Nonlinear Control and Servo systems
Lecture 1

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Overview Lecture 1

- Practical information
- Course contents
- Nonlinear control systems phenomena
- Nonlinear differential equations

Course Goal

To provide students with solid theoretical foundations of nonlinear control systems combined with good engineering ability

You should after the course be able to

- recognize common nonlinear control problems,
- use some powerful analysis methods, and
- use some practical design methods.

Today's Goal

- *Recognize some common nonlinear phenomena*
- *Transform differential equations to autonomous form, first-order form, and feedback form*
- *Describe saturation, dead-zone, relay with hysteresis, backlash*
- *Calculate equilibrium points*

Course Material

- Textbook
  - Glad and Ljung, *Reglerteori, flervariabla och olinjra metoder*, 2003, Studentlitteratur,ISBN 9-14-403003-7 or the English translation *Control Theory*, 2000, Taylor & Francis Ltd, ISBN 0-74-840878-9. The course covers Chapters 11-16,18. (MPC and optimal control not covered in the other alternative textbooks.)
  - H. Khalil, *Nonlinear Systems* (3rd ed.), 2002, Prentice Hall, ISBN 0-13-122740-8. A good, a bit more advanced text.
  - `ALTERNATIVE:` Slotine and Li, *Applied Nonlinear Control*, Prentice Hall, 1991. The course covers chapters 1-3 and 5, and sections 4.7-4.8, 6.2, 7.1-7.3.

Course Material, cont.

- Handouts (Lecture notes + extra material)
- Exercises (can be downloaded from the course home page)
- Lab PMs 1, 2 and 3
- Home page
  `http://www.control.lth.se/course/FRTN05/`
- Matlab/Simulink other simulation software
  see home page

Lectures and labs

The lectures (28 hours) are given in E:C as follows:

Mon 13–15	week 44, 45, 46,	48, 49
Wed 8–10	week 44, 45, 46,	48, 49
Thu 8–10	week	49
Frid 8-10	week 44, 45,	47

Lectures are given in English.

The three laboratory experiments are mandatory.

Sign-up lists are posted on the web at least one week before the first laboratory experiment. *The lists close one day before the first session.*

The Laboratory PMs are available at the course homepage.

Before the lab sessions some home assignments have to be done. No reports after the labs.

Exercise sessions and TAs

The exercises (28 hours) are offered twice a week in M:2112B (the seminar room of Automatic Control LTH, M-building, second floor)

Tue 15:15-17:00 Wed 15:15-17:00

Martin Karlsson Mattias Fält

The Course

- 14 lectures
- 14 exercises
- 3 laboratories
- 5 hour exam: **January 10, 2017, 14:00-19:00, MA10 H-J**.
  Open-book exam: Lecture notes but no old exams/exercises.

Course Outline

Lecture 1-3	Modelling and basic phenomena (linearization, phase plane, limit cycles)
Lecture 2-6	Analysis methods (Lyapunov, circle criterion, describing functions))
Lecture 7-8	Common nonlinearities (Saturation, friction, backlash, quantization))
Lecture 9-13	Design methods (Lyapunov methods, Backstepping, Optimal control)
Lecture 14	Summary

Todays lecture

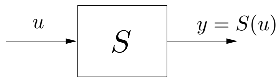
Common nonlinear phenomena

- Input-dependent stability
- Stable periodic solutions
- Jump resonances and subresonances

Nonlinear model structures

- Common nonlinear components
- State equations
- Feedback representation

Linear Systems



Definitions: The system S is *linear* if

$$\begin{aligned} S(\alpha u) &= \alpha S(u), & \text{scaling} \\ S(u_1 + u_2) &= S(u_1) + S(u_2), & \text{superposition} \end{aligned}$$

A system is *time-invariant* if delaying the input results in a delayed output:

$$y(t - \tau) = S(u(t - \tau))$$

Linear time-invariant systems are easy to analyze

Different representations of same system/behavior

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \quad y(t) = Cx(t), \quad x(0) = 0 \\ y(t) &= g(t) \star u(t) = \int g(r)u(t - r)dr \\ Y(s) &= G(s)U(s) \end{aligned}$$

Local stability = global stability:

Eigenvalues of A (= poles of $G(s)$) in left half plane

Superposition:

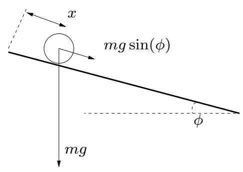
Enough to know step (or impulse) response

Frequency analysis possible:

Sinusoidal inputs give sinusoidal outputs

Linear models are not always enough

Example: Ball and beam



Linear model (acceleration along beam) :
Combine $F = m \cdot a = m\frac{d^2x}{dt^2}$ with $F = mg\sin(\phi)$:

$$\ddot{x}(t) = g\sin(\phi(t))$$

Linear models are not enough

x = position (m) ϕ = angle (rad) $g = 9.81$ (m/s^2)

Can the ball move 0.1 meter in 0.1 seconds with constant ϕ?

Linearization: $\sin\phi \sim \phi$ for $\phi \sim 0$

$$\begin{cases} \ddot{x}(t) = g\phi \\ x(0) = 0 \end{cases}$$

Solving the above gives $x(t) = \frac{t^2}{2}g\phi$

For $x(0.1) = 0.1$, one needs $\phi = \frac{2*0.1}{0.1^2*g} \geq 2$ rad

Clearly outside linear region!

Contact problem, friction, centripetal force, saturation

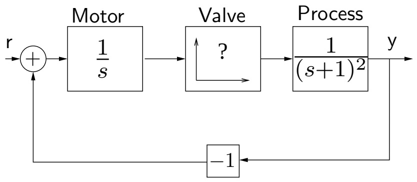
How fast can it be done? (Optimal control)

Warm-Up Exercise: 1-D Nonlinear Control System

$$\dot{x} = x^2 - x + u$$

- stability for $u = 0$?
- stability for constant $u = b$?
- stability with linear feedback $u = ax + b$?
- stability with non-linear feedback $u(x) = ?$

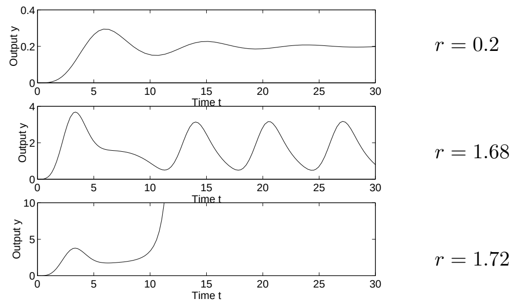
Stability Can Depend on Amplitude



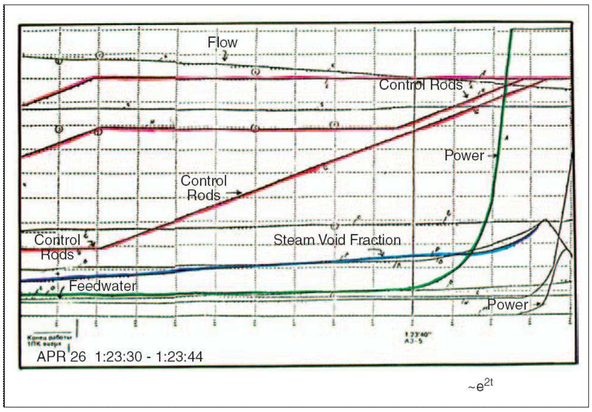
Valve characteristic $f(x) = ???$

Step changes of amplitude, $r = 0.2$, $r = 1.68$, and $r = 1.72$

Step Responses

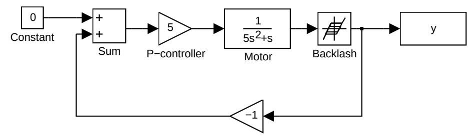


Stability depends on amplitude!



Stable Periodic Solutions

Example: Motor with back-lash

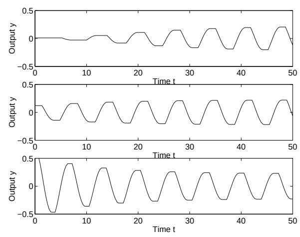


Motor: $G(s) = \frac{1}{s(1+5s)}$

Controller: $K = 5$

Stable Periodic Solutions

Output for different initial conditions:

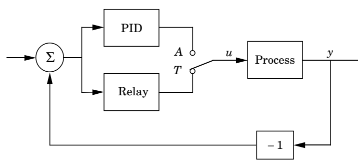


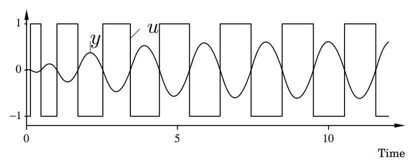
Frequency and amplitude independent of initial conditions!

Several systems use the existence of such a phenomenon

Relay Feedback Example

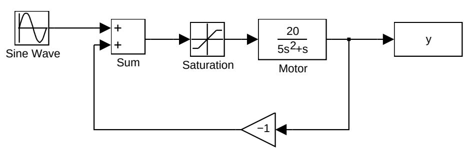
Period and amplitude of limit cycle are used for autotuning





[patent: T Hgglund and K J strm]

Jump Resonances



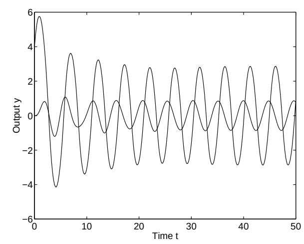
Response for sinusoidal depends on initial condition

Problem when doing frequency response measurement

Jump Resonances

$$u = 0.5\sin(1.3t), \quad \text{saturation level} = 1.0$$

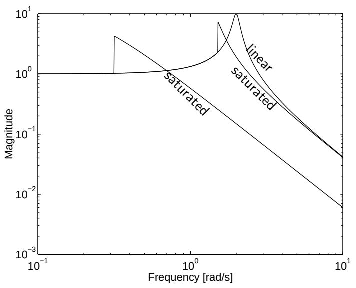
Two different initial conditions



give two different amplifications for same sinusoid!

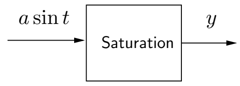
Jump Resonances

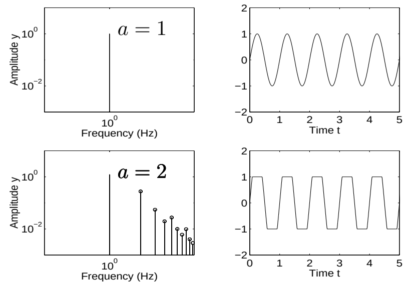
Measured frequency response (many-valued)



New Frequencies

Example: Sinusoidal input, saturation level 1





New Frequencies

Example: Electrical power distribution

THD = Total Harmonic Distortion = $\frac{\sum_{k=2}^{\infty} \text{energy in tone } k}{\text{energy in tone } 1}$

Nonlinear loads: Rectifiers, switched electronics, transformers

Important, increasing problem

Guarantee electrical quality

Standards, such as $THD < 5\%$

New Frequencies

Example: Mobile telephone

Effective amplifiers work in nonlinear region

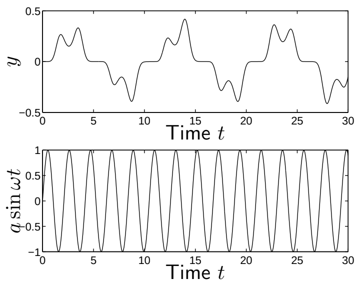
Introduces spectrum leakage

Channels close to each other

Trade-off between effectivity and linearity

Subresonances

Example: Duffing's equation $\ddot{y} + \dot{y} + y - y^3 = a \sin(\omega t)$



When is Nonlinear Theory Needed?

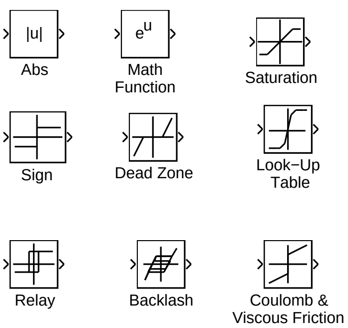
- Hard to know when - Try simple things first!
- Regulator problem versus servo problem
- Change of working conditions (production on demand, short batches, many startups)
- Mode switches
- Nonlinear components

How to detect? Make step responses, Bode plots

- Step up/step down
- Vary amplitude
- Sweep frequency up/frequency down

Some Nonlinearities

Static – dynamic



Nonlinear Differential Equations

Problems

- No analytic solutions
- Existence?
- Uniqueness?
- etc

Finite escape time

Example: The differential equation

$$\frac{dx}{dt} = x^2, \qquad x(0) = x_0$$

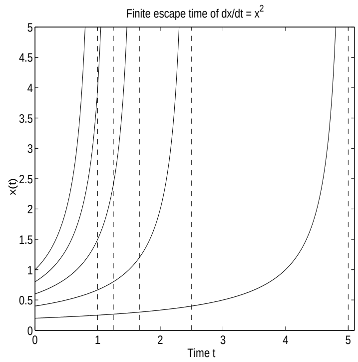
has solution

$$x(t) = \frac{x_0}{1 - x_0 t}, \qquad 0 \le t < \frac{1}{x_0}$$

Finite escape time

$$t_f = \frac{1}{x_0}$$

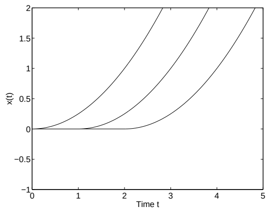
Finite Escape Time



Uniqueness Problems

Example: The equation $\dot{x} = \sqrt{x}$, $x(0) = 0$ has many solutions:

$$x(t) = \begin{cases} (t - C)^2/4 & t > C \\ 0 & t \le C \end{cases}$$



Compare with water tank:

$$dh/dt = -a\sqrt{h}, \qquad h: \text{height (water level)}$$

Change to backward-time: "If I see it empty, when was it full?")

Local Existence and Uniqueness

For $R > 0$, let Ω_R denote the ball $\Omega_R = \{z : \|z - a\| \le R\}$.

Theorem
If, f is Lipschitz-continuous in Ω_R, i.e.,

$$\|f(z) - f(y)\| \le K\|z - y\|, \qquad \text{for all } z, y \in \Omega_R\,,$$

then

$$\begin{cases} \dot{x}(t) = f(x(t)) \\ x(0) = a \end{cases}$$

has a unique solution

$$x(t)\,, \qquad 0 \le t < R/C_R,$$

where $C_R = \max_{x \in \Omega_R} \|f(x)\|$

Global Existence and Uniqueness

Theorem
If f is Lipschitz-continuous in R^n, i.e.,

$$\|f(z) - f(y)\| \le K\|z - y\|, \qquad \text{for all } z, y \in R^n\,,$$

then

$$\dot{x}(t) = f(x(t)), x(0) = a$$

has a unique solution

$$x(t)\,, \qquad t \ge 0\,.$$

State-Space Models

- State vector x
- Input vector u
- Output vector y

general:	$f(x, u, y, \dot{x}, \dot{u}, \dot{y}, \ldots) = 0$
explicit:	$\dot{x} = f(x, u), \quad y = h(x)$
affine in u:	$\dot{x} = f(x) + g(x)u, \quad y = h(x)$
linear time-invariant:	$\dot{x} = Ax + Bu, \quad y = Cx$

Transformation to Autonomous System

Nonautonomous:

$$\dot{x} = f(x, t)$$

Always possible to transform to autonomous system

Introduce x_{n+1} = time

$$\begin{aligned} \dot{x} &= f(x, x_{n+1}) \\ \dot{x}_{n+1} &= 1 \end{aligned}$$

Transformation to First-Order System

Assume $\frac{d^k y}{dt^k}$ highest derivative of y

Introduce $x = \begin{bmatrix} y & \frac{dy}{dt} & \ldots & \frac{d^{k-1}y}{dt^{k-1}} \end{bmatrix}^T$

Example: Pendulum

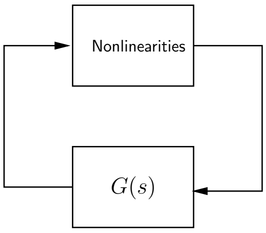
$$MR\ddot{\theta} + k\dot{\theta} + MgR\sin\theta = 0$$

$x = \begin{bmatrix} \theta & \dot{\theta} \end{bmatrix}^T$ gives

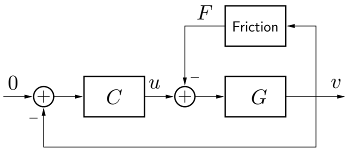
$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{k}{MR}x_2 - \frac{g}{R}\sin x_1 \end{aligned}$$

A Standard Form for Analysis

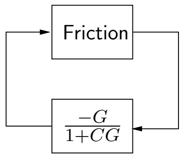
Transform to the following form



Example, Closed Loop with Friction



$\Longleftrightarrow$



Equilibria (=singular points)

Put all derivatives to zero!

General: $f(x_0, u_0, y_0, 0, 0, 0, \ldots) = 0$

Explicit: $f(x_0, u_0) = 0$

Linear: $Ax_0 + Bu_0 = 0$ (has analytical solution(s)!)

Multiple Equilibria

Example: Pendulum

$$MR\ddot{\theta} + k\dot{\theta} + MgR\sin\theta = 0$$

Equilibria given by $\ddot{\theta} = \dot{\theta} = 0 \Longrightarrow \sin\theta = 0 \Longrightarrow \theta = n\pi$

Alternatively,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{k}{MR}x_2 - \frac{g}{R}\sin x_1 \end{aligned}$$

gives $x_2 = 0$, $\sin(x_1) = 0$, etc

Next Lecture

- Linearization
- Stability definitions
- Simulation in Matlab