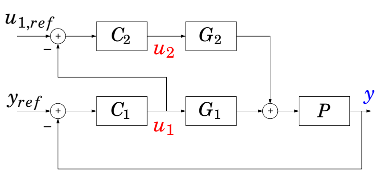
"Ordinary" Robot control:

Often cascaded PI-controllers for each joint
(inner velocity and outer position loop)

Feedforward for
- **disturbance rejection** between joints
- velocity and torque reference (improved tracking!)

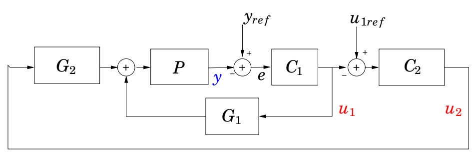
Mid-ranging Control

- Mid-ranging control structure is used for processes with two inputs and only one output to control.
- A classical application is valve position control
- Fast process input u_1 (Example: fast but small ranged valve)
- Slow process input u_2 (Example: slow but but large ranged valve)



Q: What should $u_{1,ref}$ be?
How does the midranging controller work?

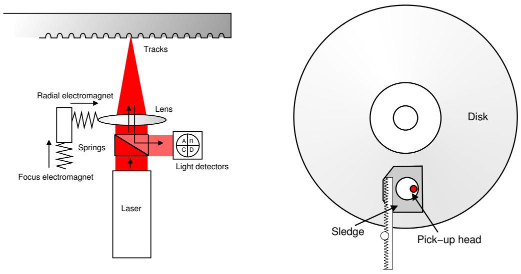
Mid-ranging control - a dual to cascade control



- First tune the fast inner loop, then the slower outer loop
- Controllers have separate time scales to avoid interaction

Mid-ranging cont'd

Example: Radial control of pich-up-head of DVD-player

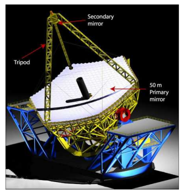


The pick-up-head has two electromagnets for fast positioning of the lens (left). Larger radial movements are taken care of by the sledge (right).

Many actuators and measurements

Example: Control of Large Deformable Telescope Mirror

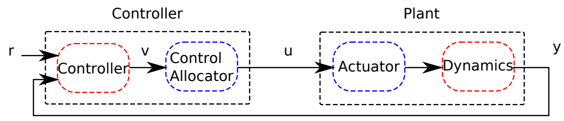
- Large number of sensors and actuators (500-3000)
- Computational limitations (1kHz)
- Tolerance ≈ 1 nano-meter
- Control accuracy crucial for telescope performance!

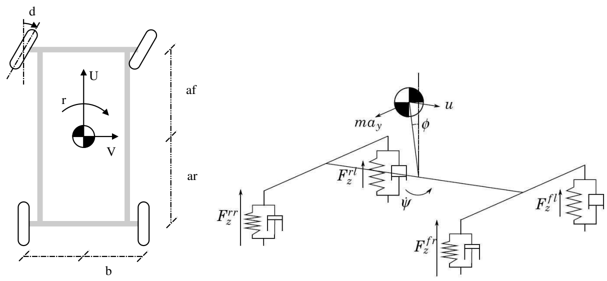


See more at e.g., `http://www.tmt.org/`
`http://www.astro.lu.se/~torben/euro50/index.html`

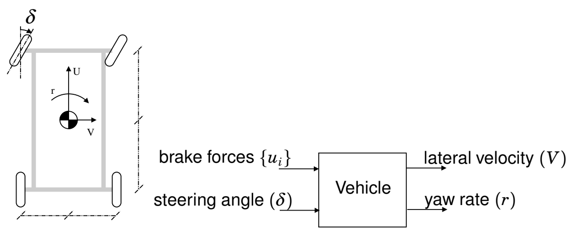
Example: Rollover protection needed

Rollover Control





Car dynamics



State space model

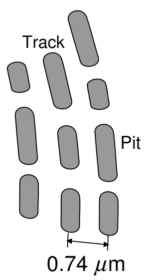
$$\begin{bmatrix} \dot V \\ \dot r \end{bmatrix} = A \begin{bmatrix} V \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ b_1 \end{bmatrix} (u_1 + u_2 - u_3 - u_4) + \begin{bmatrix} b_2 \\ b_3 \end{bmatrix} \delta$$

Fredrik Arp (Volvo) on Environmental Issues

[Sydsvenskan 2007]:
"Genom effektivisering av de konventionella bensin- och dieselmotorerna kan vi hämta hem en besparing på 20 procent i emissioner och bränsleekonomi de närmaste fem-sex åren"

Med andra ord: Bättre reglering ska göra jobbet.

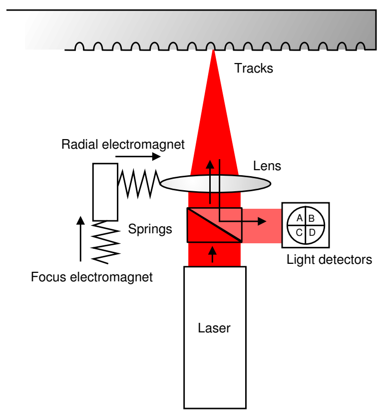
The DVD reader tracking problem



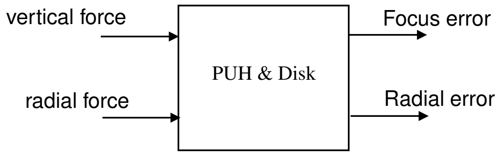
- 3.5 m/s speed along track
- 0.022 μm tracking tolerance
- 100 μm deviations at 23 Hz due to asymmetric discs

DVD Digital Versatile Disc, 4.7 Gb
CD Compact Disc, 650 Mb, mostly audio and software

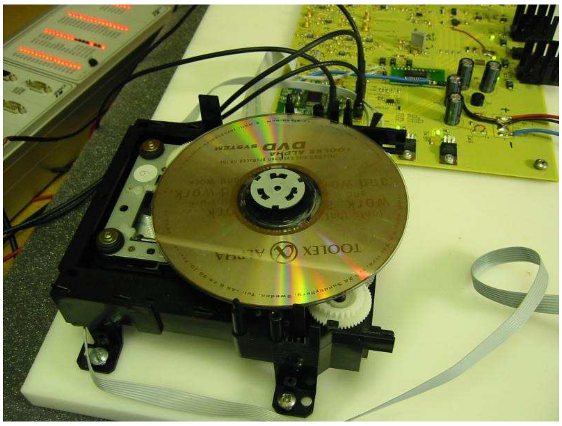
The DVD pick-up head



Input-output diagram for DVD control



The DVD reader in our lab



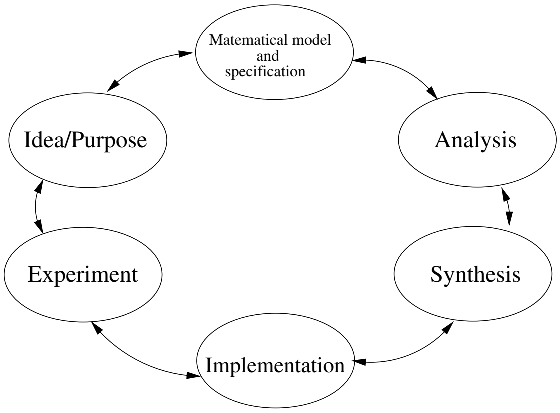
DVD in the course

- Focus control and tracking control lectured as a design example (Case study lecture 5)

What do we learn?

- Challenging design excercises
- Respect fundamental limitations
- Sampling frequency critical
- The use of observers

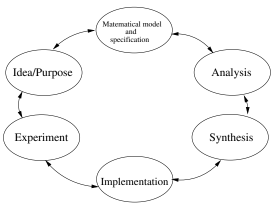
The design process



Contents of the course

- Single–input–single–output control revisited
- Multi–input-multi–output control
  - example: LQ/LQG
- Fundamental limitations
- Controller structures
- Control synthesis by optimization

Lectures, exercises and labs

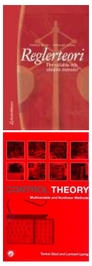


Course home page

`http://www.control.lth.se/Education/EngineeringProgram/FRTN10.html`

Literature

- T. Glad and L. Ljung:
  - Svensk utgåva: *Reglerteori – Flervariabla och olinjära metoder* , *2nd ed* Studentlitteratur, 2004
  - English translation: *Control Theory – Multivariable and Nonlinear Methods*, Taylor and Francis
- Lecture Slides/Notes on the web
- Exercise problems with solutions on the web
- Laboratory PMs
- Swedish-English control dictionary on homepage



KFS sells the book

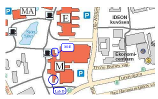
Course web page:

`http://www.control.lth.se/course/FRTN10`

Lectures

The lectures (30 hours) are given as follows:

Mondays 8-10,	M:E,	Aug 29 to Oct 10
Tuesdays 10-12,	M:E,	Aug 30 to Oct 4
Thursdays 13-15,	M:E,	Sep 1 to Sep 8



All course material is in English.

The lectures are given by

Anders Robertsson + some guest lecturers.

Exercise sessions and TAs

The exercises (28 hours) are taught according to the schedule

Group 1	Mon 10–12	Wed 13–15	Lab A
Group 2	Mon 13–15	Wed 10–12	Lab A

They are all held in the department laboratory on the bottom floor in the south end of the Mechanical Engineering building (Reglerteknik: Lab A).

Karl Berntorp Alfred Theorin Daria Madjidian Mikael Lindberg




Laboratory experiments

The three laboratory experiments are mandatory.

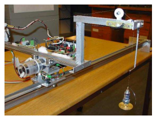
Sign-up lists are posted on the web at least one week before the first laboratory experiment. The lists close one day before the first session.

The Laboratory PMs are available at the course homepage.

Before the lab sessions some home assignments have to be done. No reports after the labs.

Lab	Week	Booking Starts	Responsible	Content
Lab 1	w 38	Sep 5	Alfred Theorin	Flex-servo
Lab 2	w 40	Sep 19	Karl Berntorp	Quad-tank
Lab 3	w 41	Sep 26	Mikael Lindberg	Crane





Exam

The exam (5 hours) will be given

- Wednesday Oct 19, 8am-1pm, Eden 25.

Lecture notes and text book are allowed, but no exercises material or extra hand-written notes.

Next time Monday Jan 9, 2012 (pre-register on web `http://www.control.lth.se/Education/EngineeringProgram`).

Use of computers in the course

- Use personal student-account or a common course account
- Matlab in exercises and laboratories (!!)
- `http://www.control.lth.se/Education/EngineeringProgram/FRTN10/`
- Email to `anders.robertsson@control.lth.se`

Feedback is important

For each course LTH use the following feedback mechanisms

- CEQ (reporting / longer time scale)
- Student representatives (fast feedback)
  - Election of student representative ("kursombud")

Help us close the loop for better performance!

Registration

New rules from HT 2011:

You **must register for the course by signing the form available** upfront during the break (will be passed around also during the 2nd hour).

If your name is not in the form please fill in an empty row.

LADOK registration will be done immediately.

If you decide to abort/skip the course within three weeks from today you should inform me and then the LADOK registration will be removed.

Lecture 1

- Description of linear systems (different representations)
  - Review of time-domain models
  - Review of frequency-domain models
- Norm of signals
- Gain of systems

State Space Equations

State-space and time-solution

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

$$y(t) = Ce^{At}x(0) + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$$

Example

$$\dot{x}_1 = -x_1 + 2x_2 + u_1 + u_2 - u_3$$
$$\dot{x}_2 = -5x_2 + 3u_2 + u_3$$
$$y_1 = x_1 + x_2 + u_3$$
$$y_2 = 4x_2 + 7u_1$$

How many states, inputs and outputs?

$$\dot{x} = Ax + Bu \qquad \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}* & *\\ * & *\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}* & * & *\\ * & * & *\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}$$

$$y = Cx + Du \qquad \begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}* & *\\ * & *\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}* & * & *\\ * & * & *\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}$$

Example

$$\dot{x}_1 = -x_1 + 2x_2 + u_1 + u_2 - u_3$$
$$\dot{x}_2 = -5x_2 + 3u_2 + u_3$$
$$y_1 = x_1 + x_2 + u_3$$
$$y_2 = 4x_2 + 7u_1$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 2\\ 0 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1 & 1 & -1\\ 0 & 3 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 1\\ 0 & 4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0 & 0 & 1\\ 7 & 0 & 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}$$

State space form cont'd

Exampel:
2nd order differential equation

$$\ddot{y} + 2\dot{y} + 3y = 4\dot{u} + 5u$$

Write on state space form.

How to chose states?
How to do if derivatives of input signal appears?

- Superposition
- Canonical forms
- Collection of formulae
- ...

Note: There are many different state-space representations for the same transfer function and system!

Q: How to chose states? (example)

See also "Collection of formulae" for different "canonical forms"

Change of coordinates

$$\begin{cases}\dot{x} = Ax + Bu\\ y = Cx + Du\end{cases}$$

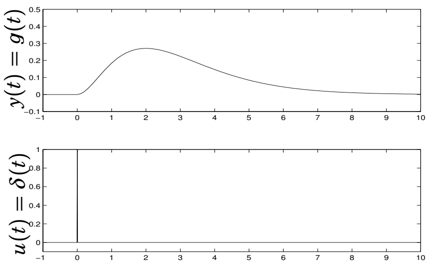
Change of coordinates

$$z = Tx$$

$$\begin{cases}\dot{z} = T\dot{x} = T(Ax + Bu) & = T(AT^{-1}z + Bu) = TAT^{-1}z + TBu\\ y = Cx + Du & = CT^{-1}z + Du\end{cases}$$

Q: What if time-varying change of coordinates?

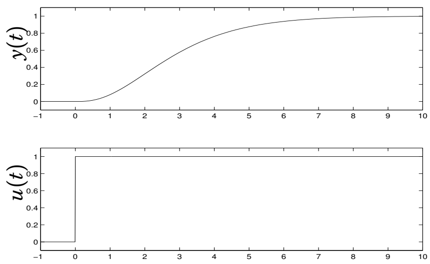
Impulse response



Common experiment in medicin and biology

$$g(t) = \int_0^t Ce^{A(t-\tau)}B\delta(\tau)d\tau + D\delta(t) = Ce^{At}B + D\delta(t)$$
$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau = [g * u](t)$$

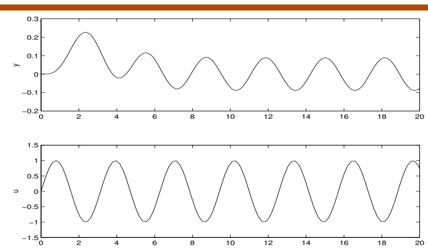
Step response



Common experiment in process industry

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau$$

Frequency response

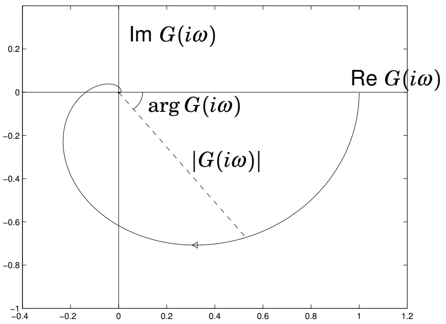


The transfer function $G(s)$ is the Laplace transform of the impulse response $G = \mathcal{L}g$. The input $u(t) = \sin \omega t$ gives

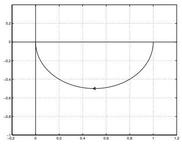
$$y(t) = \int_0^t g(\tau)u(t-\tau)d\tau = \text{Im}\left[\int_0^t g(\tau)e^{-i\omega\tau}d\tau \cdot e^{i\omega t}\right]$$
$$[t \to \infty] = \text{Im}\left(G(i\omega)e^{i\omega t}\right) = |G(i\omega)|\sin\left(\omega t + \arg G(i\omega)\right)$$

After a transient, also the output becomes sinusoidal

The Nyquist Diagram



Asymptotic formulas for first order system



$$G(s) = \frac{1}{s+1}$$

$$G(i\omega) = \frac{1}{i\omega + 1} = \frac{1 - i\omega}{\omega^2 + 1}$$

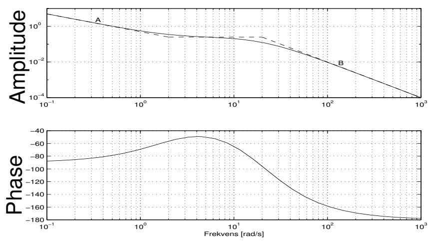
Small ω : $\quad G(i\omega) \approx 1$

Large ω : $\quad G(i\omega) \approx \frac{1}{\omega^2} - i\frac{1}{\omega}$

Matlab:

```
>> s=tf('s');
>> G=1/(s+1);
>> nyquist(G)
```

The Bode Diagram



$$G = G_1 G_2 G_3 \qquad \begin{cases} \log|G| = \log|G_1| + \log|G_2| + \log|G_3| \\ \arg G = \arg G_1 + \arg G_2 + \arg G_3 \end{cases}$$

Each new factor enter additively!

Hint: Set matlab-scales

```
>> ctrlpref
```

The L_2-norm of a signal

For $y(t) \in \mathbf{R}^n$ the "L_2-norm"

$$\|y\|_2 := \sqrt{\int_0^\infty |y(t)|^2 dt} \quad \text{is equal to} \quad \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} |\mathcal{L}y(i\omega)|^2 d\omega}$$

The equality is known as Parseval's formula

The L_2-gain of a system For a system $\mathcal{S}$ with input u and output $\mathcal{S}(u)$, the L_2-gain is defined as

$$\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2}$$

Miniproblem

What are the gains of the following systems?

1. $y(t) = -u(t)$ (a sign shift)
2. $y(t) = u(t - T)$ (a time delay)
3. $y(t) = \int_0^t u(\tau)d\tau$ (an integrator)
4. $y(t) = \int_0^t e^{-(t-\tau)}u(\tau)d\tau$ (a first order filter)

The L_2-gain from frequency data

Consider a stable system $\mathcal{S}$ with input u and output $\mathcal{S}(u)$ having the transfer function $G(s)$. Then, the system gain

$$\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2} \quad \text{is equal to} \quad \|G\|_\infty := \sup_\omega |G(i\omega)|$$

Proof. Let $y = \mathcal{S}(u)$. Then

$$\|y\|^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\mathcal{L}y(i\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(i\omega)|^2 \cdot |\mathcal{L}u(i\omega)|^2 d\omega \le \|G\|_\infty^2 \|u\|^2$$

The inequality is arbitrarily tight when $u(t)$ is a sinusoid near the maximizing frequency.

W. Wright at Western Society of Engineers 1901

"Men already know how to construct wings or airplanes, which when driven through the air at sufficient speed, will not only sustain the weight of the wings themselves, but also that of the engine, and of the engineer as well. Men also know how to build engines and screws of sufficient lightness and power to drive these planes at sustaining speed ... **Inability to balance and steer still confronts students of the flying problem.** ... When this one feature has been worked out, the age of flying will have arrived, for all other difficulties are of minor importance."

Wright was right!

Flight International, Aug 5, 2010

Control science tops list of USAF science and technology priorities

By Stephen Trimble

If the chief scientist of the US Air Force is correct, the key technology challenge for airpower over the next two decades is not directed energy, cruise missile defence or even satellite-killing weapons.

In a sweeping new 153-page report Technology Horizons, USAF chief scientist Werner Dahm instead identifies advances in "control science", an obscure niche of the software industry, as potentially the most important breakthrough for airpower between now and 2030.

Control science tops list ... cont'd

Control science develops verification and validation tools to allow humans to trust decisions made by autonomous systems, which, Dahm writes, must make huge leaps in capability over the next decade for the USAF's budgets to remain affordable.

Although too primitive to unleash the inherent power of modern autonomous systems, Dahm's report could make control science a major funding priority for at least the next 10 years.

Read article at

`http://www.flightglobal.com/articles/2010/08/05/345765/control-science-tops-list-of-usaf-science-and-technology.html`

Next lecture

- Stability
- Robustness
- Small Gain theorem