Lecture 7: Anti-windup and friction compensation

- Compensation for saturations (anti-windup)
- Friction models
- Friction compensation

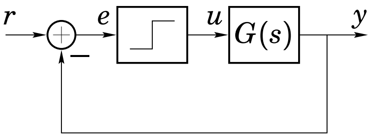
Material

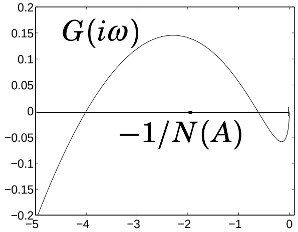
- Lecture slides

Course Outline

Lecture 1-3	Modelling and basic phenomena (linearization, phase plane, limit cycles)
Lecture 2-6	Analysis methods (Lyapunov, circle criterion, describing functions)
Lecture 7-8	Common nonlinearities (Saturation, friction, backlash, quantization)
Lecture 9-13	Design methods (Lyapunov methods, Backstepping, Optimal control)
Lecture 14	Summary

Last lecture: Stable periodic solution



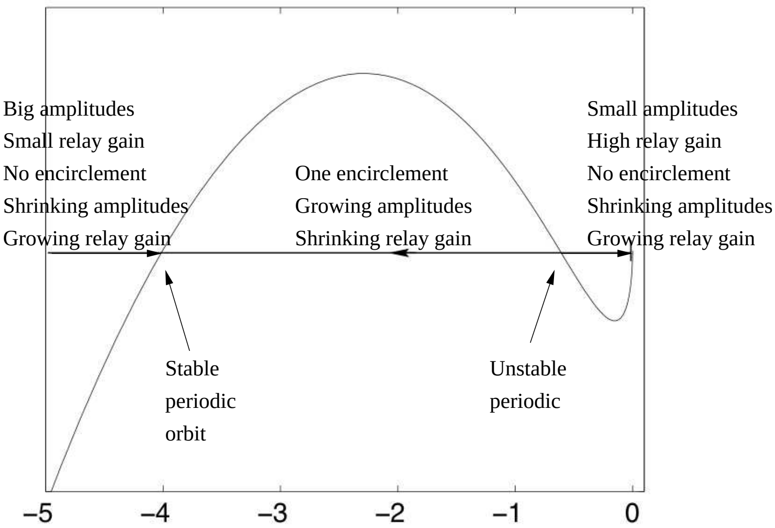


$$G(s) = \frac{(s+10)^2}{(s+1)^3} \quad \text{with feedback} \quad u = -\operatorname{sgn} y$$

gives one stable and one unstable limit cycle. The left most intersection corresponds to the stable one.

Periodic Solutions in Relay System

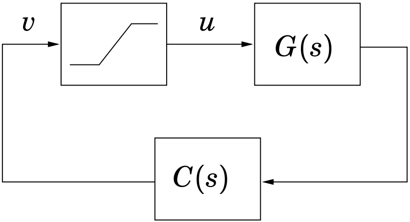
The relay gain $N(A)$ is higher for small A:



Today's Goal

- *To be able to design and analyze antiwindup schemes for*
  - *PID*
  - *state-space systems*
  - *and Kalman filters (observers)*
- *To understand common models of friction*
- *To design and analyze friction compensation schemes*

Windup – The Problem



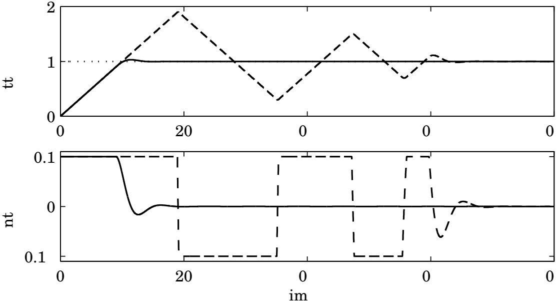
The feedback path is broken when u saturates

The controller $C(s)$ is a dynamic system

Problems when controller is unstable (or stable but not AS)

Example: I-part in PID-controller

Example-Windup in PID Controller

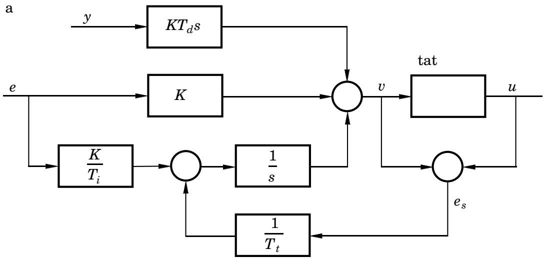


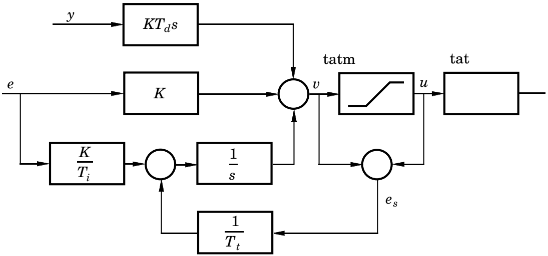
Dashed line: ordinary PID-controller

Solid line: PID-controller with anti-windup

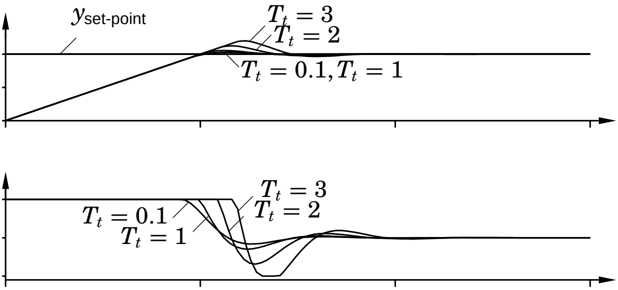
Anti-windup for PID-Controller (“Tracking”)

Anti-windup (a) with actuator output available and (b) without





Choice of Tracking Time T_t

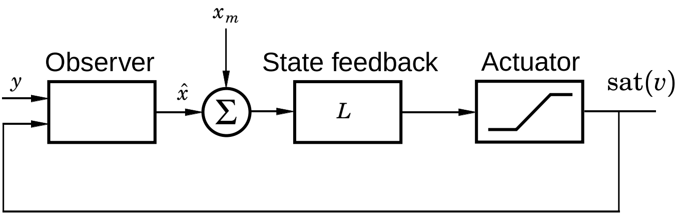


With very small T_t (large gain $1/T_t$), spurious errors can saturate the output, which leads to accidental reset of the integrator. Too large T_t gives too slow reaction (little effect).

The tracking time T_t is the design parameter of the anti-windup.

Common choices: $T_t = T_i$ or $T_t = \sqrt{T_i T_d}$.

State feedback with Observer



$$\dot{\hat{x}} = A\hat{x} + B\,\text{sat}(v) + K(y - C\hat{x})$$
$$v = L(x_m - \hat{x})$$

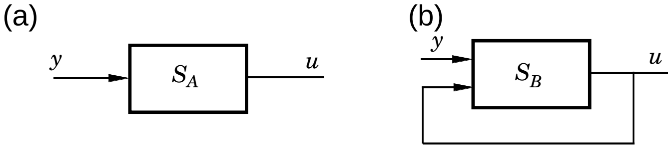
$\hat{x}$ is estimate of process state, x_m desired (model) state.
Need model of saturation if $\text{sat}(v)$ is not measurable

Antiwindup – General State-Space Controller

State-space controller:

$$
\begin{aligned}
\dot{x}_c(t) &= Fx_c(t) + Gy(t) \\
u(t) &= Cx_c(t) + Dy(t)
\end{aligned}
$$

Windup possible if F is unstable and u saturates.



Idea:

Rewrite representation of control law from (a) to (b) such that:

(a) and (b) have same input-output relation

(b) behaves better when feedback loop is broken, if S_B stable

Antiwindup – General State-Space Controller

Mimic the observer-based controller:

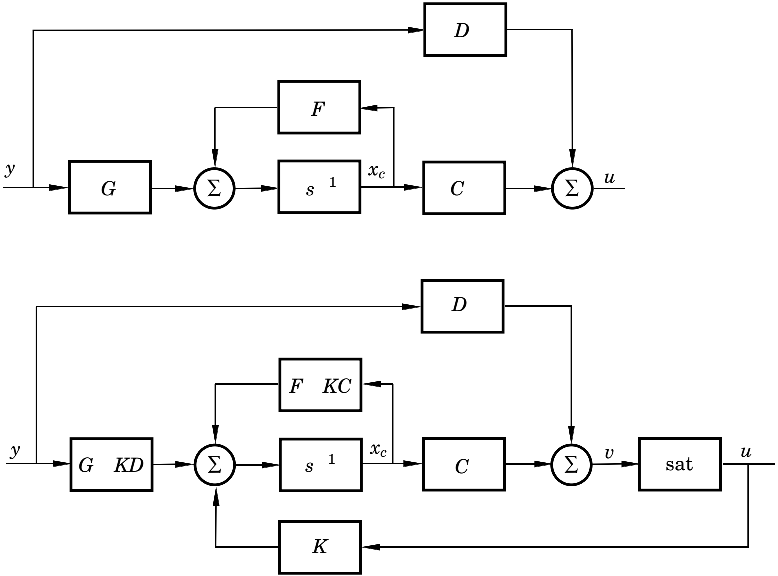
$$\begin{aligned}
\dot{x}_c &= F x_c + G y + K \underbrace{(u - C x_c - D y)}_{=0} \\
&= (F - KC) x_c + (G - KD) y + K u \\
&= F_0 x_c + G_0 y + K u
\end{aligned}$$

Design so that $F_0 = F - KC$ has desired stable eigenvalues

Then use controller

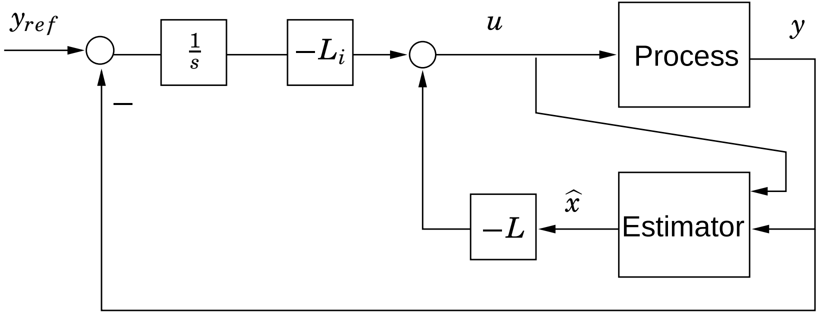
$$\begin{aligned}
\dot{x}_c &= F_0 x_c + G_0 y + K u \\
u &= \text{sat}\,(C x_c + D y)
\end{aligned}$$

State-space controller without and with anti-windup:



5 Minute Exercise

How would you do antiwindup for the following state-feedback controller with observer and integral action ?



Saturation

Optimal control theory (later)

Multi-loop Anti-windup (Cascaded systems):

Difficult problem, several suggested solutions

Turn off integrator in outer loop when inner loop saturates

Friction

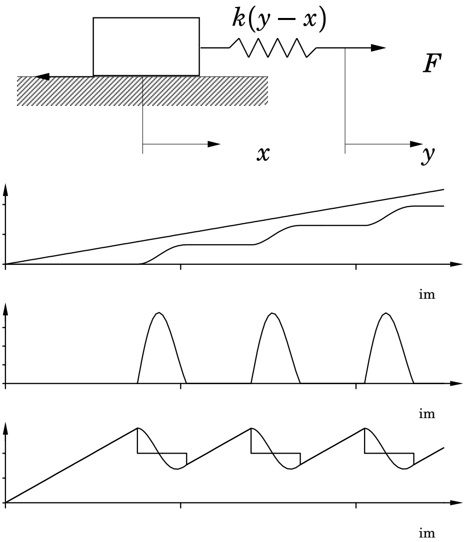
Present almost everywhere

- Often bad
  - Friction in valves and mechanical constructions
- Sometimes good
  - Friction in brakes
- Sometimes too small
  - Earthquakes

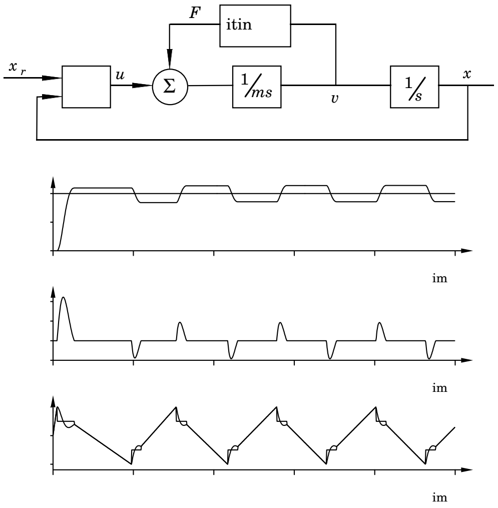
Problems

- How to model friction
- How to compensate for friction

Stick-slip Motion



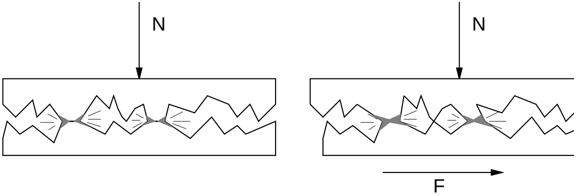
Position Control of Servo with Friction – Hunting

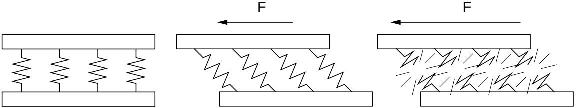


3 Minute Exercise

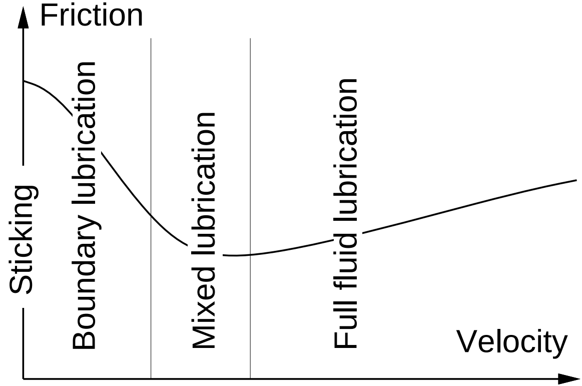
What are the signals in the previous plots? What model of friction has been used in the simulation?

Friction





Lubrication Regimes

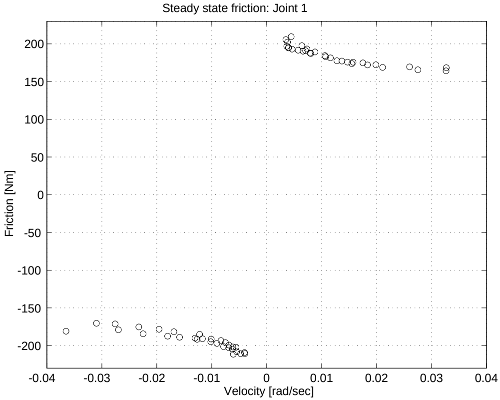


Hard to get good model at $v = 0$

Stribeck Effect

For low velocity: friction increases with decreasing velocity

Stribeck (1902)



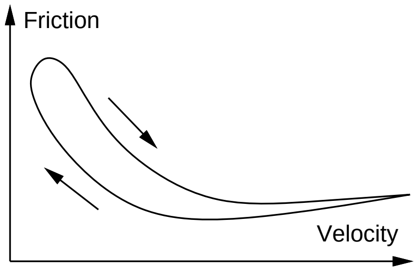
Frictional Lag

Dynamics are important also outside sticking regime

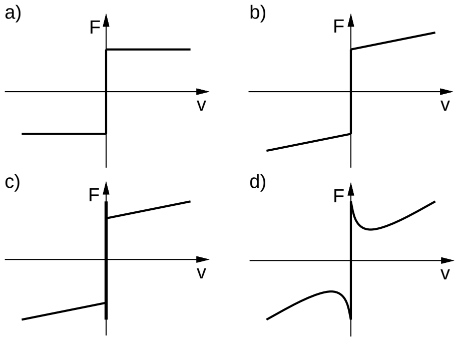
Hess and Soom (1990)

Experiment with unidirectional motion $v(t) = v_0 + a\sin(\omega t)$

Hysteresis effect!



Classical Friction Models



c) $$F(t) = \begin{cases} F_c \text{ sign } v(t) + F_v v(t) & v(t) \neq 0 \\ \max(\min(F_e(t), F_s), -F_s) & v(t) = 0 \end{cases}$$

$F_e(t) =$ external applied force , F_c, F_v, F_s constants

Advanced Friction Models

See PhD-thesis by Henrik Olsson

- Karnopp model
- Armstrong’s seven parameter model
- Dahl model
- Bristle model
- Reset integrator model
- Bliman and Sorine
- Wit-Olsson-Åström

Demands on a model

To be useful for control the model should be

- sufficiently accurate,
- suitable for simulation,
- simple, few parameters to determine.
- physical interpretations, insight

Pick the simplest model that does the job! If no stiction occurs the $v = 0$-models are not needed.

Friction Compensation

- Lubrication
- Integral action (beware!)
- Dither
- Non-model based control
- Model based friction compensation
- Adaptive friction compensation

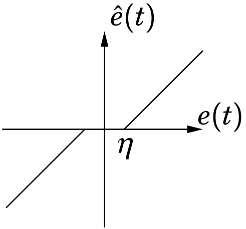
Integral Action

- The integral action compensates for any external disturbance
- Good if friction force changes slowly ($v \approx$ constant).
- To get fast action when friction changes one must use much integral action (small T_i)
- Gives phase lag, may cause stability problems etc

Deadzone - Modified Integral Action

Modify integral part to $I = \frac{K}{T_i} \int_0^t \hat{e}(t) d\tau$

where input to integrator $\hat{e} = \begin{cases} e(t) - \eta & e(t) > \eta \\ 0 & |e(t)| < \eta \\ e(t) + \eta & e(t) < -\eta \end{cases}$

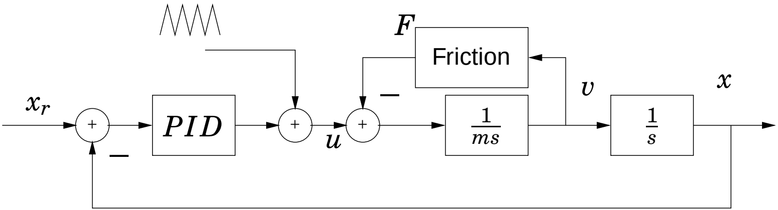


Advantage: Avoid that small static error introduces limit cycle

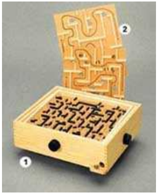
Disadvantage: Must accept small error (will not go to zero)

Mechanical Vibrator–Dither

Avoids sticking at $v = 0$ where there usually is high friction by adding high-frequency mechanical vibration (dither)

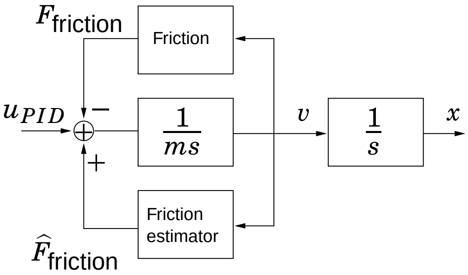


Cf., mechanical maze puzzle (labyrintspel)



Adaptive Friction Compensation

Coulomb Friction $F = a\,\text{sgn}(v)$



Assumption: v measurable.

Friction estimator:

$$\begin{aligned}
\dot{z} &= k u_{PID}\,\text{sgn}(v) \\
\widehat{a} &= z - km|v| \\
\widehat{F}_{\text{friction}} &= \widehat{a}\,\text{sgn}(v)
\end{aligned}$$

Result: $e = a - \widehat{a} \to 0$ as $t \to \infty$,

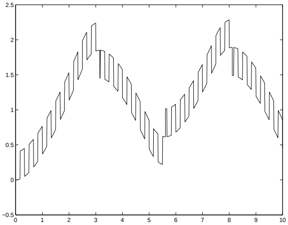
since

$$
\begin{aligned}
\frac{de}{dt} &= -\frac{d\hat{a}}{dt} = -\frac{dz}{dt} + km\frac{d}{dt}|v| \\
&= -ku_{PID}\,\mathrm{sgn}(v) + km\dot{v}\,\mathrm{sgn}(v) \\
&= -k\,\mathrm{sgn}(v)(u_{PID} - m\dot{v}) \\
&= -k\,\mathrm{sgn}(v)(F - \hat{F}) \\
&= -k(a - \hat{a}) \\
&= -ke
\end{aligned}
$$

Remark: Careful with $\frac{d}{dt}|v|$ at $v = 0$.

The Knocker

Combines Coulomb compensation and square wave dither



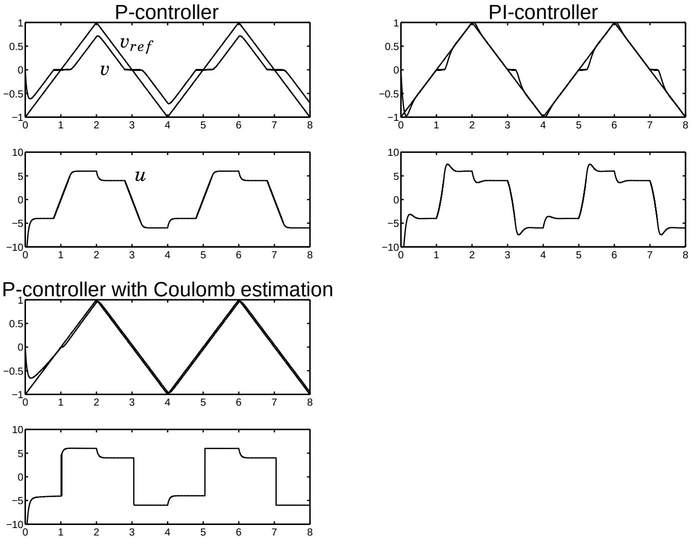
Tore Hägglund, Innovation Cup winner + patent 1997

Example–Friction Compensation

Velocity control with

a) P-controller
b) PI-controller
c) P + Coulomb estimation

Results



P-controller
PI-controller
v_{ref}
v
u
P-controller with Coulomb estimation

Next Lecture

- Backlash
- Quantization