Lecture 4

Scalar control synthesis[1]

The lectures reviews the main aspects in synthesis of scalar feedback systems. Another name for such systems is single-input-single-output (SISO) systems. The specifications include ability to follow reference signals, to attenuate load disturbances and measurement noise and to reduce the effects of process variations. In the presentation, we separate the solution into feedback control and feedforward control.

4.1 Specifications

Recall from Lecture 1 the illustration of the design process shown in Figure 4.1. While Lecture 2 was mainly concerned with analysis, we are now focusing on the three neighboring blocks: Specification, Analysis and Synthesis.

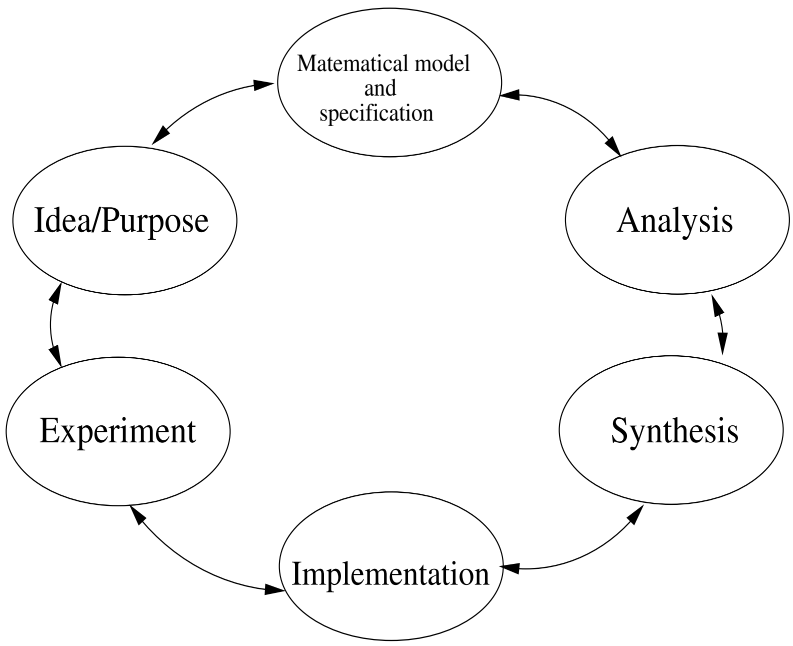

Figure 4.1 Schematic overview of the design process

We will restrict attention to the following structure (Figure 4.2), with a scalar transfer function for the plant. This setup was studied in the basic course and is sufficient for many practical situations.

The controller consists of two transfer functions, the feedback part $C(s)$ and the feedforward part $F(s)$. The control objective is to keep the process output x close to the reference signal r, in spite of load disturbances d. The measurement y is corrupted by noise n.

[1]Written by A. Rantzer with contributions by K.J. Åström

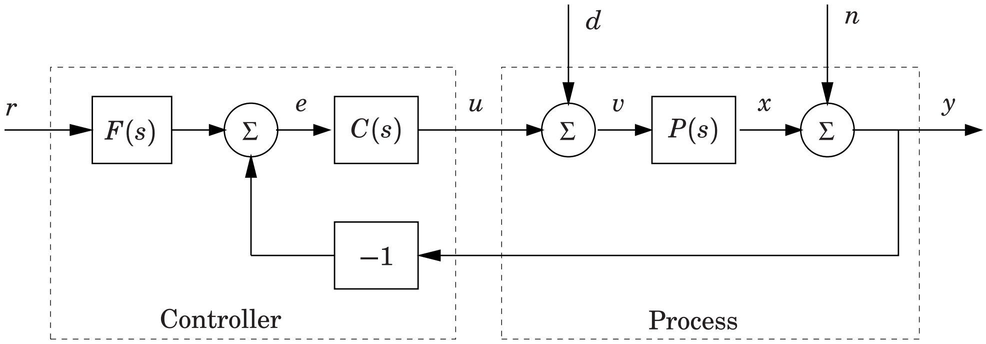

Figure 4.2 A Controller with two degrees of freedom

Several types of specifications could be relevant for this control loop.

A: Reduce the effects of load disturbances

B: Control the effects of measurement noise

C: Reduce sensitivity to process variations

D: Make the output follow command signals

A useful synthesis approach is to first design $C(s)$ to meet the specifications A, B, and C, then design $F(s)$, to deal with the response to reference changes, D. However, the two steps are not completely independent: A poor feedback design will have a negative influence also on the response to reference signals.

The following relations hold between the Laplace transforms of the signals in the closed loop system.

$$
\begin{aligned}
X(s) &= \frac{PCF}{1+PC}R(s) - \frac{PC}{1+PC}N(s) + \frac{P}{1+PC}D(s) \\
V(s) &= \frac{CF}{1+PC}R(s) - \frac{C}{1+PC}N(s) + \frac{1}{1+PC}D(s) \\
Y(s) &= \frac{PCF}{1+PC}R(s) + \frac{1}{1+PC}N(s) + \frac{P}{1+PC}D(s)
\end{aligned}
$$

Several observations can be made:

- The signals in the feedback loop are characterized by four transfer functions (sometimes called The Gang of Four)

$$
\frac{1}{1+P(s)C(s)} \qquad \frac{P(s)}{1+P(s)C(s)} \qquad \frac{C(s)}{1+P(s)C(s)} \qquad \frac{P(s)C(s)}{1+P(s)C(s)}
$$

  In particular, we recognize the first one as the sensitivity function and the last one as the complementary sensitivity.

- The total system with a controller having two degrees of freedom is characterized by *six* transfer functions (The Gang of Six).

To fully understand the properties of the closed loop system, it is necessary to look at all the transfer functions. It can be strongly misleading to only show properties of a few input-output maps, for example a step response from reference signal to process output. This is a common mistake in the literature.

The properties of the different transfer functions can be illustrated in several ways, by time- or frequency-responses. For a particular example, we show below first the six frequency response amplitudes, then the corresponding six step responses.

It is worthwhile to compare the frequency plots and the step responses and to relate their shape to the specifications A-D:

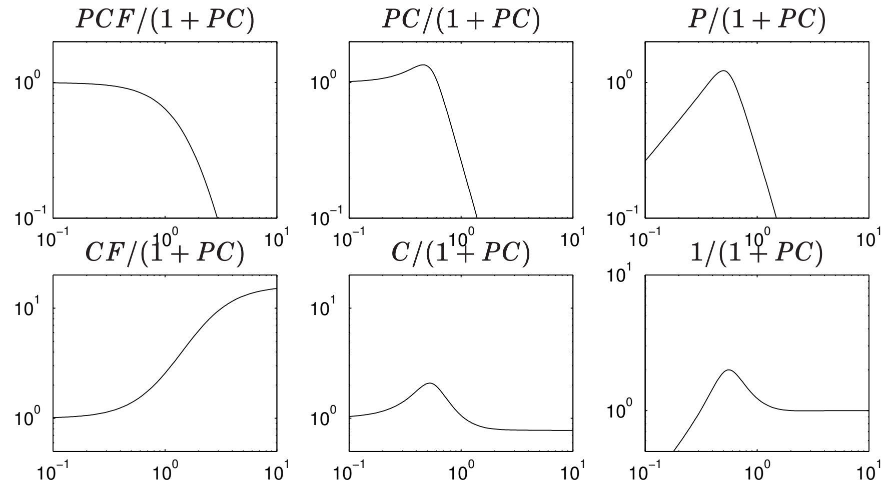

Figure 4.3 Frequency response amplitudes for $P(s) = (s+1)^{-4}$, $C(s) = 0.775(s^{-1}/2.05+1)$ when $F(s)$ is designed to give $PCF/(1+PC) = (0.5s+1)^{-4}$

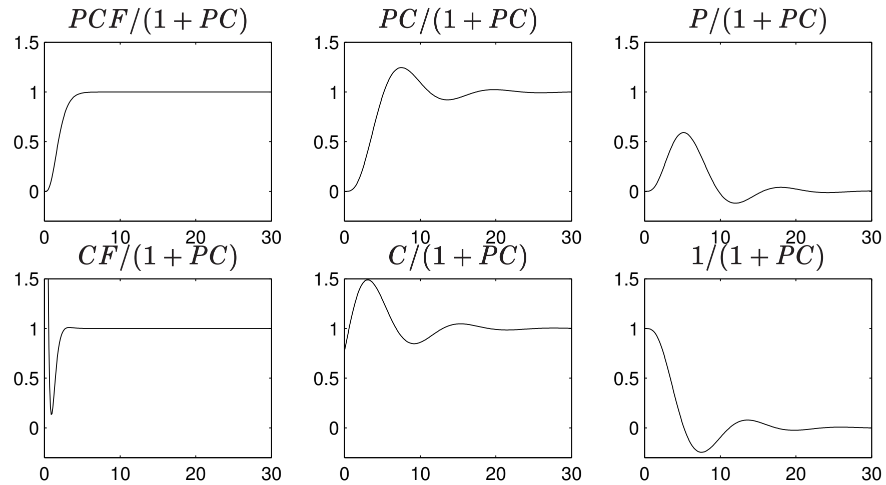

Figure 4.4 Step responses for $P(s) = (s+1)^{-4}$, $C(s) = 0.775(s^{-1}/2.05+1)$ when $F(s)$ is designed to give $PCF/(1+PC) = (0.5s+1)^{-4}$

Disturbance rejection The two upper right plots show the effect of the disturbance d in process output x and input v respectively. The resulting process error should not be too large and should settle to zero quickly enough. The control input would cancel the disturbance exactly if the mid upper step response would be an ideal step. In a short time-scale this is impossible, since the control input will not change until the effect of the disturbance has appeared in the process output and been available for measurement. However, slow disturbances should normally be cancelled by u. Equivalently, *the sensitivity function* $1/(1+PC)$ *should be small for low frequencies*. This specification is usually corresponds to an integrator in the controller.

Suppression of measurement noise The second specification was to limit the effect of measurement noise, typically a high frequency phenomenon. The mid upper frequency plot shows good attenuation of measurement noise above the “cut off” frequency of 1 Hz. In this example, this is mainly an effect of the process dynamics. A more interesting question is maybe the gain from measurement noise to control input, since fast oscillations in the control actuator are usually undesir-

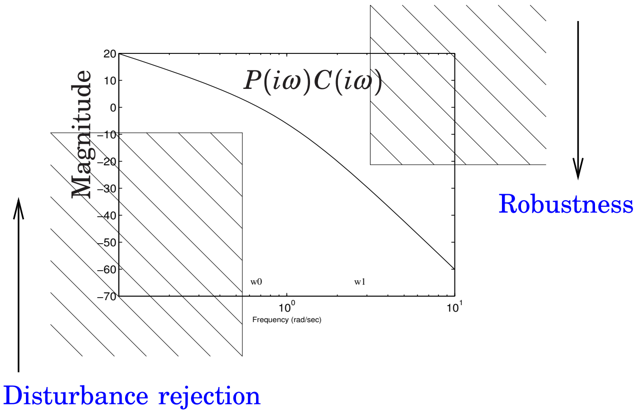

Figure 4.5 Frequency specifications for the closed loop system; for the sensitivity function $S = \frac{1}{1+L}$ and for the complementary sensitivity function $T = \frac{L}{1+L}$, can (approximately) be interpreted as frequency specifications in open loop for the *loop transfer matrix* $L = P(i\omega)C(i\omega)$, such that L should have small norm $\|P(i\omega)C(i\omega)\|$ at high frequencies, while at low the frequencies instead $\|[P(i\omega)C(i\omega)]^{-1}\|$ should be small.

able. For this aspect, the mid lower frequency plot, showing the Bode amplitude from n to v, is of interest.

Robustness to process variations As shown in the previous lecture, the robustness to process variations is determined by the sensitivity functions. In this example, the lower right frequency plot has a maximal value of 2, which shows that a small relative error in the process can give rise to a relative error of double size in the closed loop transfer function. The maximal amplitude of the frequency plot for the complementary sensitivity function is 1.35, so the small gain theorem proves stability of the closed loop system as long as the relative error in the process model is below $74\% = 1/1.35$. In fact, most process models are inaccurate at high frequencies, so *the complementary sensitivity function $PC/(1 + PC)$ should be small for high frequencies*.

Command response The upper left corner plot shows the map from reference signal r to process output x. Using the prefilter F, it is possible to get a better step response here than in the upper mid plot. The prize to pay is that the corresponding response in the control signal gets higher amplitude. This can be seen by comparing the lower left plot, showing the map from r to v, to the lower mid plot, which shows the corresponding map when $F \equiv 1$.

4.2 Loop shaping

The closed loop performance depends critically on the *loop transfer function*

$$L(s) = P(s)C(s)$$

In particular, the sensitivity functions can be written as $S = (1 + L)^{-1}$ and $T = L(1 + L)^{-1}$ respectively. A popular approach to control synthesis, known as *loop shaping*, is to focus on the shape of the loop transfer function and keep modifying $C(s)$ until the desired shape is obtained.

Recall that proper disturbance rejection requires small sensitivity S (large L) for for small frequencies, while process uncertainty requires the complementary sensitivity function to be small (small L) for high frequencies, see Fig. 4.5. On the other hand, if the amplitude of L decreases very rapidly, the phase tends to become lower than $-180°$ and the system becomes unstable. Loop shaping is therefore a trade-off between different kinds of specifications.

Many control problems can be adequately solved by PID controllers, which can be viewed a combination of one lag compensator and one lead compensator. For more advanced applications, like resonant systems, higher order controllers are desirable. An example of such a system is the flexible servo treated in lab 1.

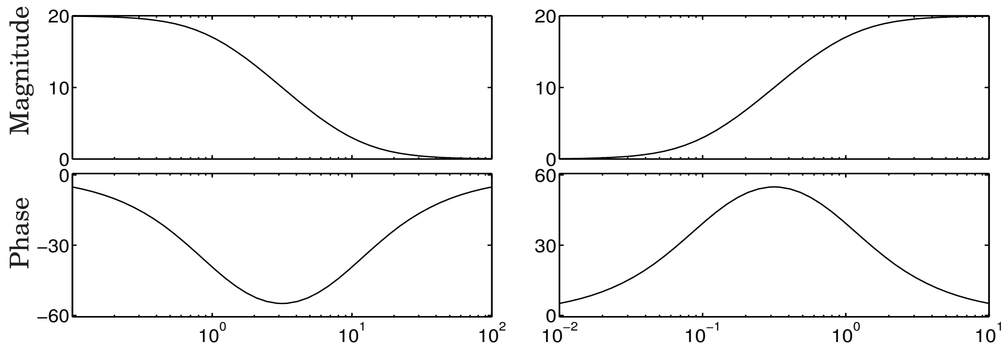

Figure 4.6 Bode diagrams for lag compensator $\frac{s+10}{s+1}$ (left) and lead compensator $\frac{10s+1}{s+1}$ (right)

Graphical illustrations in Bode- or Nichols- diagrams are typically used to support the design. These diagrams are convenient because of the logarithmic scale, where the controller contributes additively to the loop transfer function:

$$\log|L(i\omega)| = \log|P(i\omega)| + \log|C(i\omega)|$$
$$\arg|L(i\omega)| = \arg P(i\omega) + \arg C(i\omega)$$

From the basic course, recall the following essential properties of lead and lag compensators, illustrated in Figure 4.6:

Lag compensator

- Increases low frequency gain: Can be used to reduce stationary errors
- Decreases phase, which may reduce stability margins

Lead compensator

- Increases high frequency gain: Can be used for faster closed loop response
- Increases phase, which may improve stability margins

Loop shaping design of high order controllers will be exercised in lab 1. We will first design a controller $C_1(s)$ for low frequencies, then keep adding compensator links $C_2(s), C_3(s), \ldots$ to modify the dynamics at higher and higher frequencies until a satisfactory controller $C(s) = C_1(s)\cdots C_m(s)$ is obtained. Lead/lag links are often sufficient, but occasionally it is useful to also consider controllers with poles or zeros outside the real axis. The figure below shows the Bode diagram for cases with stable complex zeros (left) and complex poles (right).

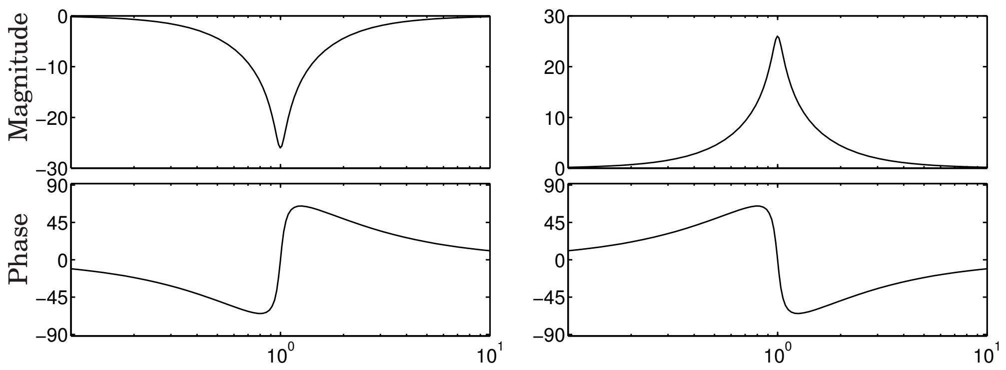

Figure 4.7 Notch compensator $\frac{s^2+0.1s+1}{(s+1)^2}$ (left) and resonant compensator $\frac{(s+1)^2}{s^2+0.1s+1}$ (right)

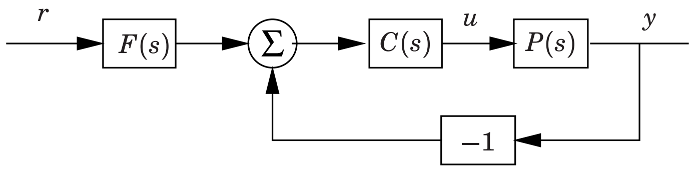

Figure 4.8 The feedforward filter $F(s)$ is used to improve the response to reference signals

4.3 Feedforward synthesis

Let us finally consider the design of $F(s)$ to shape the response to reference signals. See Figure 4.8. The usual interpretation of the reference signal r is that it specifies the desired value of y. Hence the transfer function from r to y should be as close to identity as possible, ideally

$$\frac{P(s)C(s)}{1+P(s)C(s)}F(s) = 1 \quad \text{or equivalently} \quad F(s) = \frac{1+P(s)C(s)}{P(s)C(s)}$$

Unfortunately, this is impossible to achieve because $PC/(1+PC)$ generally has more poles than zeros (pole excess) and hence $F(s)$ would not be proper.

Instead, $F(s)$ is usually chosen to approximate $(1+PC)/(PC)$ at small frequencies. The simplest (and most common) choice is to make F constant and equal to

$$\frac{1+P(0)C(0)}{P(0)C(0)}$$

A more advanced option is to choose

$$\frac{P(s)C(s)}{1+P(s)C(s)}F(s) = \frac{1}{(sT+1)^d}$$

for some suitable time constant T and with d large enough to make F proper and implementable.

Example 1 If

$$P(s) = \frac{1}{(s+1)^4} \qquad \frac{P(s)C(s)}{1+P(s)C(s)}F(s) = \frac{1}{(sT+1)^4}$$

then the closed loop transfer function from r to u becomes

$$\frac{C(s)}{1+P(s)C(s)}F(s) = \frac{(s+1)^4}{(sT+1)^4}$$

The gain is 1 for low frequencies ($s \approx 0$) but $1/T^4$ for $s = 1$. Hence, fast response (T small) requires high controller gain. Bounds on the control signal therefore limit how fast response we can obtain. □

In servo systems and motion control of mechanical systems like industrial robots, it is very common to have a cascaded controller structure with an inner velocity control and an outer position control. Not only does it simplify tuning of the servos, but it also lends itself to a natural feedforward structure; when generating trajectories for the position reference the corresponding velocity and torque references should also be generated and used as feedforward signals to the inner velocity controller and actuator, respectively, see Fig 4.9.

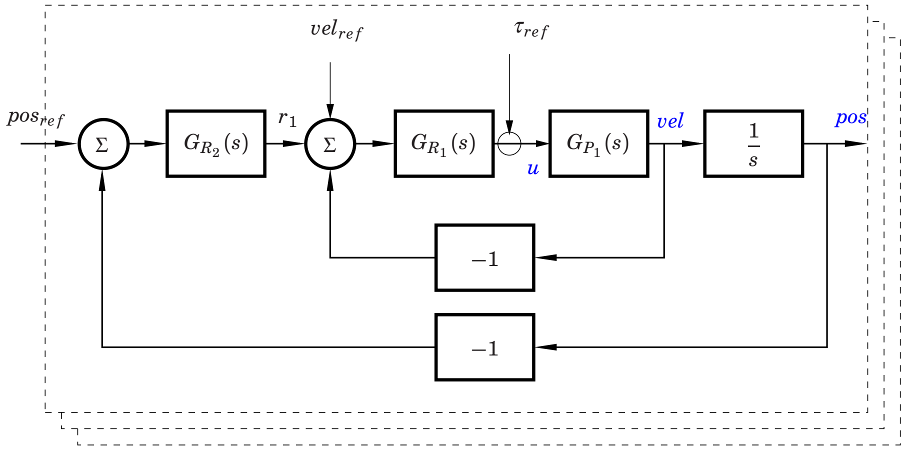

Figure 4.9 Cascaded controller for servo control with feedforward control of velocity references and torque references, consistent with the desired position references (generated together).